CSCI 5561: Assignment #1
Histogram of Oriented Gradients (HOG)

1 Submission

• Assignment due: Feb 5 (11:55pm)
• Individual assignment
• Submission through Canvas.
• You will complete HOG_ver1.py that contains the following functions:
  - extract_hog
  - get_differential_filter
  - filter_image
  - get_gradient
  - build_histogram
  - get_block_descriptor
  - face_detection

The code can be downloaded from

• The function that does not comply with its specification will not be graded (no credit).

• The code must be run with Python 3 interpreter.

• Required python packages: numpy, matplotlib, and opencv.
  - numpy & matplotlib: https://scipy.org/install.html
  - opencv: https://pypi.org/project/opencv-python/

• You are not allowed to use any high level python functions of image processing and computer vision, e.g., cv2.filter2D. Please consult with TA if you are not sure about the list of allowed functions.

• We provide a visualization code for HOG. The resulting HOG descriptor must be able to be visualized with the provided code:
  def visualize_hog(im, hog, cell_size, block_size)

• We provide a visualization code for face detection. The detected faces must be able to be visualized with the provided code:
  visualize_face_detection(I_target, bounding_boxes, bb_size)

• Please place the code (HOG_ver1.py) as well as a one-page summary write-up with resulting visualization(in pdf format; more than 1 page assignment will be automatically returned.) into a folder, compress it, and submit.
2 HOG

Figure 1: Histogram of oriented gradients. HOG feature is extracted and visualized for (a) the entire image and (b) zoom-in image. The orientation and magnitude of the red lines represents the gradient components in a local cell.

In this assignment, you will implement a variant of HOG (Histogram of Oriented Gradients) in Python proposed by Dalal and Trigg [1] (2015 Longuet-Higgins Prize Winner). It had been long standing top representation (until deep learning) for the object detection task with a deformable part model by combining with a SVM classifier [2]. Given an input image, your algorithm will compute the HOG feature and visualize as shown in Figure 1 (the line directions are perpendicular to the gradient to show edge alignment). The orientation and magnitude of the red lines represents the gradient components in a local cell.

```python
def extract_hog(im):
    ...
    return hog
```

**Input:** input gray-scale image with uint8 format.
**Output:** HOG descriptor.
**Description:** You will compute the HOG descriptor of input image `im`. The pseudo-code can be found below:

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**Algorithm 1 HOG**

1. Convert the gray-scale image to float format and normalize to range [0, 1].
2. Get differential images using `get_differential_filter` and `filter_image`
3. Compute the gradients using `get_gradient`
4. Build the histogram of oriented gradients for all cells using `build_histogram`
5. Build the descriptor of all blocks with normalization using `get_block_descriptor`
6. Return a long vector (`hog`) by concatenating all block descriptors.
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2.1 Image filtering

Figure 2: (a) Input image dimension. (b-c) Differential image along $x$ and $y$ directions.

```python
def get_differential_filter():
    ...
    return filter_x, filter_y

Input: none.
Output: filter_x and filter_y are 3×3 filters that differentiate along $x$ and $y$ directions, respectively.
Description: You will compute the gradient by differentiating the image along $x$ and $y$ directions. This code will output the differential filters.
```

```python
def filter_image(im, filter):
    ...
    return im_filtered

Input: im is the gray scale $m \times n$ image (Figure 2(a)) converted to float format and filter is a filter ($k \times k$ matrix)
Output: im_filtered is $m \times n$ filtered image. You may need to pad zeros on the boundary on the input image to get the same size filtered image.
Description: Given an image and filter, you will compute the filtered image. Given the two functions above, you can generate differential images by visualizing the magnitude of the filter response as shown in Figure 2(b) and 2(c).```
2.2 Gradient Computation

![Magnitude](image1.png) ![Angle](image2.png) ![Gradient](image3.png) ![Zoomed Eye](image4.png) ![Zoomed Neck](image5.png)

Figure 3: Visualization of (a) magnitude and (b) orientation of image gradients. (c-e) Visualization of gradients at every 3rd pixel (the magnitudes are re-scaled for illustrative purpose.).

```python
def get_gradient(im_dx, im_dy):
    ...
    return grad_mag, grad_angle
```

**Input:** `im_dx` and `im_dy` are the x and y differential images (size: \( m \times n \)).

**Output:** `grad_mag` and `grad_angle` are the magnitude and orientation of the gradient images (size: \( m \times n \)). Note that the range of the angle should be \([0, \pi]\), i.e., unsigned angle \((\theta = \theta + \pi)\).

**Description:** Given the differential images, you will compute the magnitude and angle of the gradient. Using the gradients, you can visualize and have some sense with the image, i.e., the magnitude of the gradient is proportional to the contrast (edge) of the local patch and the orientation is perpendicular to the edge direction as shown in Figure 3.
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2.3 Orientation Binning

![Cell size and orientation binning diagram]

Figure 4: (a) Histogram of oriented gradients can be built by (b) binning the gradients to corresponding bin.

```python
def build_histogram(grad_mag, grad_angle, cell_size):
    ...
    return ori_histo
```

**Input:** `grad_mag` and `grad_angle` are the magnitude and orientation of the gradient images (size: $m \times n$); `cell_size` is the size of each cell, which is a positive integer.

**Output:** `ori_histo` is a 3D tensor with size $M \times N \times 6$ where $M$ and $N$ are the number of cells along $y$ and $x$ axes, respectively, i.e., $M = \lceil m / \text{cell\_size} \rceil$ and $N = \lceil n / \text{cell\_size} \rceil$ where $\lceil \cdot \rceil$ is the round-off operation as shown in Figure 4(a).

**Description:** Given the magnitude and orientation of the gradients per pixel, you can build the histogram of oriented gradients for each cell.

$$
\text{ori\_histo}(i,j,k) = \sum_{(u,v) \in C_{i,j}} \text{grad\_mag}(u,v) \text{ if } \text{grad\_angle}(u,v) \in \theta_k \quad (1)
$$

where $C_{i,j}$ is a set of $x$ and $y$ coordinates within the $(i,j)$ cell, and $\theta_k$ is the angle range of each bin, e.g., $\theta_1 = [165^\circ, 180^\circ) \cup [0^\circ, 15^\circ)$, $\theta_2 = [15^\circ, 45^\circ)$, $\theta_3 = [45^\circ, 75^\circ)$, $\theta_4 = [75^\circ, 105^\circ)$, $\theta_5 = [105^\circ, 135^\circ)$, and $\theta_6 = [135^\circ, 165^\circ)$. Therefore, `ori_histo(i,j,:)` returns the histogram of the oriented gradients at $(i,j)$ cell as shown in Figure 4(b).

Using the `ori_histo`, you can visualize HOG per cell where the magnitude of the line proportional to the histogram as shown in Figure 1. Typical `cell\_size` is 8.
Figure 5: HOG is normalized to account illumination and contrast to form a descriptor for a block. (a) HOG within (1,1) block is concatenated and normalized to form a long vector of size 24. (b) This applies to the rest block with overlap and stride 1 to form the normalized HOG.

```python
def get_block_descriptor(ori_histo, block_size):
    ...
    return ori_histo_normalized
```

**Input:** `ori_histo` is the histogram of oriented gradients without normalization. `block_size` is the size of each block (e.g., the number of cells in each row/column), which is a positive integer.

**Output:** `ori_histo_normalized` is the normalized histogram (size: $(M-(\text{block\_size}-1)) \times (N-(\text{block\_size}-1)) \times (6 \times \text{block\_size})$).

**Description:** To account for changes in illumination and contrast, the gradient strengths must be locally normalized, which requires grouping the cells together into larger, spatially connected blocks (adjacent cells). Given the histogram of oriented gradients, you apply $L_2$ normalization as follow:

1. Build a descriptor of the first block by concatenating the HOG within the block. You can use `block_size`=2, i.e., $2 \times 2$ block will contain $2 \times 2 \times 6$ entries that will be concatenated to form one long vector as shown in Figure 6(c).
2. Normalize the descriptor as follow:

   $$\hat{h}_i = \frac{h_i}{\sqrt{\sum_i h_i^2 + e^2}} \quad (2)$$

   where $h_i$ is the $i^{th}$ element of the histogram and $\hat{h}_i$ is the normalized histogram. $e$ is the normalization constant to prevent division by zero (e.g., $e = 0.001$).
3. Assign the normalized histogram to `ori_histo_normalized`(1,1) (white dot location in Figure 6(c)).
4. Move to the next block `ori_histo_normalized`(1,2) with the stride 1 and iterate 1-3 steps above.
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The resulting ori_histo_normalized will have the size of \((M - 1) \times (N - 1) \times 24\).
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2.5 Application: Face Detection

Figure 6: You will use (a) a single template image to detect faces in (b) the target image using HOG descriptors. (c) HOG descriptors from the template and target image patches can be compared by using the measure of normalized cross-correlation (NCC). (d) Thresholding on NCC score will produce many overlapping bounding boxes. (e) Correct bounding boxes for faces can be obtained by using non-maximum suppression.

Using the HOG descriptor, you will design a face detection algorithm. You can download the template and target images from here: https://www-users.cs.umn.edu/~hspark/csci5561_S2020/hw1.zip

```python
def face_recognition(I_target, I_template):
    ...
    return bounding_boxes
```

**Input:** I_target is the image that contains multiple faces. I_template is the template face image that will be matched to the image to detect faces.

**Output:** bounding_boxes is $n \times 3$ array that describes the $n$ detected bounding boxes. Each row of the array is \([x_i, y_i, s_i]\) where \((x_i, y_i)\) is the left-top corner coordinate of the $i^{th}$ bounding box, and $s_i$ is the normalized cross-correlation (NCC) score between the bounding box patch and the template:

$$s = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||\,||\mathbf{b}||} \quad (3)$$
where \( \mathbf{a} \) and \( \mathbf{b} \) are two normalized descriptors, i.e., zero mean:

\[
    a_i = a_i - \bar{a}
\]

where \( a_i \) is the \( i^{th} \) element of \( \mathbf{a} \), and \( a_i \) is the \( i^{th} \) element of the HOG descriptor. \( \bar{a} \) is the mean of the HOG descriptor.

**Description:** You will use thresholding and non-maximum suppression with IoU 50\% to localize the faces. You may use `visualize_face_detection(I\_target, bounding\_boxes, bb\_size)` to visualize your detection.

**References**
