A Unified Framework for Nearby and Distant Landmarks in Bearing-Only SLAM

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***Abstract*— Bearing-Only SLAM describes the process of simultaneously localizing a mobile robot while building a map of the unknown surroundings, using bearing measurements to landmarks as the only available exteroceptive sensor information. Commonly, the position of map features is estimated along with the robot pose. However, consistent initialization of these positions is a difficult problem in Bearing-Only SLAM, in particular for distant landmarks. In previous approaches, measurements to remote landmarks often had to be discarded, thus losing valuable orientation information. In this paper, we present for the first time a unifying framework allowing for non-delayed initialization of both nearby and distant features. This is made possible by a four-element landmark parametrization, combined with a constraint-based inferred measurement.**

I. Introduction

In recent research on Simultaneous Localization and Mapping (SLAM), attention has been drawn to using sensors that do not provide sufficient information to estimate landmark positions from a single measurement. This is the case for range-only or bearing-only observations. Cameras are typical examples of sensors that provide bearing-only measurements. Due to their lower cost, size, weight and power consumption compared to laser scanners, they are becoming increasingly popular in robotics. Unfortunately, the problem of feature-based SLAM using a bearing-only sensor (so-called Bearing-Only SLAM) is more difficult than regular SLAM, since at least two observations of the same landmark from two sufficiently spaced locations are necessary to initialize its position in the state vector. This initialization is normally accomplished by triangulating the feature from two robot positions. The landmark's position is estimated to lie on the intersection of the two lines defined by the robot poses and bearing measurements. When these two lines become almost parallel, the position estimate is ill-conditioned and its accuracy deteriorates. This situation can occur when (i) the baseline (the distance between the points of landmark observations) is small, (ii) the observed feature is very far away, or (iii) when the robot moves towards a landmark.

Existing approaches usually discard measurements to a landmark that cannot be reliably initialized. However, bearing measurements to distant landmarks can provide extremely valuable information for improving the orientation accuracy of a robot. Therefore, having to discard these measurements should be avoided. We are able to address this challenge by using a different representation of the landmark in the state vector and a constraint-based measurement model. In essence, instead of encoding landmarks by their Euclidean coordinates, we employ two perspective projections in one real and one virtual camera. To the best of our knowledge, this is the first framework that allows for non-delayed landmark initialization of both *nearby* and *distant* landmarks.

II. Related Work

The problem of landmark initialization in Bearing-Only SLAM has been addressed by several researchers. Generally, one can distinguish *delayed* vs. *non-delayed* approaches. In delayed initialization schemes, robot observations are accumulated until a criterion is fulfilled that allows for well-conditioned initialization of the landmark position in the state vector. Deans and Hebert [1] use a nonlinear bundle-adjustment procedure to initialize the landmarks from several past observations. Bailey [2] proposes constrained initialization within the EKF framework. In this approach, measurements to not-yet-initialized features are stored in the state vector, together with the corresponding robot positions. At a later stage, when the probability density of the landmark position has become sufficiently Gaussian, it is initialized in a batch EKF-update. Davison [3] uses a separate particle filter to estimate the distance of a landmark independently from the map, and initializes a candidate landmark only when the range uncertainty has become sufficiently small.

Non-delayed approaches initialize a landmark into the state-vector directly after the first observation. They make use of the fact that each landmark, after the first observation, is constrained to lie in a cone-shaped region centered around the line that emanates from the current robot position towards the direction of the landmark. The range to the feature along this line is at this point completely uncertain and can be modeled as uniformly distributed in the interval between the minimum and maximum sensor range. In their multiple hypotheses filter, Kwok and Dissanayake [4] initialize several instances of one landmark, each with a different range hypothesis. Invalid hypotheses are later eliminated from the state vector by means of sequential probability testing. Solà *et al.* [5] use an approximate sum of Gaussians to represent the uniform range distribution. The number of Gaussians is a function of the sensor range.

One limitation of all aforementioned approaches is their inability to deal with landmarks that are effectively at "infinite distance", or, equivalently, to allow for a sensor with sensing range significantly larger than the range of motion of the robot. For a landmark at "infinite distance", sufficient baseline for a well-conditioned initialization cannot be established, and a sensible partitioning of the range span by a tractable number of hypotheses (Gaussian kernels or particles) is difficult. In his particle-filter approach, Davison [3] acknowledges without further elaboration that, compared to the 100 particles in the range between 0.5 m and 5.0 m which he proposes for indoor scenarios, “quite a different type of prior” may be required for outdoor environments. Solá *et al.* [5] suggest a heuristic that yields seven Gaussians for covering the range between 1 m and 1000 m, but do not consider landmarks further away.

It is well known that very distant landmarks provide invaluable orientation information. Intuition suggests that a prominent feature on the horizon, e.g., a mountain peak or a tall building, can serve to “get one’s bearings”. If the positions of these landmarks are known in a global frame of reference, then the bearing measurements are equivalent to unit vector observations with which one’s pose can be estimated [6]. In the case of sufficiently distant landmarks, so that the position displacement during operation yields only negligible change in the unit vector towards that feature, these landmarks can be used for orientation estimation without explicit knowledge of their position. It suffices to determine the corresponding unit vector in a desired fixed reference frame. An example of the latter case is the magnetic compass. If, for example, it is decided that the unit vector provided by the compass should coincide with the global x-axis, the compass information can henceforth be used directly as an absolute orientation measurement. Sun sensors and star trackers apply this principle in the field of 3D spacecraft attitude determination [7].

Apart from observations of very distant landmarks, the use of features that lie along the direction of travel of the robot is another challenging issue [5]. A delayed initialization scheme as proposed by Bailey [2] would not be able to initialize such a landmark within reasonable time due to insufficient baseline. However, since this scenario could arise frequently for a forward-sensing robot moving in a straight line, an efficient algorithm should be able to use such landmarks for pose updates as well.

In this paper, we present for the first time a unified formulation capable of incorporating information from nearby and distant landmarks as well as those lying in the direction of travel of the robot.

III. Bearing Measurements to nearby vs. distant landmarks

In this section, we outline the differences in the treatment of nearby and distant features in 2D Bearing-Only SLAM (BOSLAM). We first present the commonly used method for close-by features, where a landmark’s (x, y)-position is estimated in the state vector, followed by procedures for handling measurements to landmarks at infinite distance. This will provide the necessary background for the development of our novel algorithm capable of dealing with both cases simultaneously. In particular, we will address the issue of feature initialization.

Another very challenging issue in BOSLAM is data association, since multiple features can lie on a single ray. For non-initialized landmarks, any two bearing measurements may intersect and produce tentative feature candidates, some of which are phantom landmarks that have no physical meaning. The issue of data association is addressed for example by Costa *et al.* [8]. However, for the purpose of this paper, we will assume data association to be solved and focus on the description of a new formulation for BOSLAM that allows us to represent and process measurements to both nearby and distant features.

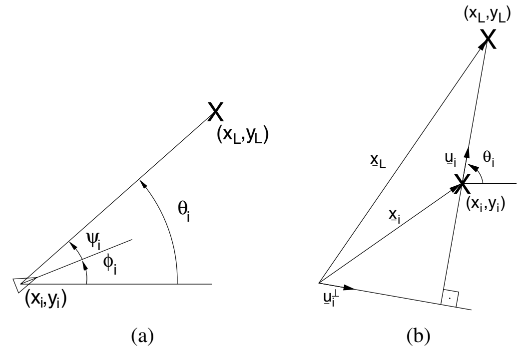

Fig. 1. (a): Illustration of robot orientation ϕ_i, relative bearing angle ψ_i and global bearing angle θ_i towards a landmark. (b): The projections of the robot location and the landmark location onto the unit vector perpendicular to the measurement are equal (cf. Eq. (10)).

A. Nearby Landmarks

In SLAM, the state vector to estimate consists of the robot pose $\mathbf{x}_r$ and the map $\mathbf{x}_M$

$$\mathbf{x} = \begin{bmatrix}\mathbf{x}_r^{\mathrm{T}} & \mathbf{x}_M^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \tag{1}$$

where the robot pose is defined as $\mathbf{x}_r(t_i) = \begin{bmatrix}x_i & y_i & \phi_i\end{bmatrix}^{\mathrm{T}}$ and the map as the set of k landmarks $\mathbf{x}_M = \begin{bmatrix}x_{L_1} & y_{L_1} & \dots & x_{L_k} & y_{L_k}\end{bmatrix}^{\mathrm{T}}$. In the remainder of the paper, we will develop the equations for only one landmark in order to facilitate notation.

As shown in Fig. 1(a), the bearing-only sensor measures the relative bearing angle ψ between the landmark and the robot at time i, corrupted by noise. In the case of an already initialized landmark, the measurement model is

$$z_\psi(t_i) = \psi_i + n = \operatorname{atan2}(y_L - y_i,\ x_L - x_i) - \phi_i + n \tag{2}$$

where n represents white, zero-mean Gaussian noise.

In addition to the relative bearing angle, we can also define the global bearing angle θ_i

$$\theta_i = \psi_i + \phi_i \tag{3}$$

From two positions $\mathbf{x}_i$ and $\mathbf{x}_j$ and corresponding global bearing angles θ_i and θ_j, we can compute the landmark position according to [2]

$$\begin{bmatrix}x_L \\ y_L\end{bmatrix} = \frac{1}{s_i c_j - s_j c_i} \cdot \begin{bmatrix}x_i s_i c_j - x_j s_j c_i + (y_j - y_i) c_i c_j \\ y_j s_i c_j - y_i s_j c_i + (x_i - x_j) s_i c_j\end{bmatrix} \tag{4}$$

where we have used the abbreviated notation $c_i = \cos(\theta_i)$ and $s_i = \sin(\theta_i)$.

It is important to note that this equation becomes singular when $s_i c_j - s_j c_i = 0$, that is, when $\theta_i = \theta_j$. This can occur in cases of zero baseline, movement in landmark direction or when a landmark is at infinite distance. Evidently, there are a number of cases when initialization fails, even though a landmark was sighted repeatedly. Without successful initialization, or appropriate representation of these observations, the information contained therein is inevitably lost.

B. Known Landmarks at Infinite Distance (Compass)

Let us now consider a *known* landmark at infinite distance. Its global bearing angle will remain constant, independent of the robot position. In the case of a magnetic compass, for example, this bearing angle with respect to the global map is known a priori. Therefore, the compass measurement can be used directly as an absolute measurement of the robot orientation

$$z_c(t_i) = c - \phi_i + n \tag{5}$$

where c is a deterministic constant depending on the magnetic declination.

C. Unknown Landmarks at Inf. Distance (Acquired Compass)

A slightly different scenario develops, when the direction to a previously *unknown* landmark at infinite distance is measured for the first time only after a certain period of robot operation. In this case, the global angle θ towards that landmark is a random variable with associated uncertainty due to accumulated odometry errors. Correlations between this and future measurements to the same feature can be used in order to improve the localization accuracy of a robot. Such an estimation process requires this global direction to be initialized in the state vector. In a sense, initializing this landmark corresponds to acquiring a compass. For the acquired compass, the measurement model is

$$z_{ac}(t_i) = \theta - \phi_i + n \tag{6}$$

The initialization process works as follows: We first add a new variable θ to the state vector that is completely uncorrelated with any other variable and has infinite variance $P_{\theta\theta}$. It can be shown that after one Kalman filter update, this variable will be initialized[1] to

$$\hat{\theta} = z_{ac} + \hat{\phi}_i \tag{7}$$

and the updated covariance matrix will be

$$\mathbf{P}^{\oplus} = \begin{bmatrix} P^{\ominus}_{xx} & P^{\ominus}_{xy} & P^{\ominus}_{x\phi} & P^{\oplus}_{x\theta} \\ P^{\ominus}_{yx} & P^{\ominus}_{yy} & P^{\ominus}_{y\phi} & P^{\oplus}_{y\theta} \\ P^{\ominus}_{\phi x} & P^{\ominus}_{\phi y} & P^{\ominus}_{\phi\phi} & P^{\oplus}_{\phi\theta} \\ P^{\oplus}_{\theta x} & P^{\oplus}_{\theta y} & P^{\oplus}_{\theta\phi} & P^{\oplus}_{\theta\theta} \end{bmatrix} \tag{8}$$

where $P^{\oplus}_{x\theta} = P^{\ominus}_{x\phi}, P^{\oplus}_{y\theta} = P^{\ominus}_{y\phi}, P^{\oplus}_{\phi\theta} = P^{\ominus}_{\phi\phi}$ and $P^{\oplus}_{\theta\theta} = P^{\ominus}_{\phi\phi} + R$. R denotes the measurement noise variance. Obviously, in order to minimize uncertainty it is desirable to incorporate distant landmarks into the state vector as early as possible.

From the previous sections, we can conclude that two parameters are the minimal representation for nearby landmarks, whereas only one parameter suffices to represent very distant features. At the beginning, when only a limited number of measurements are available, it is usually impossible to make the decision whether a landmark is close-by or far away. In the next section, we develop a new algorithm that can treat both cases simultaneously, *without* having to commit to either representation.

[1]From now on, estimated quantities are denoted by "$\hat{\ }$", and errors by "$\tilde{\ }$".

IV. Problem Formulation

We now present the main contribution of our work, namely a unified framework to handle *both* nearby and distant landmarks. The key underlying idea is a *non-minimal* representation of the landmark observations within the state vector, along with a *constraint-based* inferred measurement. Instead of using the relative bearing measurement z_ψ directly (cf. Eq. (2)), we incorporate it into a constraint between three robot positions and the corresponding global bearing measurements towards a landmark.

We first describe the landmark position in terms of each robot position $\mathbf{x}_i$ and global bearing angle θ_i

$$\mathbf{x}_L = \mathbf{x}_i + \rho_i \mathbf{u}_i\,, \qquad i = 1,2,3 \tag{9}$$

where ρ_i stands for the unknown distance to the landmark, and $\mathbf{u}_i$ denotes the unit vector towards the landmark, $\mathbf{u}_i = \begin{bmatrix} c_i & s_i \end{bmatrix}^{\mathrm{T}}$.

In order to eliminate the distance ρ_i from these equations, we multiply both sides of the above equations with the perpendicular unit vector $\mathbf{u}_i^{\perp} = \begin{bmatrix} -s_i & c_i \end{bmatrix}^{\mathrm{T}}$ (cf. Fig. 1(b))

$$\mathbf{x}_L^{\mathrm{T}} \mathbf{u}_i^{\perp} = \mathbf{x}_i^{\mathrm{T}} \mathbf{u}_i^{\perp}\,, \qquad i = 1,2,3 \tag{10}$$

By adding the three above equations (Eq. (10)) after having multiplied with $\sin(\theta_3 - \theta_2)$ for $i = 1$, $\sin(\theta_1 - \theta_3)$ for $i = 2$, and $\sin(\theta_2 - \theta_1)$ for $i = 3$, we obtain the constraint

$$\begin{aligned} 0 = \;& h(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \theta_1, \theta_2, \theta_3) \\ = \;& (x_3 - x_1)s_1c_2s_3 + (x_2 - x_3)c_1s_2s_3 + (x_1 - x_2)s_1s_2c_3 \\ & + (y_3 - y_1)c_1s_2c_3 + (y_2 - y_3)s_1c_2c_3 + (y_1 - y_2)c_1c_2s_3 \end{aligned} \tag{11}$$

In order to incorporate this constraint into a filter algorithm, we could at first store two triplets (x_i, y_i, θ_i) corresponding to landmark observations at different robot positions in the state vector. Subsequent bearing measurements would then be used to update the state vector via the constraint of Eq. (11), with the current robot pose serving as the third point in the equation. Such a procedure would require six elements per landmark in the state vector. As we have seen in the previous section, a minimal representation consists of two or even only one variable per feature. The 6-element representation will lead to four zero eigenvalues in the steady state covariance matrix for a nearby landmark, or five in the case of a distant feature. Additionally, it raises the question how to select the two position-bearing sets that best represent a landmark. What is more, for larger environments, this six-elements per landmark representation will further exacerbate the computational complexity of SLAM.

We therefore introduce a different representation. When the landmark is observed for the second time, we will not simply record the position-bearing triplet in the state vector, but instead *project* it onto an artificial position, a new vantage point (x_2, y_2) completely correlated with (x_1, y_1) (cf. Fig. 2).

$$\begin{aligned} x_2 &= x_1 + \rho\cos\varphi \\ y_2 &= y_1 + \rho\sin\varphi \end{aligned} \tag{12}$$

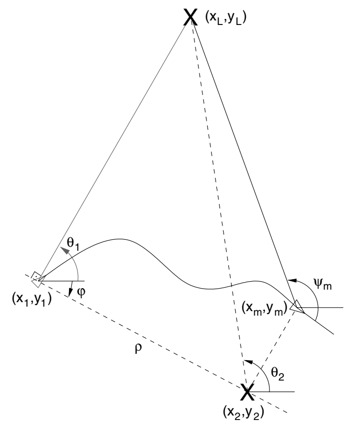

Fig. 2. Instead of storing the robot pose (x_m, y_m) at the second landmark observation, we project it onto an artificial position (x_2, y_2) that is fully correlated with (x_1, y_1).

In order to locally maximize the baseline, we choose the point (x_2, y_2) to lie on a line orthogonal to the direction towards the landmark, i.e.,

$$\varphi = \theta_1 - \pi/2 \tag{13}$$

The distance is determined by a scaled projection of the current robot position (x_m, y_m) onto that line

$$\rho = \eta \cdot (\mathbf{x}_m - \mathbf{x}_1)^{\mathrm{T}} \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix} \tag{14}$$

where η is a scaling factor and $\mathbf{x}_m$ denotes the robot position at the time of the second landmark observation. For the stability of the algorithm it is important that $\rho \neq 0$. If moving towards the landmark (that is, if the angle between $\begin{bmatrix}\cos(\theta_1) & \sin(\theta_1)\end{bmatrix}^{\mathrm{T}}$ and $(\mathbf{x}_m - \mathbf{x}_1)$ is below a certain threshold) we modify the projection strategy such that the projection will not change the location of the vantage point.

Effectively, the projection of the robot position onto a new vantage point completely correlated with (x_1, y_1) corresponds to a conversion of position uncertainty to orientation uncertainty. Imagine selecting ρ, ϕ such that x_2 and y_2 coincide with (x_m, y_m), but (x_1, y_1) (and thus (x_2, y_2)) have a considerably lower position uncertainty than (x_m, y_m). In this case, θ_2 would be initialized to $\theta_2 = \theta_m$ (i.e., the estimate would not change), but its uncertainty can be shown to grow [9].

We can now express x_2 and y_2 in terms of the state variables x_1 and y_1, as well as the deterministic variables ρ and ϕ, which do *not* need to be kept in the state vector. We thus require only four state vector elements per landmark, namely x_1, y_1, θ_1 and θ_2. This representation, albeit non-minimal, allows us to use landmark observations to update the robot pose without having to decide whether the landmark is nearby or very distant. In comparison, delayed initialization schemes require storing *three elements per observation* in the state vector, until the landmark can be initialized. Depending on the landmark's distance and direction, this may take considerable time, and a large number of observations and robot poses need to be included in the state vector. In ill-conditioned cases where initialization fails completely, all this stored information has to be discarded. Our approach, in contrast, requires a low, fixed number of variables per landmark and allows *immediate* use of bearing measurements to update the estimated state.

V. The Filter Algorithm

Having derived the new parametrization and constraint for bearing-only measurements to a landmark, we will now demonstrate how to incorporate them into a filter algorithm.

A. Initialization

Before a landmark is initialized in the state vector, we can assume that its four state variables x_1, y_1, θ_1 and θ_2 have infinite uncertainty and are uncorrelated with any other variable. In this section, we outline the procedure to initialize their estimates and their covariance matrix.

1) Initialization of x_1, y_1 and θ_1: Let the current robot pose be denoted by $\mathbf{x}_\ell = \begin{bmatrix} x_\ell & y_\ell & \phi_\ell \end{bmatrix}^{\mathrm{T}}$ and the current state vector by $\mathbf{x} = \begin{bmatrix} \mathbf{x}_\ell^{\mathrm{T}} & x_1 & y_1 & \theta_1 & \theta_2 \end{bmatrix}^{\mathrm{T}}$. Furthermore, let an estimate of a variable x prior to (after) performing an update be denoted by $\hat{x}^\ominus$ $(\hat{x}^\oplus)$. Upon first observation of a landmark, the current robot pose estimate $\hat{\mathbf{x}}_\ell$ and the relative bearing measurement z_{ψ_ℓ} are used to initialize the first three parameters x_1, y_1 and θ_1 according to

$$\hat{x}_1^\oplus = \hat{x}_\ell^\ominus \qquad \hat{y}_1^\oplus = \hat{y}_\ell^\ominus \qquad \hat{\theta}_1^\oplus = \hat{\phi}_\ell^\ominus + z_{\psi_\ell} \tag{15}$$

The position coordinates $(\hat{x}_1, \hat{y}_1)$ are completely correlated with the current robot position estimate $(\hat{x}_\ell, \hat{y}_\ell)$, and the error in $\hat{\theta}_1$ is correlated with that of the robot orientation. The measurement error, assumed uncorrelated with the state, only enters in the variance of θ_1, that is, in the term $\mathbf{P}(6,6)$ of the covariance matrix in the following equation. Thus, the updated covariance matrix can be shown to be [9]

$$\mathbf{P}^\oplus = \lim_{\mu\to\infty} \begin{bmatrix} P_{xx}^\ominus & P_{xy}^\ominus & P_{x\phi}^\ominus & P_{xx}^\ominus & P_{xy}^\ominus & P_{x\phi}^\ominus & 0 \\ P_{yx}^\ominus & P_{yy}^\ominus & P_{y\phi}^\ominus & P_{yx}^\ominus & P_{yy}^\ominus & P_{y\phi}^\ominus & 0 \\ P_{\phi x}^\ominus & P_{\phi y}^\ominus & P_{\phi\phi}^\ominus & P_{\phi x}^\ominus & P_{\phi y}^\ominus & P_{\phi\phi}^\ominus & 0 \\ P_{xx}^\ominus & P_{xy}^\ominus & P_{x\phi}^\ominus & P_{xx}^\ominus & P_{xy}^\ominus & P_{x\phi}^\ominus & 0 \\ P_{yx}^\ominus & P_{yy}^\ominus & P_{y\phi}^\ominus & P_{yx}^\ominus & P_{yy}^\ominus & P_{y\phi}^\ominus & 0 \\ P_{\phi x}^\ominus & P_{\phi y}^\ominus & P_{\phi\phi}^\ominus & P_{\phi x}^\ominus & P_{\phi y}^\ominus & P_{\phi\phi}^\ominus + R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

Note that θ_2 has not yet been initialized and therefore still has infinite variance μ.

2) Initialization of θ_2: Assume the robot has moved to a new pose $\mathbf{x}_m$ and a new relative bearing measurement to the landmark z_{ψ_m} is available (cf. Fig. 2). We can now initialize θ_2. First, we project the current robot position onto the new vantage point (x_2, y_2), using Eqs. (13) and (14) to compute ρ and φ.

In order to compute an estimate for θ_2 and its covariance, we will use a modified form of the constraint Eq. (11), substituting (x_2, y_2) with the expression from Eq. (12).

$$\begin{aligned} 0 = \;& h(\mathbf{x}_m, \theta_m, \mathbf{x}_1, \theta_1, \theta_2) \qquad (16) \\ = \;& (x_1 - x_m)s_m \sin(\theta_1 - \theta_2) + \rho\cos(\varphi)s_2\sin(\theta_1 - \theta_m) \\ & -(y_1 - y_m)c_m \sin(\theta_1 - \theta_2) \; - \rho\sin(\varphi)c_2\sin(\theta_1 - \theta_m) \end{aligned}$$

Here, $\theta_m = \phi_m + \psi_m$ denotes the current global bearing angle to the landmark. According to this constraint equation, the inferred measurement is

$$\zeta = h(\mathbf{x}_m, \psi_m) = 0 \tag{17}$$

and therefore deterministically known. The expected measurement, however, is based on the current state estimate and the measured bearing angle z_{ψ_m}.

$$\hat{\zeta} = h(\hat{\mathbf{x}}_m, z_{\psi_m}) \tag{18}$$

In order to compute the covariance of the residual, we need to linearize the constraint equation

$$\tilde{\zeta} = \zeta - \hat{\zeta} \simeq \nabla_{\mathbf{x}}^{\mathrm{T}} h \cdot \tilde{\mathbf{x}} - \nabla_{\psi}^{\mathrm{T}} h \cdot n = \mathbf{h}^{\mathrm{T}} \tilde{\mathbf{x}} - \gamma n \tag{19}$$

The covariance S of the residual is

$$S = \mathbf{h}^{\mathrm{T}} \mathbf{P}^{\ominus} \mathbf{h} + \gamma^2 R \tag{20}$$

while the remaining EKF update equations are [10]

$$\hat{\mathbf{x}}^{\oplus} = \hat{\mathbf{x}}^{\ominus} - \mathbf{k}\hat{\zeta}\,, \qquad \mathbf{P}^{\oplus} = \mathbf{P}^{\ominus} - \mathbf{k} S \mathbf{k}^{\mathrm{T}} \tag{21}$$

$$\text{with Kalman Gain} \quad \mathbf{k} = \mathbf{P}^{\ominus} \mathbf{h} S^{-1} \tag{22}$$

After one update to initialize θ_2, only the elements in the last row and column of the covariance matrix change. Using Matlab notation, we can compute the first six elements of the last column of $\mathbf{P}$ as

$$\mathbf{P}^{\oplus}(1:6,7) = -\frac{1}{h_{\theta_2}} \mathbf{P}^{\ominus}(1:6,1:6)\mathbf{h}(1:6,1) \tag{23}$$

and $\mathbf{P}^{\oplus}(7,1:6) = \mathbf{P}^{\oplus\mathrm{T}}(1:6,7)$. The variance of θ_2 is

$$\mathbf{P}^{\oplus}(7,7) = \frac{1}{h_{\theta_2}^2}\Big(\mathbf{h}^{\mathrm{T}}(1,1:6)\mathbf{P}^{\ominus}(1:6,1:6)\mathbf{h}(1:6,1) + \gamma^2 R\Big) \tag{24}$$

In these expressions

$$h_{\theta_2} = \frac{\partial h}{\partial \theta_2}\,, \qquad \gamma = \frac{\partial h}{\partial n} = \frac{\partial h}{\partial \theta_m} = \frac{\partial h}{\partial \psi_m} = \frac{\partial h}{\partial \phi_m} \tag{25}$$

Due to the highly non-linear constraint, one EKF update will not yield a sufficiently accurate estimate for θ_2. Instead, we solve the constraint (Eq. (16)) for θ_2 and obtain

$$\begin{aligned}\theta_2 = \operatorname{atan2}\big(&\Delta x_{1m} s_1 s_m - \Delta y_{1m} s_1 c_m - \rho \sin(\varphi)\sin(\theta_1 - \theta_m), \\ &\Delta x_{1m} c_1 s_m - \Delta y_{1m} c_1 c_m - \rho \cos(\varphi)\sin(\theta_1 - \theta_m)\big)\end{aligned} \tag{26}$$

where $\Delta x_{1m} = x_1 - x_m$, and $\Delta y_{1m} = y_1 - y_m$. This formula is correct up to $\pm\pi$, and we can determine the appropriate value by constraining θ_2 to lie in the same half-plane as θ_1. In the singular case where $\theta_1 = \theta_m$, i.e., both robot positions and the landmark location are aligned, or the landmark is at infinite distance, the above formula initializes $\theta_2 = \theta_1$ as desired. It can be shown that the linearized variance $P_{\theta_2\theta_2}$ of this estimate is equal to the one computed by the Kalman filter update [9].

All four state vector elements for this landmark are now initialized, and we can use subsequent bearing measurements via the constraint Eq. (16) to update the state estimate. This process has been outlined in Eqs. (17)-(22). Instead of having to store observations until a landmark's position estimate becomes sufficiently well-conditioned, we are able to make immediate use of all available sensor information, regardless of the distance to the observed landmarks.

B. Increasing the baseline

After a sufficient number of regular updates, the uncertainty of θ_2 will become increasingly small. Due to the small baseline accumulated at initialization (i.e., small value of ρ), the filter is prone to numerical instability and divergence. It seems therefore advisable to increase the baseline (distance between the two stored vantage points) artificially. The criterion used for this purpose is the value of $P_{\theta_2\theta_2}$. When it drops below a threshold, we increase the baseline, thereby in effect causing the uncertainty of the new θ_2 to increase as well. In this section, we outline the corresponding procedure, which is similar to the initialization of θ_2. An increase of ρ to ρ_n defines a new position along the same line perpendicular to the direction towards the landmark

$$x_n = x_1 + \rho_n \cos(\varphi) \tag{27}$$

$$y_n = y_1 + \rho_n \sin(\varphi) \tag{28}$$

with a corresponding new global bearing angle θ_n. The latter is initialized based on $x_1, y_1, \theta_1, x_2, y_2$ and θ_2 as

$$\begin{aligned}\theta_n = \operatorname{atan2}\big(&\rho s_1 \sin(\varphi - \theta_2) - \rho_n \sin(\varphi)\sin(\theta_1 - \theta_2), \\ &\rho c_1 \sin(\varphi - \theta_2) - \rho_n \cos(\varphi)\sin(\theta_1 - \theta_2)\big)\end{aligned} \tag{29}$$

As this is again ambiguous up to $\pm\pi$, we compare the value to θ_2 to decide on the correct half plane.

Making the appropriate substitutions, the constraint is formulated as

$$\begin{aligned}0 = \;& h(\mathbf{x}_1, \theta_1, \mathbf{x}_2, \theta_2, \theta_n) \\ =& -\rho_n \cos(\varphi) s_1 c_2 s_n - \Delta\rho \cos(\varphi) c_1 s_2 s_n + \rho \cos(\varphi) s_1 s_2 c_n \\ &- \rho_n \sin(\varphi) c_1 s_2 c_n - \Delta\rho \sin(\varphi) s_1 c_2 c_n + \rho \sin(\varphi) c_1 c_2 s_n\end{aligned} \tag{30}$$

where $\Delta\rho = \rho - \rho_n$. As in the previous derivations, we assume that θ_n has an uninformative prior, and is uncorrelated with any other variable before initialization. Initialization corresponds to performing one Kalman filter update step.

Note that the only non-zero partial derivatives are $h_{\theta_1}, h_{\theta_2}$ and h_{θ_n}. The updated covariance matrix only changes in the row/column pertaining to θ_n, according to

$$\mathbf{P}^{\oplus}(1:7,8) = \mathbf{P}^{\oplus\mathrm{T}}(8,1:7) = -\frac{1}{h_{\theta_n}} \mathbf{P}^{\ominus}(1:7,6:7) \begin{bmatrix} h_{\theta_1} \\ h_{\theta_2} \end{bmatrix}$$

$$\mathbf{P}^{\oplus}(8,8) = \frac{1}{h_{\theta_n}^2} \begin{bmatrix} h_{\theta_1} & h_{\theta_2} \end{bmatrix} \mathbf{P}^{\ominus}(6:7,6:7) \begin{bmatrix} h_{\theta_1} \\ h_{\theta_2} \end{bmatrix} \tag{31}$$

After this step, we have successfully increased the baseline. We can now replace θ_2 by θ_n, ρ by ρ_n and discard the row and column corresponding to θ_2 in the covariance matrix.

VI. Simulation Results

The new algorithm was tested extensively in simulations. Contrary to existing methods, it proved able to successfully incorporate observations even of faraway landmarks. Interestingly, simulation results showed that our formulation performs particularly well with distant landmarks, whereas very close-by landmarks can sometimes lead to inconsistent estimates for the corresponding θ_2 angles (however, with surprisingly little adverse effect on the robot's pose estimate). Computation of landmark position estimates and their covariances provided an intriguing argument in favor of our new formulation. In

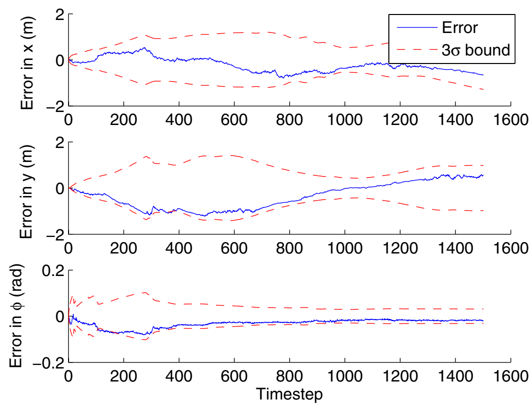

Fig. 3. Bearing-Only SLAM when estimating the positions of two nearby landmarks in the state vector (S-BOSLAM). Error and 3σ-bounds for the robot pose.

fact, the results showed that even though all four parameters x_1, y_1, θ_1 and θ_2 were estimated consistently, the landmark position estimate frequently was not. This can be partly explained by the error incurred by employing a linearized approximation to compute the covariance of the position estimate. The inconsistency hints at the possible problems if one were to attempt to represent distant landmarks by a regular position parametrization instead of our new approach.

The two main concerns of our algorithm are the high non-linearity of the measurement model, coupled with numerical instability due to the non-minimal landmark parametrization. Although the constraint is mathematically correct, the aforementioned problems are difficult to handle with an Extended Kalman Filter. We therefore employed the Unscented Kalman Filter (UKF) introduced by Julier *et al.* [11], that is considerably better suited to deal with nonlinearities than the regular EKF. Experiments showed that for this particular problem it also outperformed the Square Root Filter, a formulation of the Kalman filter which improves numerical accuracy [10]. When used in scenarios with only one landmark, our new algorithm exhibits signs of instability. It performs robustly, however, if several landmarks with sufficient angular spacing are included in the map.

In order to demonstrate the algorithm's performance, we present results for a sample, random trajectory in an environment containing only two landmarks. We compare the estimates produced by an EKF using the standard landmark position parametrization (from now on referred to as S-BOSLAM[2]) with those generated through a UKF with our proposed four parameter representation (henceforth denoted as NDL-BOSLAM[3]). While the first algorithm is limited to nearby features ($\sim$20 m), our algorithm was able to utilize observations of landmarks at much greater distances ($\sim$3500 m). Both algorithms produce consistent estimates and increase the localization accuracy by an order of magnitude compared to dead-reckoning only.

Closer inspection of the pose errors and error bounds (cf.

[2]S-BOSLAM for "Standard Bearing-Only SLAM"
[3]NDL-BOSLAM for "Near-and-Distant-Landmarks Bearing-Only SLAM"

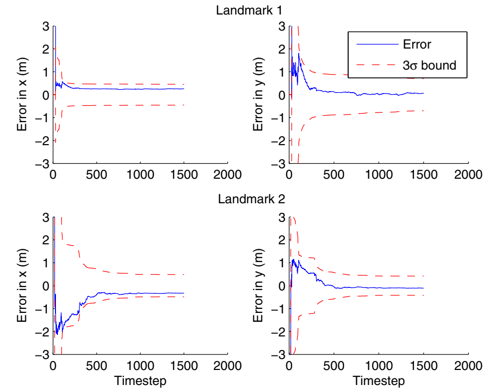

Fig. 4. Bearing-Only SLAM when estimating the positions of two nearby landmarks in the state vector (S-BOSLAM). Error and 3σ-bounds for the landmark positions.

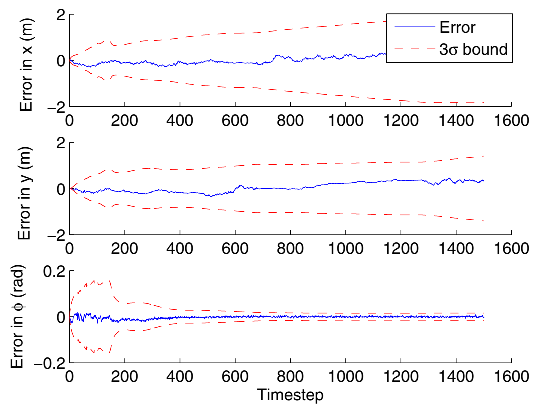

Fig. 5. Bearing-Only SLAM using our proposed framework, allowing usage of two more distant landmarks instead (NDL-BOSLAM). Error and 3σ-bounds for the robot pose.

Figs. 3 and 5) reveals, however, that NDL-BOSLAM has a significantly better steady state orientation accuracy. Its 3σ-bound is approximately half of that produced by S-BOSLAM, leveling out at ± 0.016rad. As expected, the use of very distant landmarks improved mainly the orientation estimate. In terms of position accuracy, on the other hand, our filter's performance is only slightly inferior to the one obtained using the position representation. The robot's position estimate benefits in particular from measurements to landmarks at close distance whose relative bearings change quickly [6].

Fig. 4 shows the estimate of the landmark positions generated by the S-BOSLAM algorithm. The position error reaches steady state around timestep 300. In comparison, Figs. 6 and 7 show the errors associated with the global bearing angles θ_1 and θ_2 used in the new NDL-BOSLAM. While θ_1 was estimated at the very beginning of the robot trajectory, and therefore remained accurate and almost constant, θ_2 exhibits periodic jumps in the error bounds, caused by baseline extensions. As discussed in Section V, these serve to avoid filter divergence and are a mechanism to store indirect information about a landmark's position.

The effects of these baseline extensions are visible in the

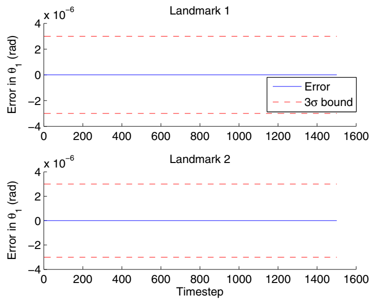

Fig. 6. Results of NDL-BOSLAM. Error and 3σ-bounds for θ_1.

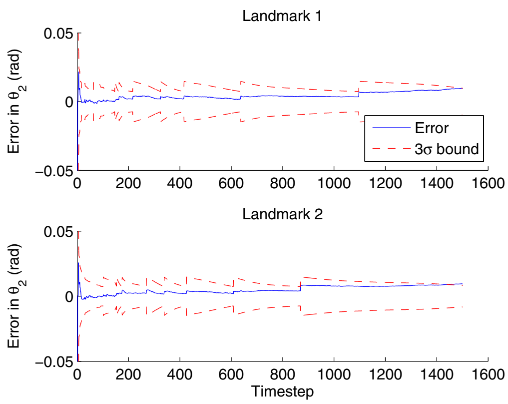

Fig. 7. Results of NDL-BOSLAM. Error and 3σ-bounds for θ_2.

robot's orientation estimate. Fig. 5 shows an increase of the error bound in ϕ approximately up to timestep 110. During this initial phase, the covariance of the residuals was very small and many observations were discarded in order to avoid destabilizing the filter. Once a few measurements were incorporated, the uncertainty of $\hat{\theta}_2$ decreased quickly and multiple baseline extensions took place, as can be seen in Fig. 7. When the baseline had grown sufficiently large, orientation accuracy improved drastically (around timestep 180) and soon reached steady state. Note that this is no hidden delayed initialization. Measurements are incorporated as soon as θ_2 is initialized and affect the orientation accuracy immediately. However, the effect of the measurement updates heightens after the baseline has been increased several times.

TABLE I

SIMULATION PARAMETERS

Δt = 0.1 s	Timestep
V = 1 m/s	Robot Velocity
σ_V = 0.2 m/s	Velocity Msmt. Noise
σ_ω = 0.08 rad/s	Turn Rate Msmt. Noise
$R = 2.7 \cdot 10^{-5}$ rad^2	Rel. Bearing Msmt. Noise
x_{L_1} = 5 m, y_{L_1} = 20 m x_{L_2} = 15 m, y_{L_2} = -10 m	Nearby Landmark Positions
x_{L_1} = 2500 m, y_{L_1} = -2960 m x_{L_2} = -190 m, y_{L_2} = -3252 m	Distant Landmark Positions

VII. CONCLUSION

In the previous sections we have outlined a novel, unified framework to allow for non-delayed initialization of near and distant landmarks in Bearing-Only SLAM. Contrary to existing algorithms, that either had to delay landmark initialization until a sufficient baseline was established, or were only able to address objects within limited range, our algorithm can initialize landmarks almost immediately and use information even from features at infinite distance. Measurements of the latter are particularly interesting, since they provide very accurate attitude information, crucial for precise navigation. Our algorithm works by employing a four-element landmark representation in combination with a constraint-based measurement model. The procedure was tested in extensive simulations, some of which were presented in this paper.

A major aspect of past and ongoing work is to cope efficiently with the numerical sensitivity of the approach due to the non-minimal state-representation and the highly nonlinear measurement model. One possible future way to avoid the issues related to non-minimal parametrization is to use the formulation presented in this paper as an intermediate representation until we can determine, with sufficient certainty, whether the landmark is close or far away. We could then switch to a two- or one-parameter representation as discussed in Section III. This would allow for non-delayed initialization of all classes of landmarks while enhancing stability of the algorithm.

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