1 Submission

- Assignment due: Feb 28 (11:55pm)
- Individual assignment
- Submission through Canvas.
- You will complete Registration.py that contains the following functions:
  - find_match
  - align_image_using_feature
  - warp_image
  - align_image
  - track_multi_frames

The code can be downloaded from https://www-users.cs.umn.edu/~hspark/csci5561_S2020/Registration.zip.

- The function that does not comply with its specification will not be graded (no credit).
- The code must be run with Python 3 interpreter.
- You are not allowed to use computer vision related package functions unless explicitly mentioned here. Please consult with TA if you are not sure about the list of allowed functions.
- Place code and a two-page summary write-up with resulting visualization (in pdf format; more than 2 pages will be automatically returned.) into a folder, compress it, and submit.
2 Sift Feature Extraction

Figure 1: Given an image (a), you will extract SIFT features using OpenCV.

One of key skills to learn in computer vision is the ability to use other open source code, which allow you not to re-invent the wheel. We will use OpenCV library for SIFT extraction given your images.

(Note) You will use this library only for SIFT feature extraction and its visualization. All following visualizations and algorithms must be done by your code. Using OpenCV, you can extract keypoints and associated descriptors as shown in Figure 1.

(Note) The function for SIFT feature extraction lie in the contrib module of OpenCV library, so you need to install opencv-contrib-python package additionally. Also, in newer versions of OpenCV, SIFT module is not available due to patent issue. But you can reinstall opencv-python and opencv-contrib-python package to an earlier version 3.4.2.16 easily through following two command lines.

- pip3 install opencv-python==3.4.2.16
- pip3 install opencv-contrib-python==3.4.2.16

(SIFT visualization) Use OpenCV to visualize SIFT features with scale and orientation as shown in Figure 1 (OpenCV may different colors to visualize). You may want to follow the following tutorial:
https://docs.opencv.org/3.4.2/da/df5/tutorial_py_sift_intro.html
3 SIFT Feature Matching

Figure 2: You will match points between the template and target image using SIFT features.

The SIFT is composed of scale, orientation, and 128 dimensional local feature descriptor (integer), $f \in \mathbb{Z}^{128}$. You will use the SIFT features to match between two images, $I_1$ and $I_2$. Use two sets of descriptors from the template and target, find the matches using nearest neighbor with the ratio test. You may use NearestNeighbors function imported from sklearn.neighbors (You can install sklearn package easily by "pip3 install -U scikit-learn").

```python
def find_match(img1, img2):
    ...
    return x1, x2
```

**Input:** two input gray-scale images with uint8 format.

**Output:** $x1$ and $x2$ are $n \times 2$ matrices that specify the correspondence.

**Description:** Each row of $x1$ and $x2$ contains the $(x, y)$ coordinate of the point correspondence in $I_1$ ad $I_2$, respectively, i.e., $x1(i,:) \leftrightarrow x2(i,:)$.  

(Note) You can only use SIFT module of OpenCV for the SIFT descriptor extraction. Matching with the ratio test needs to be implemented by yourself.
4 Feature-based Image Alignment

Figure 3: You will compute an affine transform using SIFT matches filtered by RANSAC. Blue: outliers; Orange: inliers; Red: the boundary of the transformed template.

(Note) From this point, you cannot use any function provided by OpenCV, except for purely visualization purpose.

The noisy SIFT matches can be filtered by RANSAC with an affine transformation as shown in Figure 3.

```python
def align_image_using_feature(x1, x2, ransac_thr, ransac_iter):
    ...
    return A
```

Input: $x_1$ and $x_2$ are the correspondence sets ($n \times 2$ matrices). $\text{ransac_thr}$ and $\text{ransac_iter}$ are the error threshold and the number of iterations for RANSAC.

Output: $3 \times 3$ affine transformation.

Description: The affine transform will transform $x_1$ to $x_2$, i.e., $x_2 = Ax_1$. You may visualize the inliers and the boundary of the transformed template to validate your implementation.
5 Image Warping

Figure 4: You will use the affine transform to warp the target image to the template using the inverse mapping. Using the warped image, the error map $|I_{\text{tpl}} - I_{\text{wrp}}|$ can be computed to validate the correctness of the transformation where $I_{\text{tpl}}$ and $I_{\text{wrp}}$ are the template and warped images.

Given an affine transform $A$, you will write a code to warp an image $I(x) \rightarrow I(Ax)$.

```python
def warp_image(img, A, output_size):
    ...
    return img_warped
```

**Input:** $I$ is an image to warp, $A$ is the affine transformation from the original coordinate to the warped coordinate, `output_size=[h,w]` is the size of the warped image where $w$ and $h$ are the width and height of the warped image.

**Output:** `img_warped` is the warped image with the size of `output_size`.

**Description:** The inverse mapping method needs to be applied to make sure the warped image does not produce empty pixel. You are allowed to use `interp` function imported from `scipy.interpolate` for bilinear interpolation (scipy package can be easily installed through ”pip3 install scipy” if you have not install it yet).

(Validation) Using the warped image, the error map $|I_{\text{tpl}} - I_{\text{wrp}}|$ can be computed to validate the correctness of the transformation where $I_{\text{tpl}}$ and $I_{\text{wrp}}$ are the template and warped images.
6 Inverse Compositional Image Alignment

Figure 5: You will use the initial estimate of the affine transform to align (i.e., track) next image. (a) Template image from the first frame image. (b) The second frame image with the initialization of the affine transform. (c) The second frame image with the optimized affine transform using the inverse compositional image alignment.

Given the initial estimate of the affine transform $A$ from the feature based image alignment (Section 4) as shown in Figure 5(b), you will track the next frame image using the inverse compositional method (Figure 5(c)). You will parametrize the affine transform with 6 parameters $p = (p_1, p_2, p_3, p_4, p_5, p_6)$, i.e.,

$$W(x; p) = \begin{bmatrix} p_1 + 1 & p_2 & p_3 \\ p_4 & p_5 + 1 & p_6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = A(p)x$$ (1)

where $W(x; p)$ is the warping function from the template patch to the target image. $x = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$ is the coordinate of the point before warping, and $A(p)$ is the affine transform parametrized by $p$.

```python
def align_image(template, target, A):
    ...
    return A_refined
```

**Input:** gray-scale template `template` and target image `target`; the initialization of $3 \times 3$ affine transform $A$, i.e., $x_{tgt} = Ax_{tpl}$ where $x_{tgt}$ and $x_{tpl}$ are points in the target and template images, respectively.

**Output:** `A_refined` is the refined affine transform based on inverse compositional image alignment.

**Description:** You will refine the affine transform using inverse compositional image alignment, i.e., $A \rightarrow A_{refined}$. The pseudo-code can be found in Algorithm 1.

**Tip:** You can validate your algorithm by visualizing their error map as shown in Figure 6(d) and 6(h). Also you can visualize the error plot over iterations, i.e., the error must decrease as shown in Figure 6(i).
Figure 6: (a,e) Template images of the first frame. (b) Warped image based on the initialization of the affine parameters. (c) Template image is overlaid by the initialization. (d) Error map of the initialization. (f) Optimized warped image using the inverse compositional image alignment. (g) Template image is overlaid by the optimized warped image. (h) Error map of the optimization. (i) An error plot over iterations.

Algorithm 1 Inverse Compositional Image Alignment

1: Initialize $p = p_0$ from input $A$.
2: Compute the gradient of template image, $\nabla I_{tpl}$
3: Compute the Jacobian $\frac{\partial W}{\partial p}$ at $(x; 0)$.
4: Compute the steepest decent images $\nabla I_{tpl} \frac{\partial W}{\partial p}$
5: Compute the $6 \times 6$ Hessian $H = \sum_x \left[ \nabla I_{tpl} \frac{\partial W}{\partial p} \right]^T \left[ \nabla I_{tpl} \frac{\partial W}{\partial p} \right]$
6: while $\|p\| > \epsilon$ do
7: Warp the target to the template domain $I_{tgt}(W(x; p))$.
8: Compute the error image $I_{err} = I_{tgt}(W(x; p)) - I_{tpl}$.
9: Compute $F = \sum_x \left[ \nabla I_{tpl} \frac{\partial W}{\partial p} \right]^T I_{err}$.
10: Compute $\Delta p = H^{-1}F$.
11: Update $W(x; p) \leftarrow W(x; p) \circ W^{-1}(x; \Delta p) = W(W^{-1}(x; \Delta p); p)$.
12: end while
13: Return $A_{\text{refined}}$ made of $p$. 
7 Putting Things Together: Multiframe Tracking

Figure 7: You will use the inverse compositional image alignment to track 4 frames of images.

Given a template and a set of consecutive images, you will (1) initialize the affine transform using the feature based alignment and then (2) track over frames using the inverse compositional image alignment.

```python
def track_multi_frames(template, img_list):
    ...
    return A_list
```

**Input:** template is gray-scale template. image_list is a list of consecutive image frames, i.e., img_list[i] is the \( i \)th frame.

**Output:** A_list is the set of affine transforms from the template to each frame of image, i.e., A_list[i] is the affine transform from the template to the \( i \)th image.

**Description:** You will apply the inverse compositional image alignment sequentially to track over frames as shown in Figure 7. Note that the template image needs to be updated at every frame, i.e., template ← warp_image(img, A, template.shape).