The augmented Lagrangian (AL) preconditioner \cite{1}, belonging to the class of block structured preconditioners \cite{6, 7}, is originally proposed to solve saddle point systems arising from the incompressible Navier-Stokes equations discretized by the finite element method (FEM). The AL preconditioner features a purely algebraic construction and robustness with respect to the Reynolds number and mesh refinement. Because of these attractive features, recent research was devoted to the further development and extension of the AL preconditioner, notably the extension \cite{3} to the context of stabilized finite volume methods (FVM), which are widely used in industrial computational fluid dynamic applications, and the modified variant \cite{2} with reduced computational complexities.

In \cite{4} we introduce an alternative method to approximate the Schur complement for the AL preconditioner, which leads to a new variant of the AL preconditioner. This new method approximates the Schur complement through its inverse form and facilitates the utilization of the existing Schur complement approximations. Among the available candidates, the Schur complement approximation from the SIMPLE preconditioner \cite{5} is chosen and substituted into the inverse Schur complement approximation for the AL preconditioner. The so-arising new variant of the AL preconditioner reduces the number of Krylov subspace iterations by a factor up to 36 compared to the original one on the turbulent applications of the maritime industry.

Since the new method to approximate the Schur complement for the AL preconditioner use the existing Schur complement approximations, the following questions straightforwardly raise. Does the utilization of other existing Schur complement approximations deliver a better performance than that from the SIMPLE preconditioner? If so, which Schur complement approximation is the most efficient one? Does the optimal choice depend on the test problem and parameters arising from the physics and discretization, e.g. the Reynolds number and grid size? To answer these questions, in this talk we utilize the existing Schur complement approximations not only from the SIMPLE preconditioner but also from the LSC and PCD operators to construct the new Schur complement approximation in the AL preconditioner. Extensive comparisons between the considered Schur complement approximations are carried out on a wide range of numerical experiments in the laminar context to evaluate the effect of the Reynolds number, mesh anisotropy and refinement on the optimal choice. Furthermore, the advantage over the traditional Schur complement approximation in terms of the reduced number of the Krylov subspace iterations is exhibited. This work is expected to provide an alternative to efficiently apply the AL preconditioner for solving saddle point systems arising

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from the Navier-Stokes equations and provide a fundamental guideline for the more complicated turbulent flow calculations.

References


