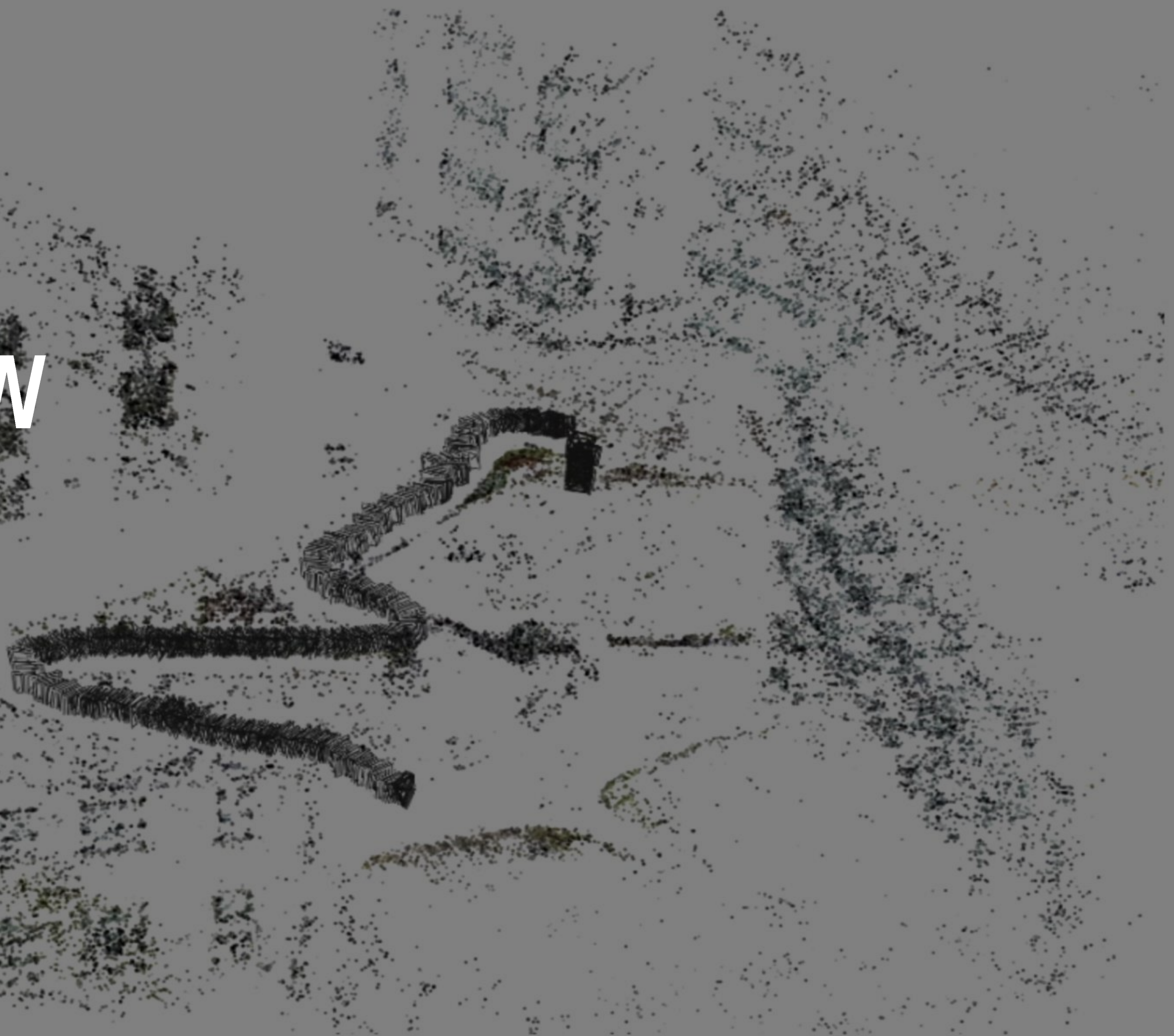
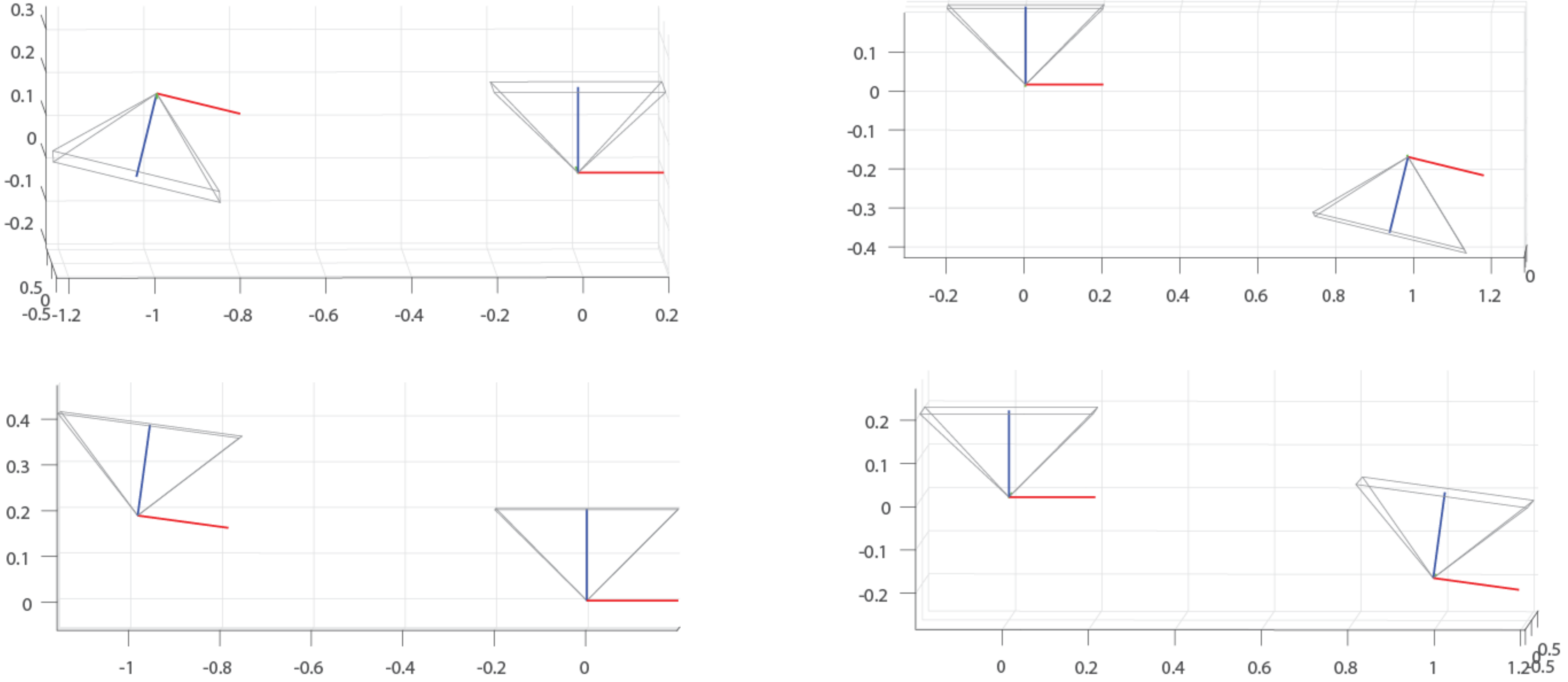
Triangulation

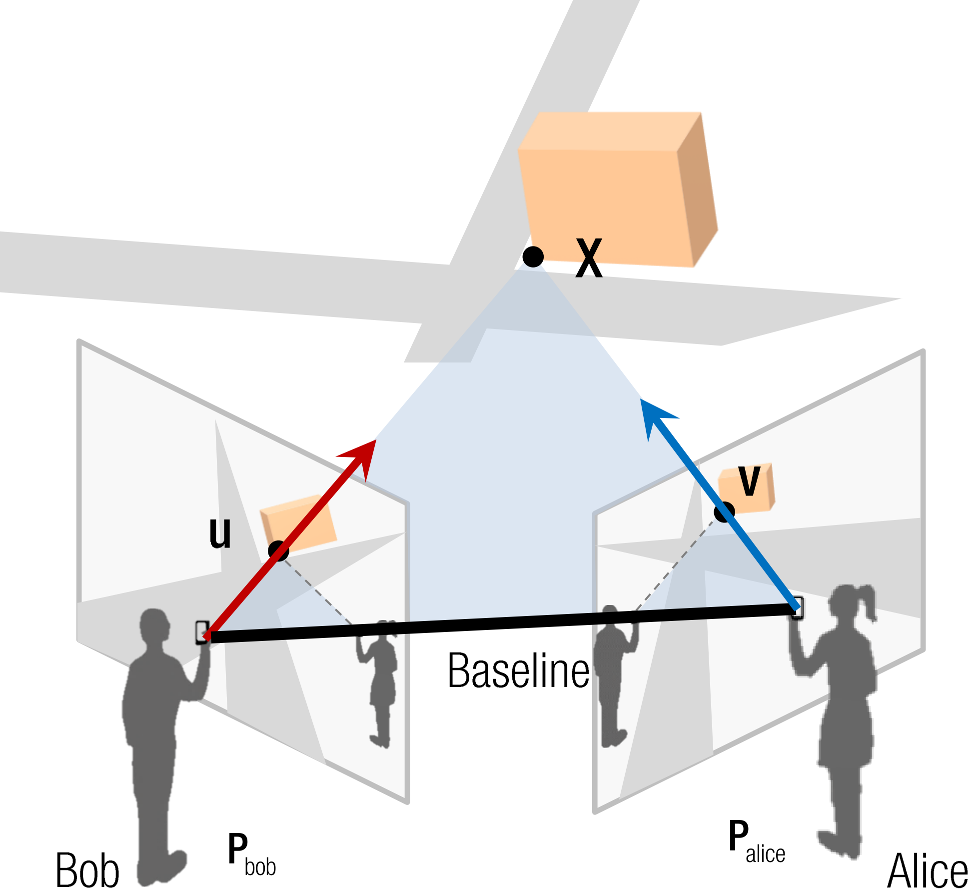
Hyun Soo Park



POSE DISAMBIGUATION

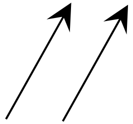


TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



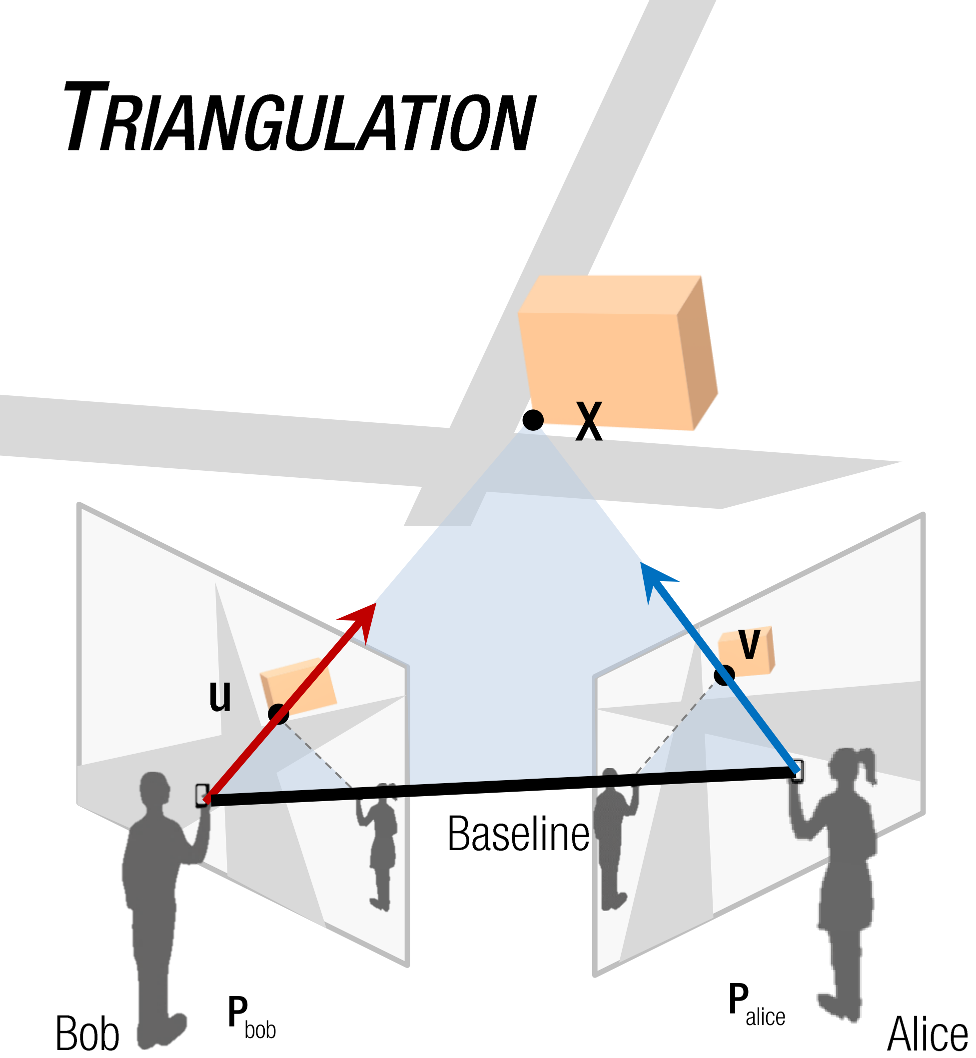
Two 3D vectors are parallel.

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{\text{bob}} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

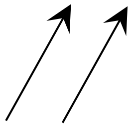
Skew-symmetric matrix

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



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$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

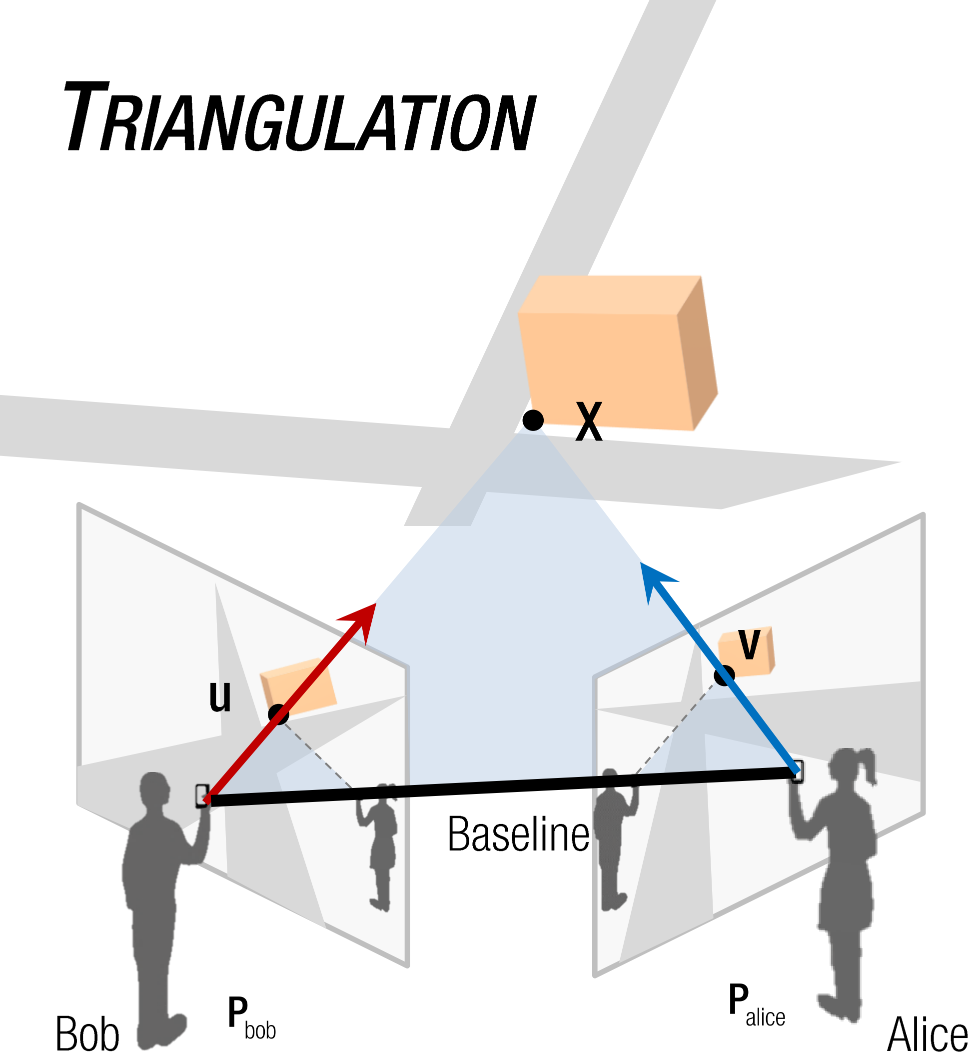
: Knowns

: Unknowns

Skew-symmetric matrix

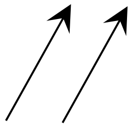
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}_{\times} = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}$$

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix}\mathbf{u}\\1\end{bmatrix} = \mathbf{P}_{bob}\begin{bmatrix}\mathbf{X}\\1\end{bmatrix}$$



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$$\rightarrow \begin{bmatrix}\mathbf{u}\\1\end{bmatrix}_{\times} \mathbf{P}_{bob}\begin{bmatrix}\mathbf{X}\\1\end{bmatrix} = \mathbf{0}$$

3x4

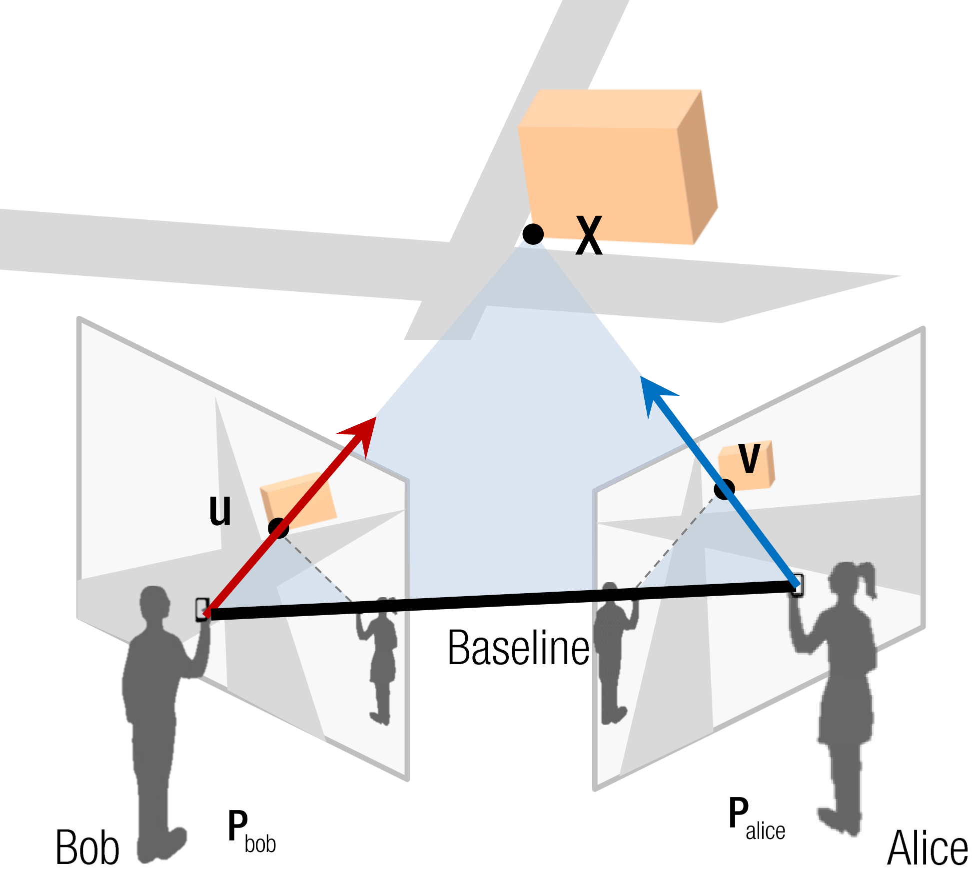
: Knowns

: Unknowns

Can we solve for **X**? (single view reconstruction)

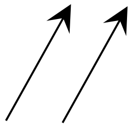
Why not?

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



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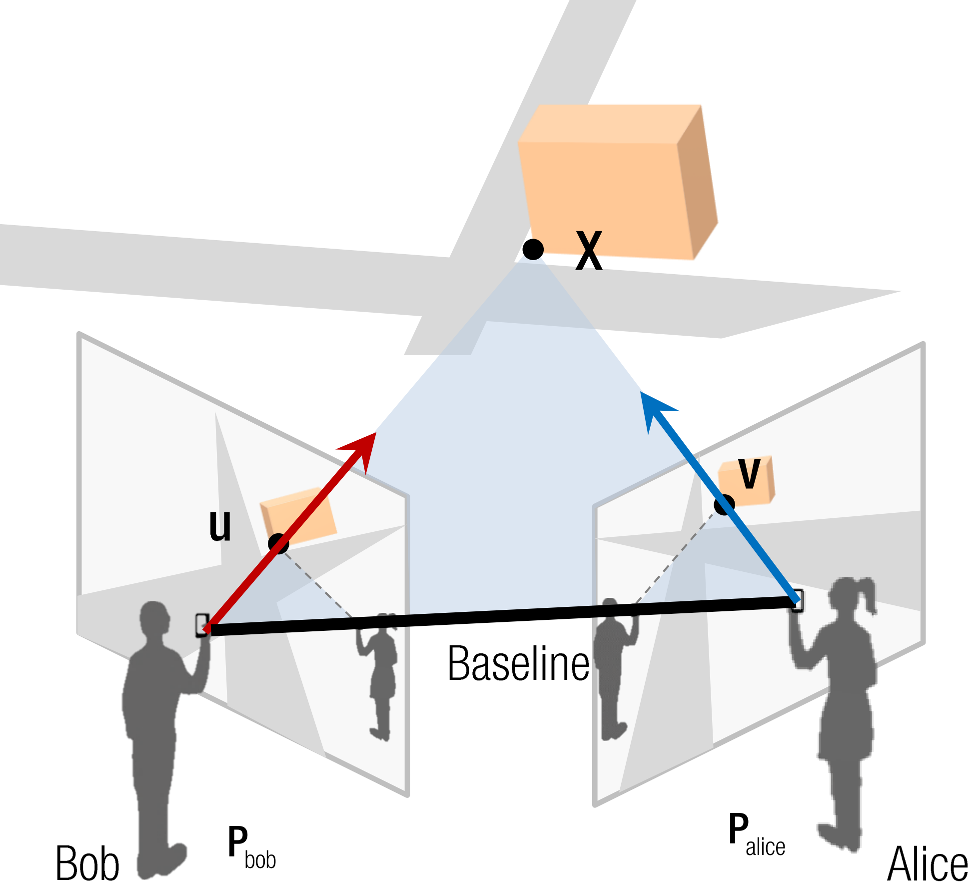
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

2x4

: Knowns

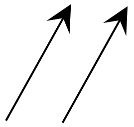
: Unknowns

TRIANGULATION



General camera pose

$$\lambda_1 \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} = \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}$$



Two 3D vectors are parallel.

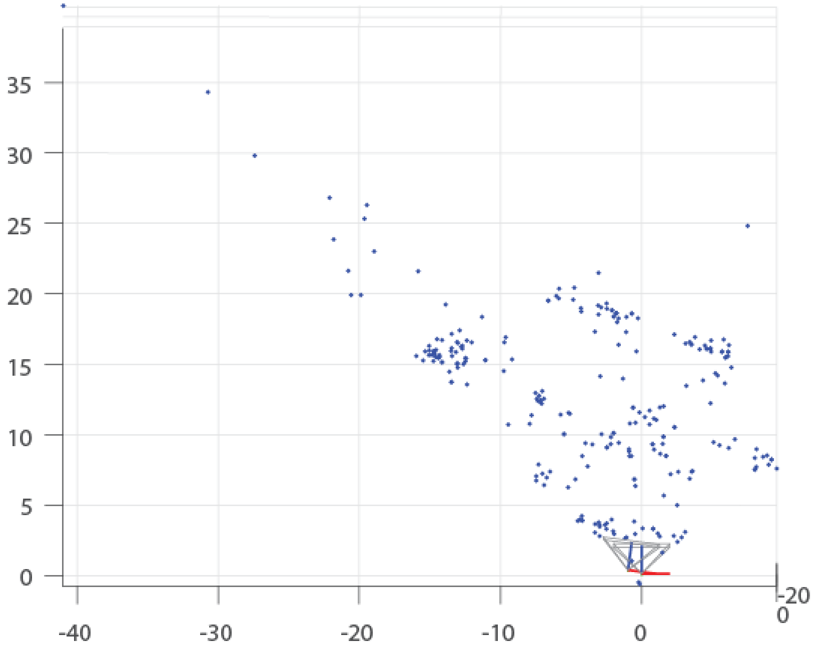
$$\rightarrow \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix} \times \mathbf{P}_{bob} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

$$\rightarrow \begin{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{bob} \\ \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_{\times} \mathbf{P}_{alice} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

4x4

: Knowns

: Unknowns



$$\begin{bmatrix} \begin{bmatrix} \mathbf{u} \\ 1 \end{bmatrix}_\times \mathbf{P}_{\mathrm{bob}} \\ \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix}_\times \mathbf{P}_{\mathrm{alice}} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix} = \mathbf{0}$$

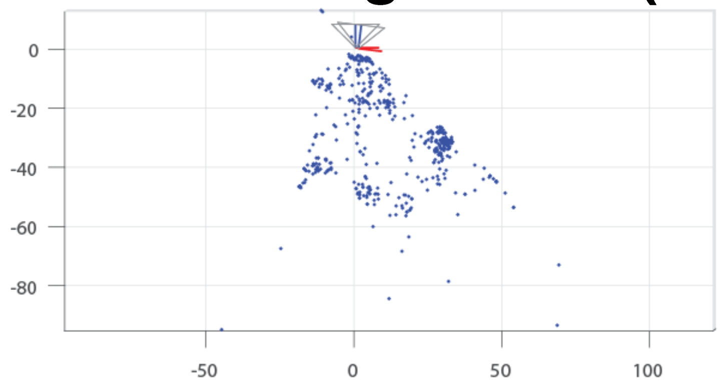
Camera Pose Disambiguation (Cheirality)

Cheirality condition:

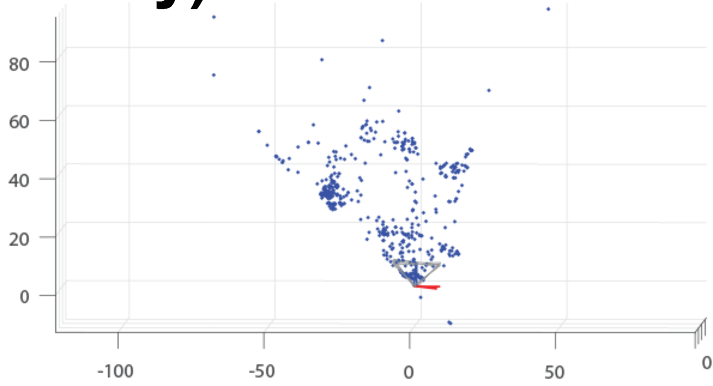
$$\mathbf{r}_3^T(\mathbf{X} - \mathbf{C}) > 0$$

where

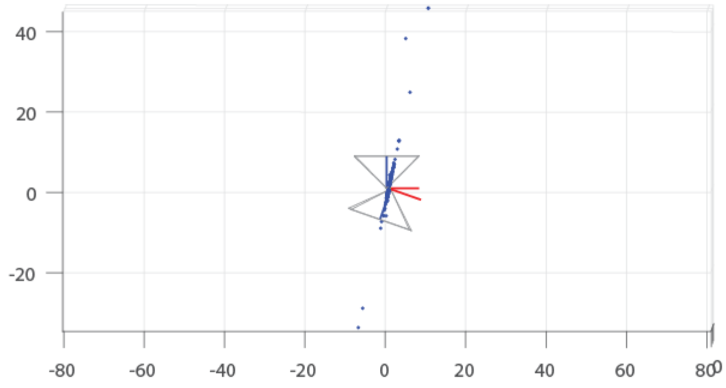
$$\mathbf{R} = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix}$$



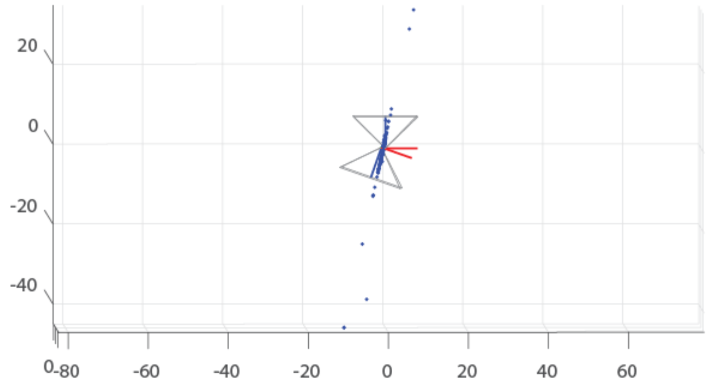
(a) nValid = 10



(b) nValid = 488



(c) nValid = 0



(d) nValid = 0