Single View Metrology

Hyun Soo Park
England vs. West Germany (1966 World Cup Final)
England vs. West Germany (1966 World Cup Final)
Vanishing line

Distance cannot be trusted.
Height from Image

Ground plane
Height from Image

Spacing is not regular.

Ground plane
Height from Image

Spacing is not regular.

Ground plane

Vanishing point
Height from Image

Spacing is not regular.

Ground plane

Vanishing point
Height from Image

Spacing is not regular.

Ground plane

Vanishing point

178 cm
Height from Image

Spacing is not regular.

Ground plane

Vanishing point

178 cm

cm

200
180
160
140
120
100
80
60
40
20
Height from Image

Spacing is not regular.

Ground plane

Vanishing point

Camera height

178 cm
Where was I (how high)?

Taken from my hotel room (6th floor)  
Taken from beach
Height from Image

It is impossible to compute the height.

\[ h = f \frac{H}{Z} \]
Height from Image

Ground plane

$H$

$H_R$

$h$
Height from Image

Ground plane
Cross Ratio
Cross Ratio

\[ \frac{AC}{BC} \]

Target points

Reference points
Cross Ratio
Cross Ratio

A

B

C

D

Target points

Reference points

AC, BD

BC, AD
Cross Ratio

\[
\frac{AC}{BD} = \frac{ac}{bd}
\]

\[
\frac{BC}{AD} = \frac{bc}{ad}
\]

Cross ratio (perspective transformation invariant)
Cross Ratio

\[
\frac{AC}{BD} = \frac{ac}{bd} = \frac{a'c'}{b'd'}
\]

\[
\frac{BC}{AD} = \frac{bc}{ad} = \frac{b'c'}{a'd'}
\]

Cross ratio (perspective transformation invariant)
**Perspective Transform ~ Homography**

Invariant properties

- Cross ratio
-Concurrency
- Colinearity

Degree of freedom

8 (9 variables – 1 scale)
Cross Ratio

\[
\frac{AC}{BD} = \frac{ac}{bd} = \frac{a'c'}{b'd'} \quad \frac{BC}{AD} = \frac{bc}{ad} = \frac{b'c'}{a'd'}
\]

Cross ratio (perspective transformation invariant)
Height from Image

Ground plane
Height from Image

\[ \frac{h_R}{h'} = \frac{H_R}{H} \]

Ground plane
Height from Image

\[
\frac{h_R}{h'} = \frac{H_R}{H}
\]
Height from Image

\[ \frac{h_R}{h_{\infty}} = \frac{H_R}{H_{\infty}} \]

Ground plane
Height from Image

\[ \frac{h_R}{h} \times \frac{h'_R}{h'_R} = \frac{H_R}{H} \times \frac{H'_R}{H'_R} = \frac{H_R}{H} = \frac{H_R}{H} \]
Height from Image

$\frac{h_R}{h} = \frac{H_R}{H} = \frac{H'}{H} = \frac{H}{H} = \frac{H_R}{H}$

$H = H_R \frac{h'h_{\infty}}{h_R h_{\infty}}$
Single View Metrology (Criminisi 1999)

Ground truth height: 187 cm
Single View Metrology (Criminisi 1999)
sh_f = [1504;1447;1];
sh_h = [1468;730;1];
jp_f = [1997;1175;1];
jp_h = [1997;695;1];
\[ \text{sh}_f = [1504; 1447; 1]; \]
\[ \text{sh}_h = [1468; 730; 1]; \]
\[ \text{jp}_f = [1997; 1175; 1]; \]
\[ \text{jp}_h = [1997; 695; 1]; \]

\[ l_{11} = \text{GetLineFromTwoPoints}(m_1, m_2); \]
\[ l_{12} = \text{GetLineFromTwoPoints}(m_3, m_4); \]
\[ l_{21} = \text{GetLineFromTwoPoints}(m_1, m_3); \]
\[ l_{22} = \text{GetLineFromTwoPoints}(m_2, m_4); \]
\[
\begin{align*}
sh_f &= [1504;1447;1]; \\
sh_h &= [1468;730;1]; \\
jp_f &= [1997;1175;1]; \\
jp_h &= [1997;695;1]; \\
l_{11} &= \text{GetLineFromTwoPoints}(m_1,m_2); \\
l_{12} &= \text{GetLineFromTwoPoints}(m_3,m_4); \\
l_{21} &= \text{GetLineFromTwoPoints}(m_1,m_3); \\
l_{22} &= \text{GetLineFromTwoPoints}(m_2,m_4); \\
v_1 &= \text{GetPointFromTwoLines}(l_{11},l_{12}); \\
v_2 &= \text{GetPointFromTwoLines}(l_{21},l_{22}); \\
l &= \text{GetLineFromTwoPoints}(v_1,v_2); \\
\end{align*}
\]
sh_f = [1504;1447;1];
sh_h = [1468;730;1];
jp_f = [1997;1175;1];
jp_h = [1997;695;1];

l_{11} = GetLineFromTwoPoints(m_1,m_2);
l_{12} = GetLineFromTwoPoints(m_3,m_4);
l_{21} = GetLineFromTwoPoints(m_1,m_3);
l_{22} = GetLineFromTwoPoints(m_2,m_4);

v_1 = GetPointFromTwoLines(l_{11},l_{12});
v_2 = GetPointFromTwoLines(l_{21},l_{22});
l = GetLineFromTwoPoints(v_1,v_2);

line_{sh.jp.f} = GetLineFromTwoPoints(sh_f,jp_f);
v = GetPointFromTwoLines(line_{sh.jp.f}, l);
sh_f = [1504;1447;1];
sh_h = [1468;730;1];
jp_f = [1997;1175;1];
jp_h = [1997;695;1];

l11 = GetLineFromTwoPoints(m1,m2);
l12 = GetLineFromTwoPoints(m3,m4);
l21 = GetLineFromTwoPoints(m1,m3);
l22 = GetLineFromTwoPoints(m2,m4);

v1 = GetPointFromTwoLines(l11,l12);
v2 = GetPointFromTwoLines(l21,l22);
l = GetLineFromTwoPoints(v1,v2);

line_sh_jp_f = GetLineFromTwoPoints(sh_f,jp_f);
v = GetPointFromTwoLines(line_sh_jp_f,l);

line_jp_h_v = GetLineFromTwoPoints(jp_head,v);
\[ sh_f = [1504;1447;1]; \]
\[ sh_h = [1468;730;1]; \]
\[ jp_f = [1997;1175;1]; \]
\[ jp_h = [1997;695;1]; \]

\[ l11 = \text{GetLineFromTwoPoints}(m1,m2); \]
\[ l12 = \text{GetLineFromTwoPoints}(m3,m4); \]
\[ l21 = \text{GetLineFromTwoPoints}(m1,m3); \]
\[ l22 = \text{GetLineFromTwoPoints}(m2,m4); \]

\[ v1 = \text{GetPointFromTwoLines}(l11,l12); \]
\[ v2 = \text{GetPointFromTwoLines}(l21,l22); \]
\[ l = \text{GetLineFromTwoPoints}(v1,v2); \]

\[ \text{line}_{sh\_jp\_f} = \text{GetLineFromTwoPoints}(sh_f,jp_f); \]
\[ v = \text{GetPointFromTwoLines}(\text{line}_{sh\_jp\_f}, l); \]

\[ \text{line}_{jp\_h\_v} = \text{GetLineFromTwoPoints}(jp\_head,v); \]
\[ \text{line}_{sh} = \text{GetLineFromTwoPoints}(sh_h,sh_f); \]
\[ p2 = \text{GetPointFromTwoLines}(\text{line}_{jp\_head\_v}, \text{line}_{sh}); \]
\[ \begin{align*}
sh_f &= [1504; 1447; 1]; \\
sh_h &= [1468; 730; 1]; \\
jp_f &= [1997; 1175; 1]; \\
jp_h &= [1997; 695; 1];
\end{align*} \]

\[ \begin{align*}
l_{11} &= \text{GetLineFromTwoPoints}(m_1, m_2); \\
l_{12} &= \text{GetLineFromTwoPoints}(m_3, m_4); \\
l_{21} &= \text{GetLineFromTwoPoints}(m_1, m_3); \\
l_{22} &= \text{GetLineFromTwoPoints}(m_2, m_4);
\end{align*} \]

\[ \begin{align*}
v_1 &= \text{GetPointFromTwoLines}(l_{11}, l_{12}); \\
v_2 &= \text{GetPointFromTwoLines}(l_{21}, l_{22}); \\
l &= \text{GetLineFromTwoPoints}(v_1, v_2);
\end{align*} \]

\[ \begin{align*}
\text{line}_{-sh\_jp\_f} &= \text{GetLineFromTwoPoints}(sh_f, jp_f); \\
v &= \text{GetPointFromTwoLines}(\text{line}_{-sh\_jp\_f}, l);
\end{align*} \]

\[ \begin{align*}
\text{line}_{-jp\_h\_v} &= \text{GetLineFromTwoPoints}(\text{jp\_head}, v); \\
\text{line}_{-sh} &= \text{GetLineFromTwoPoints}(sh_h, sh_f); \\
p_2 &= \text{GetPointFromTwoLines}(\text{line}_{-jp\_head\_v}, \text{line}_{-sh}); \\
p_3 &= sh_h; \\
p_1 &= sh_f;
\end{align*} \]
\( \text{sh}_f = [1504;1447;1]; \)
\( \text{sh}_h = [1468;730;1]; \)
\( \text{jp}_f = [1997;1175;1]; \)
\( \text{jp}_h = [1997;695;1]; \)

\( \text{l}_{11} = \text{GetLineFromTwoPoints}(m_1,m_2); \)
\( \text{l}_{12} = \text{GetLineFromTwoPoints}(m_3,m_4); \)
\( \text{l}_{21} = \text{GetLineFromTwoPoints}(m_1,m_3); \)
\( \text{l}_{22} = \text{GetLineFromTwoPoints}(m_2,m_4); \)

\( \text{v}_1 = \text{GetPointFromTwoLines}(\text{l}_{11},\text{l}_{12}); \)
\( \text{v}_2 = \text{GetPointFromTwoLines}(\text{l}_{21},\text{l}_{22}); \)
\( \text{l} = \text{GetLineFromTwoPoints}(\text{v}_1,\text{v}_2); \)

\( \text{line}_{\text{sh} \_ \text{jp} \_ f} = \text{GetLineFromTwoPoints}(\text{sh}_f, \text{jp}_f); \)
\( \text{v} = \text{GetPointFromTwoLines}(\text{line}_{\text{sh} \_ \text{jp} \_ f}, \text{l}); \)

\( \text{line}_{\text{jp} \_ \text{h} \_ \text{v}} = \text{GetLineFromTwoPoints}(\text{jp}_\text{head}, \text{v}); \)
\( \text{line}_\text{sh} = \text{GetLineFromTwoPoints}(\text{sh}_h, \text{sh}_f); \)
\( \text{p}_3 = \text{GetPointFromTwoLines}(\text{line}_{\text{jp} \_ \text{h} \_ \text{v}}, \text{line}_\text{sh}); \)
\( \text{p}_2 = \text{sh}_h; \)
\( \text{p}_1 = \text{sh}_f; \)

\( \text{l}_{31} = \text{GetLineFromTwoPoints}(m_5,m_6); \)
\( \text{l}_{32} = \text{GetLineFromTwoPoints}(m_7,m_8); \)
\( \text{v}_3 = \text{GetPointFromTwoLines}(\text{l}_{31},\text{l}_{32}); \)
\( \text{p}_4 = \text{v}_3; \)
\[ h_{\prime} = \text{norm}(p1-p2); \]
\[ h_{R} = \text{norm}(p1-p3); \]
\[ h_{\prime \infty} = \text{norm}(p4-p2); \]
\[ h_{\infty} = \text{norm}(p4-p3); \]

\[ H = H_{R} \frac{h' h_{\infty}}{h_{R} h'_{\infty}} \]
\[ h_{\text{prime}} = \text{norm}(p1 - p2); \]
\[ h_{R} = \text{norm}(p1 - p3); \]
\[ h_{\text{prime inf}} = \text{norm}(p4 - p2); \]
\[ h_{\text{inf}} = \text{norm}(p4 - p3); \]
\[ H = H_{R} * h_{\text{prime}} * h_{\text{prime inf}} / h_{R} / h_{\text{inf}} \]

\[ H = 1.6779 \quad \text{Ground truth: 1.7m} \]
Measure the height of the street lamp.

\[
H = H_R \frac{h'h_\infty}{h_R h'_\infty}
\]
Measure the height of the street lamp. $H = H_R \frac{h'h_\infty}{h_R h'_{\infty}}$
Measure the height of the street lamp.
Measure the height of the lamp.
Measure the height of the street lamp.
Measure the height of the street lamp.

\[ H = H_R \frac{h' h_\infty}{h_R h'_\infty} \]
Measure the height of the street lamp.
Measure the height of the street lamp.

\[ H = 2.8108 \text{m} \]
England vs. West Germany (1966 World Cup Final)
\[ l_1 = \mathbf{H}^T l_2 \]
\[ l_1 = H^{-T}l_2 \]
the last image pair). The answer to the question, “did the ball cross the line?” must also take the ball radius into account, and a more quantitative analysis is given in the graph of figure 8(b) which shows the distance of the ball from being a goal (taking its radius into account) plotted against frame number. The dotted lines indicate three standard deviations from the estimate, thus a conservative estimate has the ball still 6cm from being a goal.

Reid and Zisserman “Goal-directed Video Metrology”, ECCV 1996
England vs. Germany (2010 World Cup Round of 16)
England vs. Germany (2010 World Cup Round of 16)