Simulation of reservoir geomechanics and multiphase flow involves solving multi-physics problems in which multiphase flow is tightly coupled with geomechanical processes. To capture this dynamic interplay, fully implicit methods, also known as monolithic approaches, are usually preferred. The main bottleneck of this strategy is the cost of solving the linear systems resulting from discretization of the problem. Because of the strong coupling present in the continuous problem, efficient techniques such as algebraic multigrid (AMG) cannot be directly applied to the discrete linear systems. In this work, we present our efforts in developing an algebraic framework based on multigrid reduction that is suited for tightly coupled systems of PDEs. Using this framework, the decoupling between the equations is done algebraically through defining appropriate interpolation and restriction operators. One can then employ existing solvers for each of the decoupled blocks or design a new solver based on knowledge of the physics. We will demonstrate the applicability of our framework to multiphase flow coupled with geomechanics. We show that the framework is flexible to accommodate a wide range of scenarios, as well as efficient and scalable for large problems.