# Approximately Optimal Flow via Truthful Mediators

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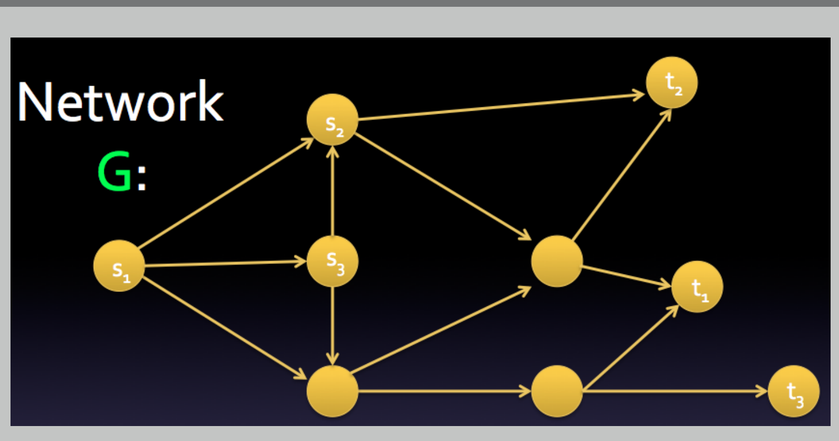
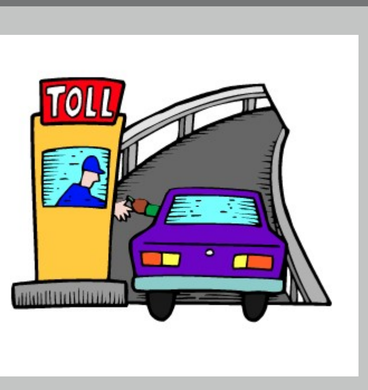
## Socially Optimal Flow in Routing Games




- Losses on each edge $\ell_e(y_e)$.
- Player $i$ routes one unit of flow from $s_i$ to $t_i$.

## Classical Approach - Impose Tolls

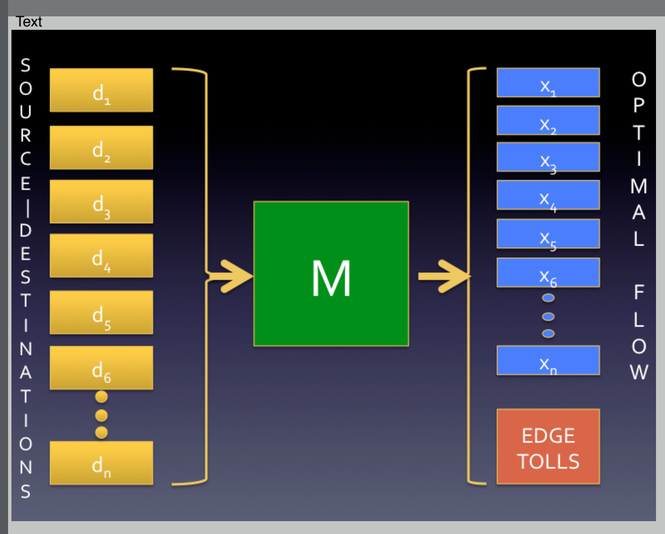



- Want a socially optimal flow
- A Mediator **M** may enforce tolls on each edge so selfish players route optimally.

## Enter Mechanism Design

- The tolls the mediator computes depends on the demands.
- Want players to truthfully report their demands so mediator can compute the correct tolls

## Introduce a Mediator that can Enforce Tolls




### Weakly Mediated Game

- May bypass **M**.
- May misreport to **M**.
- May not follow **M**'s suggested route.

## Main Result

- We develop a mediator such that for $\mathbf{n} \to \infty$
  - Reporting truthfully and following the suggested action of **M** is an (asymptotic) ex - post Nash equilibrium
  - The resulting flow has cost $\mathbf{(1 + o(1))OPT}$

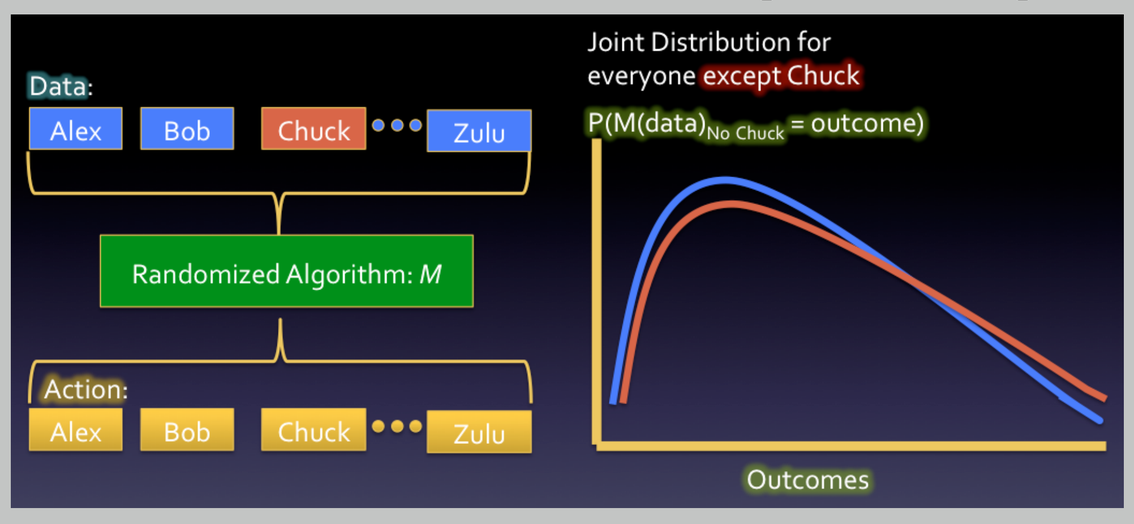
## Main Assumption - Large Games


### Large Game

Any player has a small $\mathbf{o(1)}$ impact on the costs of others as $\mathbf{n} \to \infty$.

## Useful Tool - Joint Differential Privacy

### Joint Differential Privacy [KPRU'14]

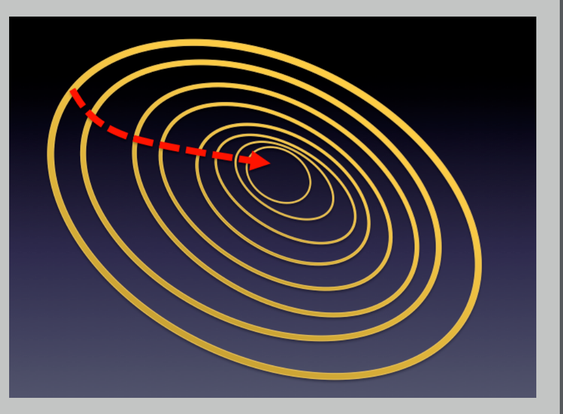



- Controls the impact a single player has on the outcome for other players.
- No real "privacy" concerns here, but still useful!

## Novel Technique - Private Gradient Descent

We need to solve the convex program in a way that is joint differentially private in data **s**.

| | |
|---|---|
| **min** | Total Cost of **x** |
| s.t. | $\mathbf{x} \in \mathcal{F}(\mathbf{s})$ |



## Conclusion

We construct a weak mediator **M** in a Large Routing Game that adds tolls to induce optimal flow, with unknown player demands.

## Open Problem

Can we make weak mediators in general large games?