

# ERRATA #2

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*Last updated: September 27, 2016*

For the most part, this errata was the one sent to SIAM at the occasion of the 3rd printing of the book (summer of 2007) so almost all errors posted here should be present only in books of the first and second printing.

Many of the errors were posted by Prof. Ian Gladwell in his web-site in Spring 06. Others were pointed out to me by Xiaoke Cui, Ken Hayami, Keiichi Morikuni, Kamaraju S Kusumanchi, Jan Mayer, Scott McLachlan and Daniel Osei-Kuffuor. Many thanks to all! If you see any other errors (as I am sure there are) please send them and I will update the errata.

1. Page 10. On line 7, after the words “consists of” add “the span of”.
2. Page 12, line 5. The 1-norm of  $x$  on the left-hand side should be replaced by the 2-norm.
3. Page 19, lines 7 to 9. All instances of  $l_1$  should be replaced by  $l_i$
4. Page 23. In the first line of the proof of Theorem 1.19 replace “or” by “of”
5. Page 41. Insert the word “nonzero” before “vectors” on the second line of question 17.
6. Page 50. In the first equation in section 2.2.2, Replace  $u(x - h_1, x_2)$  by  $u(x_1 - h_1, x_2)$ .
7. Page 54. On line -4, the inequalities should read  $1 \leq i \leq n_1$  and  $1 \leq j \leq n_2$ . On line -2, replace ‘ $n_1 = 7$  and  $n_2 = 5$ ’ by  $n_1 = 5$  and  $n_2 = 3$ .
8. Page 55. The matrix  $B$  at the bottom of the page should be  $5 \times 5$ . The equations should be

$$A = \frac{1}{h^2} \begin{pmatrix} B & -I & & & \\ -I & B & -I & & \\ & -I & B & & \\ & & & -I & B \\ & & & & -I & B \end{pmatrix} \quad \text{with} \quad B = \begin{pmatrix} 4 & -1 & & & \\ -1 & 4 & -1 & & \\ & -1 & 4 & -1 & \\ & & -1 & 4 & -1 \\ & & & -1 & 4 \end{pmatrix}.$$

9. Page 56. In the definition of  $\lambda_j$  there is a 2 missing in front of cos and the  $m$ 's and  $p$ 's must be swapped from here to 2 lines farther down: ....

$$\lambda_j = 4 - 2 \cos \left( \frac{j\pi}{p+1} \right) \quad j = 1, \dots, p$$

and, defining  $\theta_j \equiv (j\pi)/(p+1)$ , the corresponding eigenvectors are given by:

$$q_j = \sqrt{\frac{2}{p+1}} \times [\sin \theta_j, \sin(2\theta_j), \dots, \sin(p\theta_j)]^T .$$

....

10. Page 57. In equation (2.28) indices of the last elements in the solution and the right hand side vectors in equation should be  $m$  not  $p$ . In (2.29), indices next to last entries should be  $ip - 1$  not  $mp - 1$ . So, the section from the top of the page should be:

....

$$\begin{pmatrix} \Lambda & -I & & & & \\ -I & \Lambda & -I & & & \\ & & \ddots & \ddots & \ddots & \\ & & & -I & \Lambda & -I \\ & & & & -I & \Lambda \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \vdots \\ \bar{u}_{m-1} \\ \bar{u}_m \end{pmatrix} = \begin{pmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \vdots \\ \bar{b}_{m-1} \\ \bar{b}_m \end{pmatrix} \quad (2.28)$$

As it turns out, the above system disguises a set of  $m$  independent tridiagonal systems. Indeed, taking the  $i$ -th row of each block, yields

$$\begin{pmatrix} \lambda_i & -1 & & & \\ -1 & \lambda_i & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & \lambda_i & -1 \\ & & & -1 & \lambda_i \end{pmatrix} \begin{pmatrix} \bar{u}_{i1} \\ \bar{u}_{i2} \\ \vdots \\ \bar{u}_{ip-1} \\ \bar{u}_{ip} \end{pmatrix} = \begin{pmatrix} \bar{b}_{i1} \\ \bar{b}_{i2} \\ \vdots \\ \bar{b}_{ip-1} \\ \bar{b}_{ip} \end{pmatrix} \quad (2.29)$$

where  $u_{ij}$  and  $b_{ij}$  represent the  $j$ -th components of the vectors  $u_j$  and  $b_j$  respectively.

...

11. Page 57. In Algorithm 2.1 and in the two lines immediately after it,  $m$  and  $p$  should be interchanged:

...

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**ALGORITHM 2.1. FFT-based Fast-Poisson Solver**

1. Compute  $\bar{b}_j = Q^T b_j, j = 1, \dots, m$
2. Solve the tridiagonal systems (2.29) for  $i = 1, \dots, p$
3. Compute  $u_j = Q \bar{u}_j, j = 1, \dots, m$

The operations in Lines 1 and 3 are performed by FFT transforms, and require a total of  $O(p \log_2 p)$  operations each, leading to a total of  $O(m \times p \log_2 p)$  operations. Solving the  $m$  tridiagonal systems requires a total of  $8 \times p \times m$  operations. As a result, the complexity of the algorithm is  $O(N \log N)$  where  $N = p \times m$ .

...

12. Page 63. Equation in Line 8: Change  $i$  to  $j$ . Two lines below in eq. (2.44) change  $\xi_i$  into  $\xi_j$ :

...

Writing the desired solution  $u$  in the basis  $\{\phi_i\}$  as

$$u = \sum_{j=1}^n \xi_j \phi_j(x)$$

and substituting in (2.43) gives the linear problem

$$\sum_{j=1}^n \alpha_{ij} \xi_j = \beta_i \quad (2.44)$$

where

...

13. Page 63, Equation in the middle of the page misses periods in the integrands:

...

*symmetric positive definite* (SPD) matrix. Indeed, it is clear that

$$\int_{\Omega} \nabla \phi_i \cdot \nabla \phi_j \, dx = \int_{\Omega} \nabla \phi_j \cdot \nabla \phi_i \, dx,$$

...

14. In Page 95, Line -6 [corrected in latest printing] The sentence “The domain and coefficients for this problem are shown in Figure 3.12” is misplaced. It refers to Problem 2 in the next page and should go at the end of the description of this Problem. [or it can just be removed altogether].

15. Page 96. Caption for Figure 3.12 incorrectly refers to Problem 1 instead of Problem 2.

16. Page 108. The condition  $W_i^T V_i = I$  should be replaced by

$$\sum_{j=1}^p V_j W_j^T = I$$

Also in the next page,  $V_i W_i^T$  is a projector only in the non-overlapping case.

17. Page 109. In the third displayed equation the sum goes from 1 to  $p$  (not  $s$ )

18. Page 110. Line after equation (4.23), replace  $N = A - D$  by  $N = D - A$ :

...

For example, for the Jacobi iteration,  $M = D, N = D - A$ , while for the Gauss-Seidel iteration,  $M = D - E, N = M - A = F$ .

...

19. Page 112. In Theorem 4.1 and on the two lines before it, replace “any” by “every”.

20. Page 115. On line 2 replace “convergence rate can be very small” by “convergence can be very slow”.

21. Page 131. Equation in 4th line of section 5.1.2,  $x$  should be replaced by  $\tilde{x}$ :

$$\tilde{x} = x_0 + V y,$$

22. Page 138, Line 12. Replace  $v$  by  $r$ , so the 3-rd line of the displayed procedure should read:

$$x \leftarrow x + \alpha r.$$

23. Page 140, Line 6 from the beginning of the section 5.3.2. Similar correction as above, i.e., replace  $v$  by  $r$ ,

$$x \leftarrow x + \alpha r.$$

Also Line 3 of Algorithm 5.3 should be (replace  $Ar$  by  $p$ ) :

$$3. \quad \alpha \leftarrow (p, r)/(p, p)$$

24. Page 168, top. The matrix  $\Omega_1$  should be  $6 \times 6$  not  $5 \times 5$  as shown.

25. Page 176, 1st line of 2nd paragraph should say “Relating the DQGMRES and IOM residuals ...” (instead of FOM). A little farther, line before eq. (6.56),  $h_{m+1,m}/h_{mm}^{(m)} = \tan \theta_m$  should be  $h_{m+1,m}/h_{mm}^{(m-1)} = \tan \theta_m$ .

26. Page 177, Theorem 6.11., 2nd line should say “Let  $r_m^Q$  and  $r_m^G$  be the residual vectors...” (instead of ‘norms’).

27. Page 179. Equation at the bottom of page,  $m$  should be replaced by  $m + 1$ , as in:

$$\frac{1}{(\rho_m^G)^2} = \sum_{i=0}^m \frac{1}{(\rho_i^F)^2} \leq \frac{m+1}{(\rho_{m^*}^F)^2}$$

28. Page 180. Equation (6.68)  $\sqrt{m}$  should be replaced by  $\sqrt{m+1}$ , as in:

$$\rho_m^G \leq \rho_{m^*}^F \leq \sqrt{m+1} \rho_m^G \tag{6.68}$$

29. Page 182, 5 lines from the bottom. The sentence starting at the end of this line should say “The orthogonality of  $r_m^S$  with  $r_m^O - r_{m-1}^S$  implies that...”

30. Page 183, (6.79): both sums should be from  $j = 0$ .

31. Page 184, line 15:  $|c_m| = |\gamma_{m+1}|/\rho_m$  (no square).

32. Page 207, two equations at the bottom of the page. Replace the powers  $k$  by  $m$ :

...

$$\frac{C_m\left(\frac{a}{d}\right)}{C_m\left(\frac{c}{d}\right)} = \frac{\left(\frac{a}{d} + \sqrt{\left(\frac{a}{d}\right)^2 - 1}\right)^m + \left(\frac{a}{d} + \sqrt{\left(\frac{a}{d}\right)^2 - 1}\right)^{-m}}{\left(\frac{c}{d} + \sqrt{\left(\frac{c}{d}\right)^2 - 1}\right)^m + \left(\frac{c}{d} + \sqrt{\left(\frac{c}{d}\right)^2 - 1}\right)^{-m}}$$

$$\approx \left(\frac{a + \sqrt{a^2 - d^2}}{c + \sqrt{c^2 - d^2}}\right)^m.$$

...

33. Page 208. In the equation defining  $E_m$  in middle of the page,  $I_n$  should be  $I_{mp}$ :

$$E_m = \text{matrix of the last } p \text{ columns of } I_{mp}$$

34. Page 224. On line 6 of section 7.3.2,  $v_1 = \beta r_0$  should be  $v_1 = r_0/\beta$ :

.... If  $v_1$  is defined as a multiple of  $r_0$ , i.e., if  $v_1 = r_0/\beta$ , ...

35. Page 201, two lines after proof of Lemma 6.26. The statement  $|\gamma| > c$  should be  $|\gamma - c| > \rho$ .

36. Page 227. Similar corrections to the one for Pages 179, 180 (see above):  $m$  should be replaced by  $m + 1$  in the second displayed equation which should be:

$$\frac{1}{\left(\rho_m^Q\right)^2} = \sum_{i=0}^m \frac{1}{\left(\rho_i\right)^2} \leq \frac{m+1}{\left(\rho_{m^*}\right)^2}.$$

.. and in equation (7.25) which should be:

$$\rho_m^Q \leq \rho_{m^*} \leq \sqrt{m+1} \rho_m^Q \tag{7.25}$$

37. Page 229, last equation of this page, power 2 on last term is misplaced. Equation should be:

$$\pi_{j+1}^2(t) = \phi_{j+1}^2(t) + 2\beta_j \phi_{j+1}(t) \pi_j(t) + \beta_j^2 \pi_j^2(t).$$

...

38. Page 262. Equation (9.3) misses a term  $M_L^{-1}$ . It should be:

$$M_L^{-1} A M_R^{-1} u = M_L^{-1} b, \quad x \equiv M_R^{-1} u. \tag{9.3}$$

39. Page 267, Lines 14, and 15. Matrices  $E$  and  $E^T$  should be exchanged in this sentence, as follows:

... Denoting by  $N_z(X)$  the number of nonzero elements of a sparse matrix  $X$ , the total number of operations (additions and multiplications) of this procedure is  $n$  for (1),  $2N_z(E^T)$  for (2),  $2N_z(E) + 2n$  for (3), and  $n$  for (4). ...

40. Page 297, bottom equation. Replace  $a_{ij}$  by  $size(a_{ij})$ :

$$size(a_{ij}) := \epsilon^{lev_{ij}} - \epsilon^{lev_{ik}} \times \epsilon^{lev_{kj}} = \epsilon^{lev_{ij}} - \epsilon^{lev_{ik} + lev_{kj}}.$$

41. Page 306. In Example 10.3, the indices for  $\gamma$  in the formulas for  $s_i$  are incorrect:  $\gamma_{i-m+1}$  instead of  $\gamma_{m-i+1}$ .
42. Page 324. In line 4. of Algorithm 10.12,  $n_i$  should be  $n_j$ :
  4. For  $i = 1, \dots, n_j$  Do:
  - ...
43. Page 486, line 12. Replace “*negative semi-definite*” by “*positive semi-definite*”.