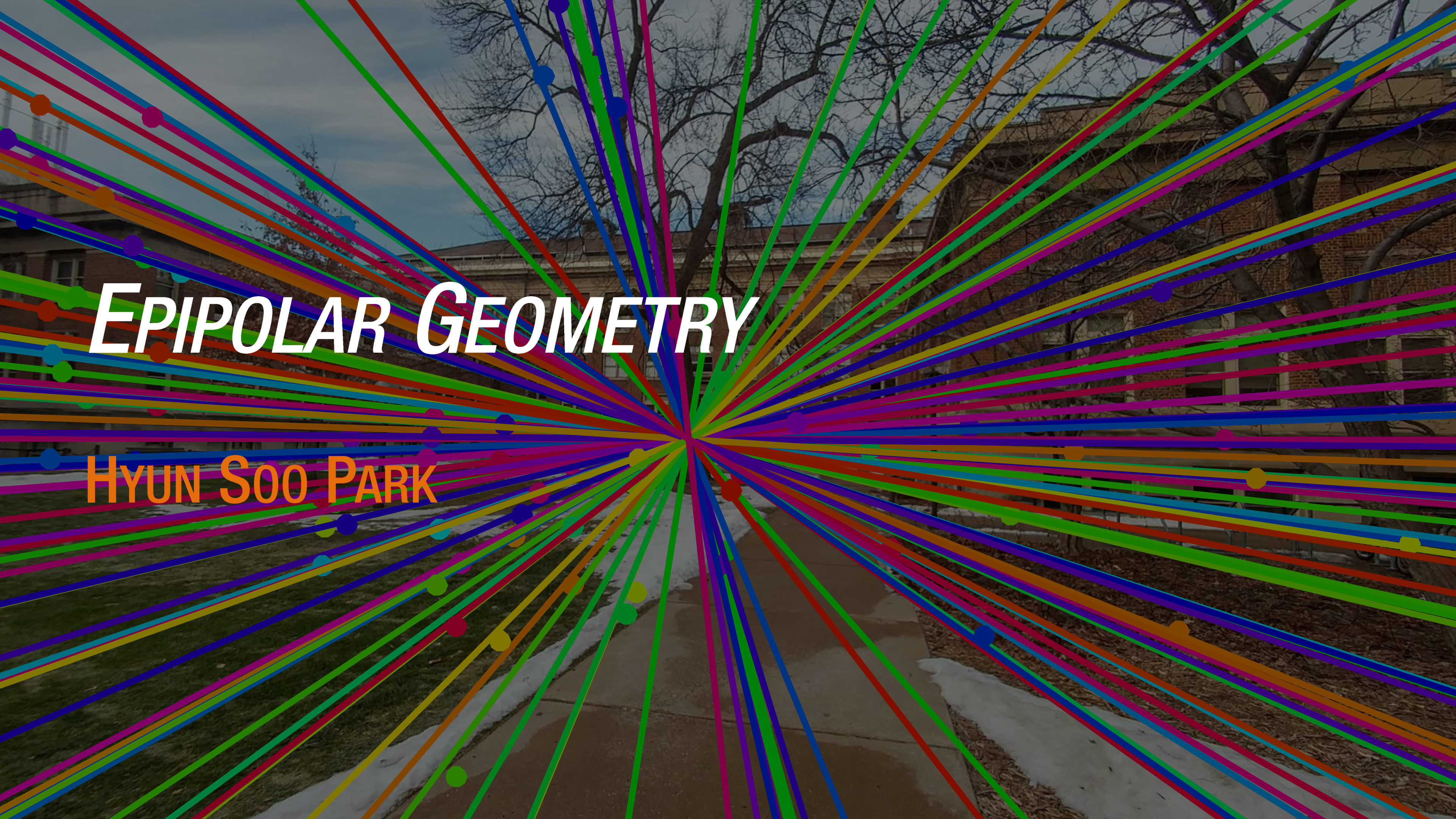
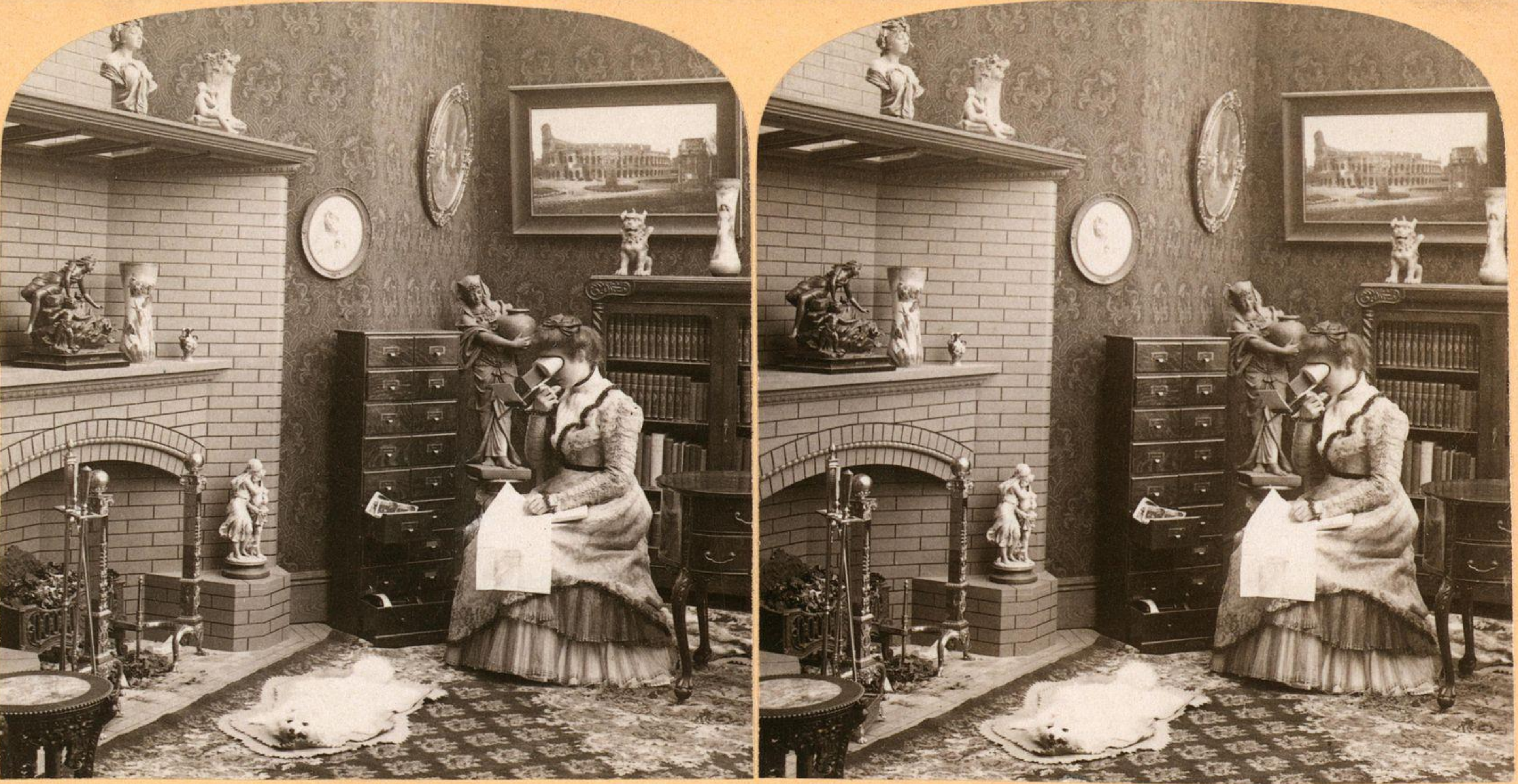


EPIPOLAR GEOMETRY

HYUN SOO PARK



The Stereograph as an Educator—Underwood Patent Extension Cabinet in a home Library. (2

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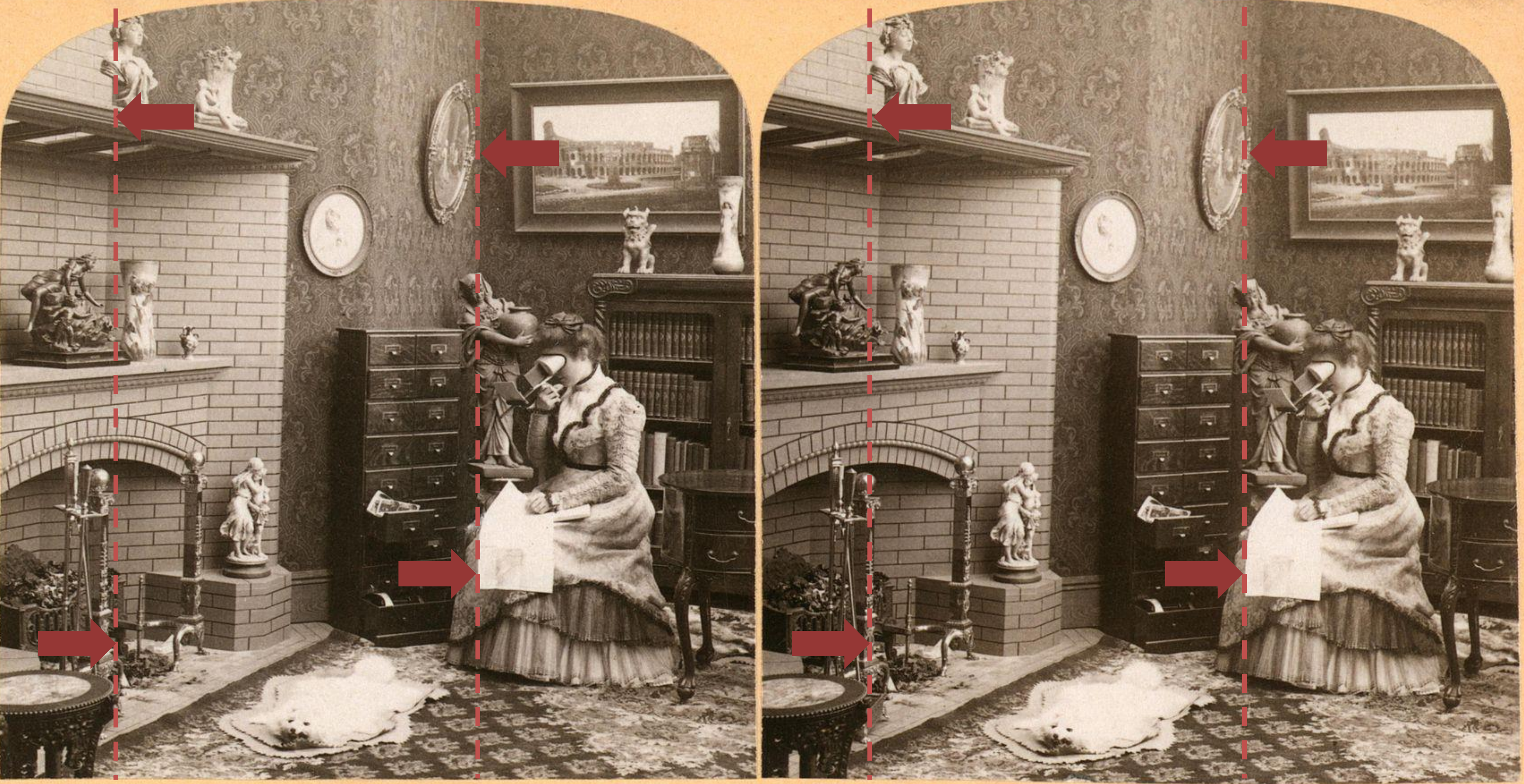

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HERO3
mode
HERO3


VITA COCO
pure coconut water
Naked
3.99

Naked
3.99


VITA
Naked
Naked
Naked
Naked
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Naked
3.99

Left image (Bob)

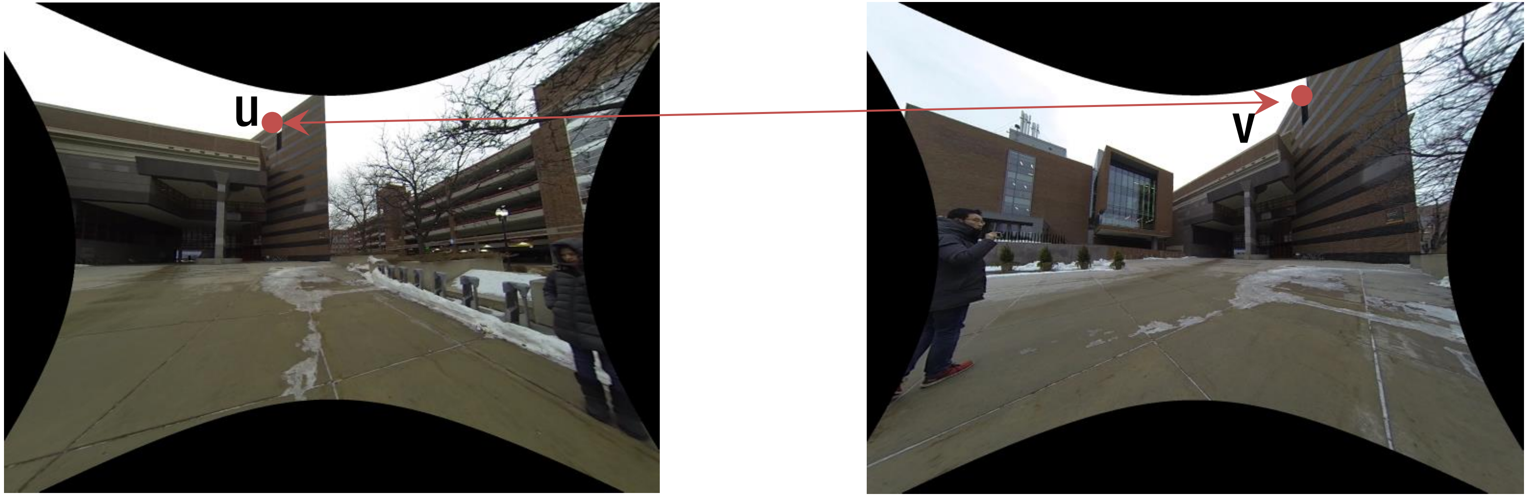
Right image (Alice)

2D CORRESPONDENCE

Left image (Bob)

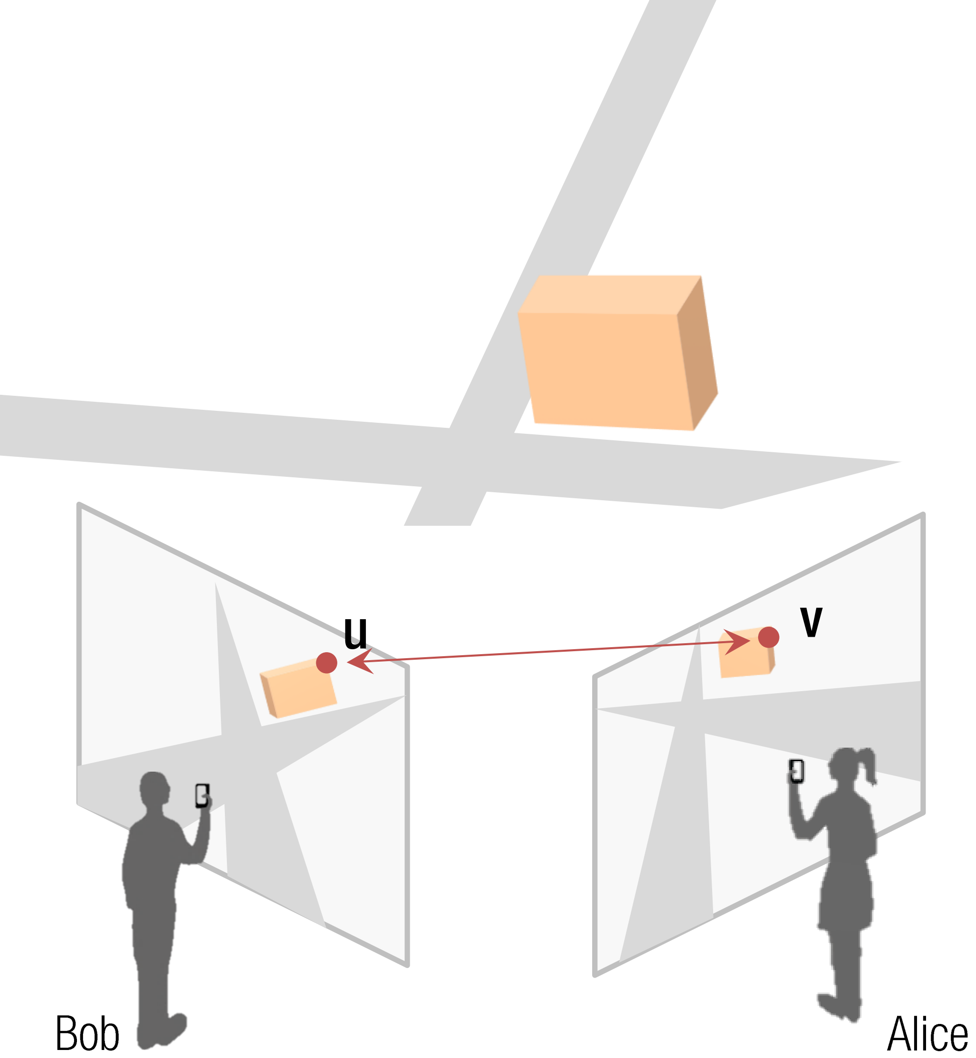
Right image (Alice)

2D Correspondence



Left image (Bob)

Right image (Alice)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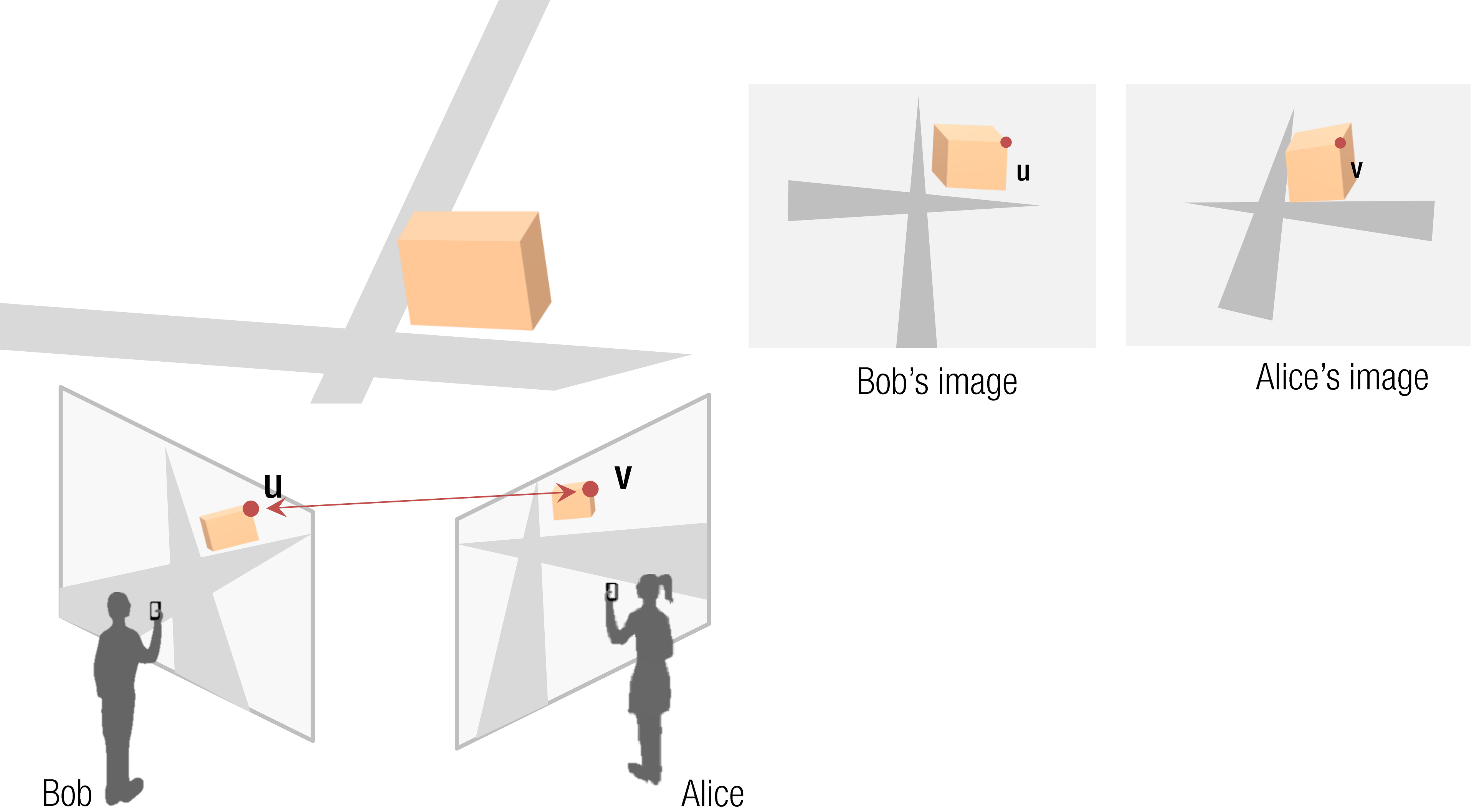
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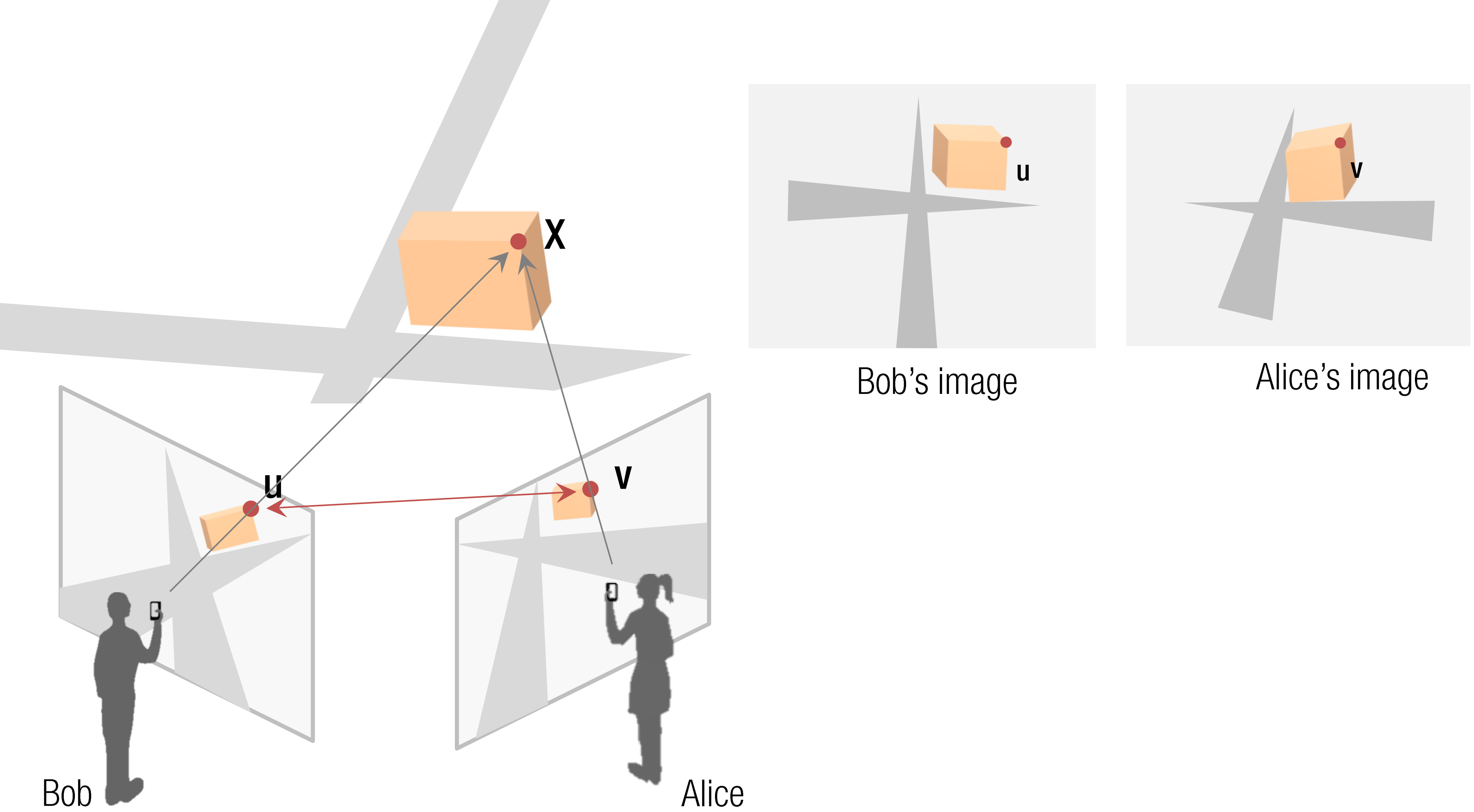

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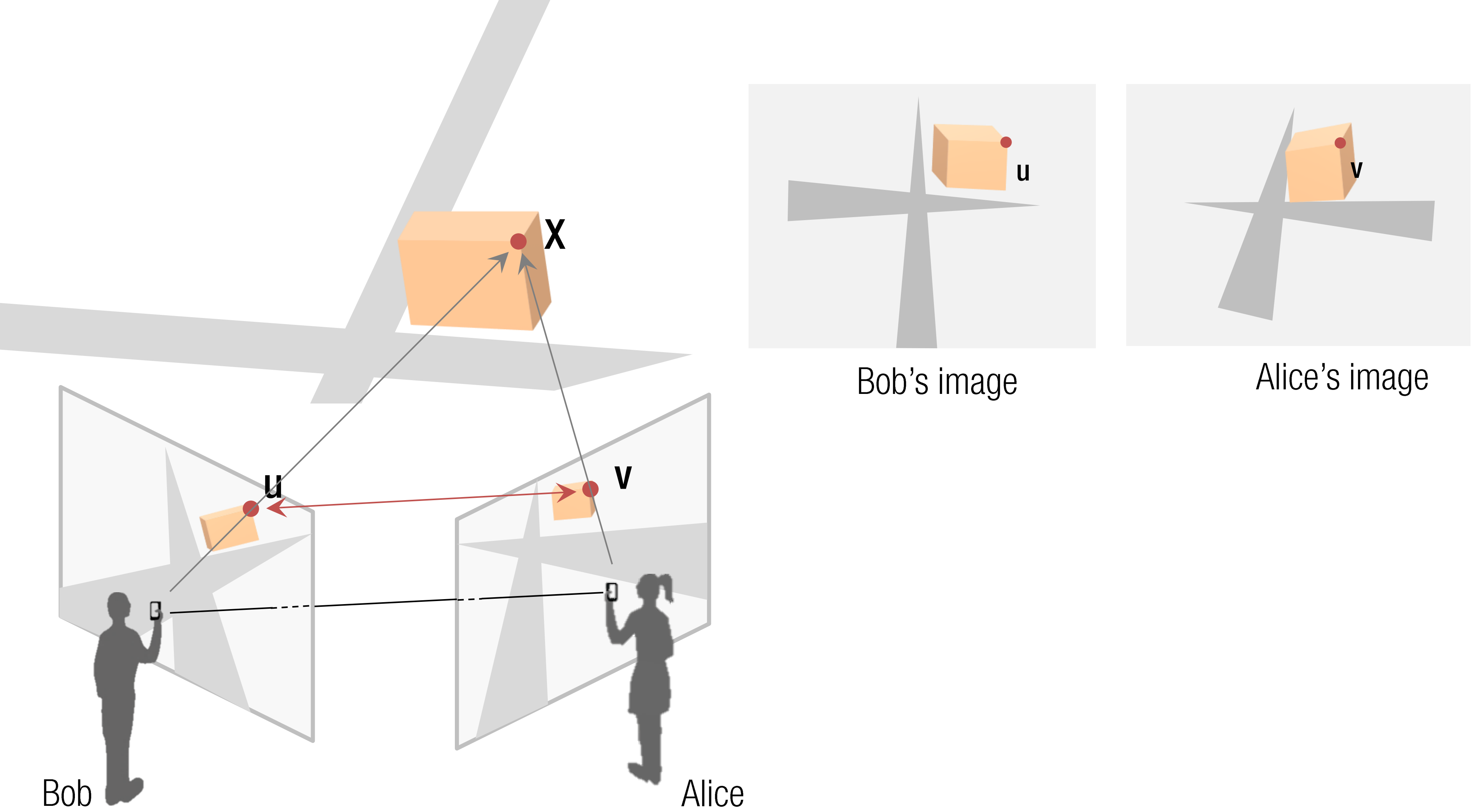
Alice's image



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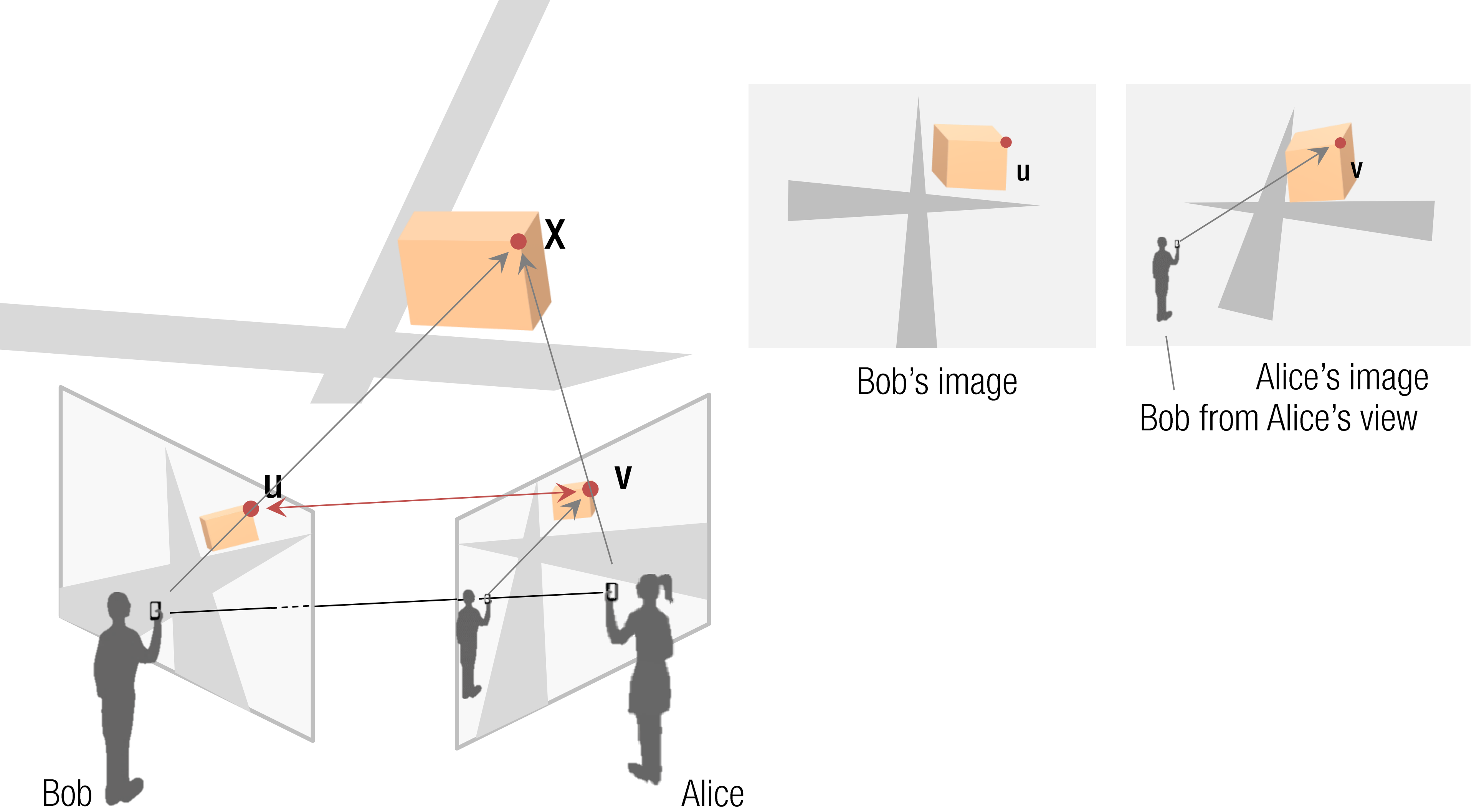
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Bob's image
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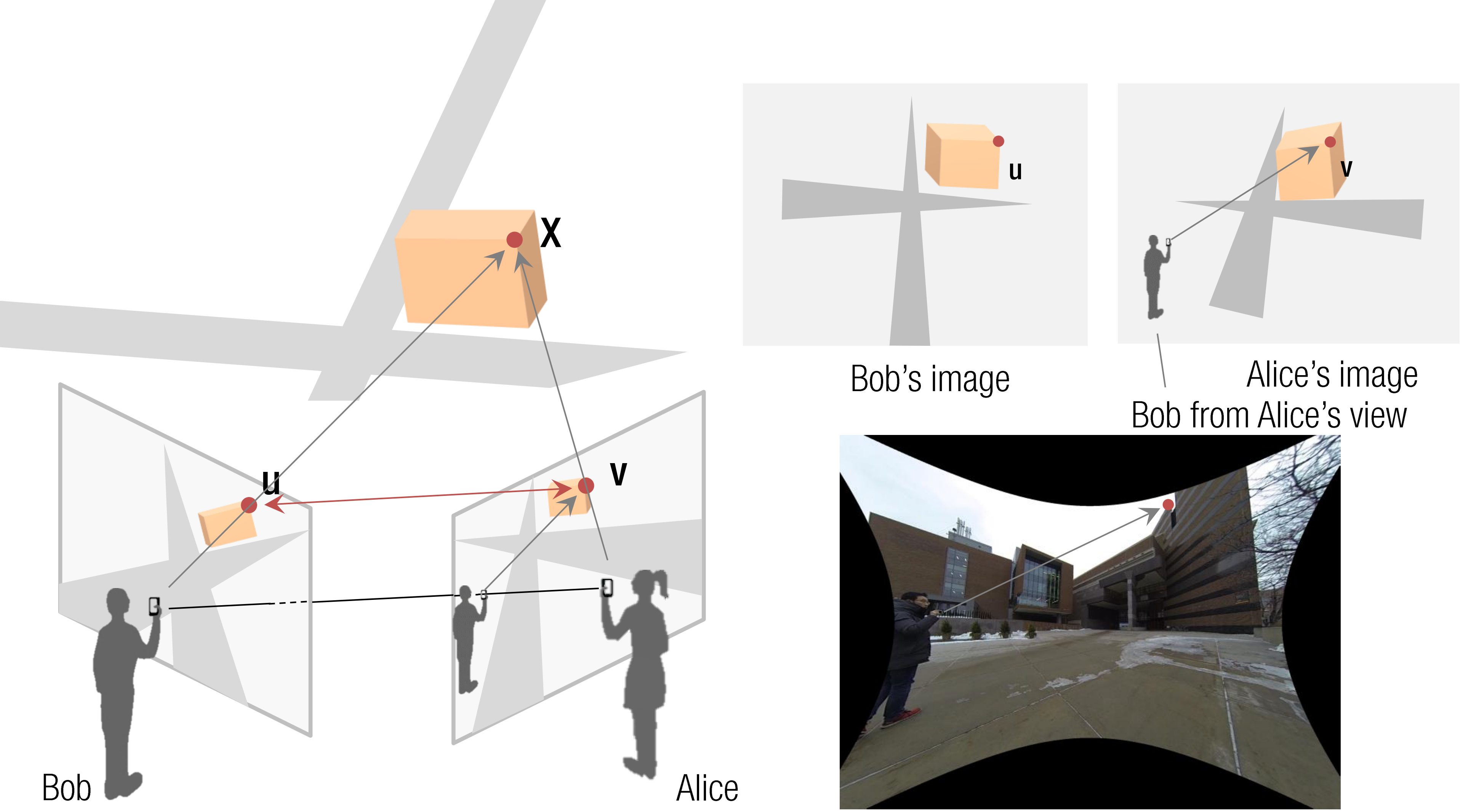
Alice's image
Bob from Alice's view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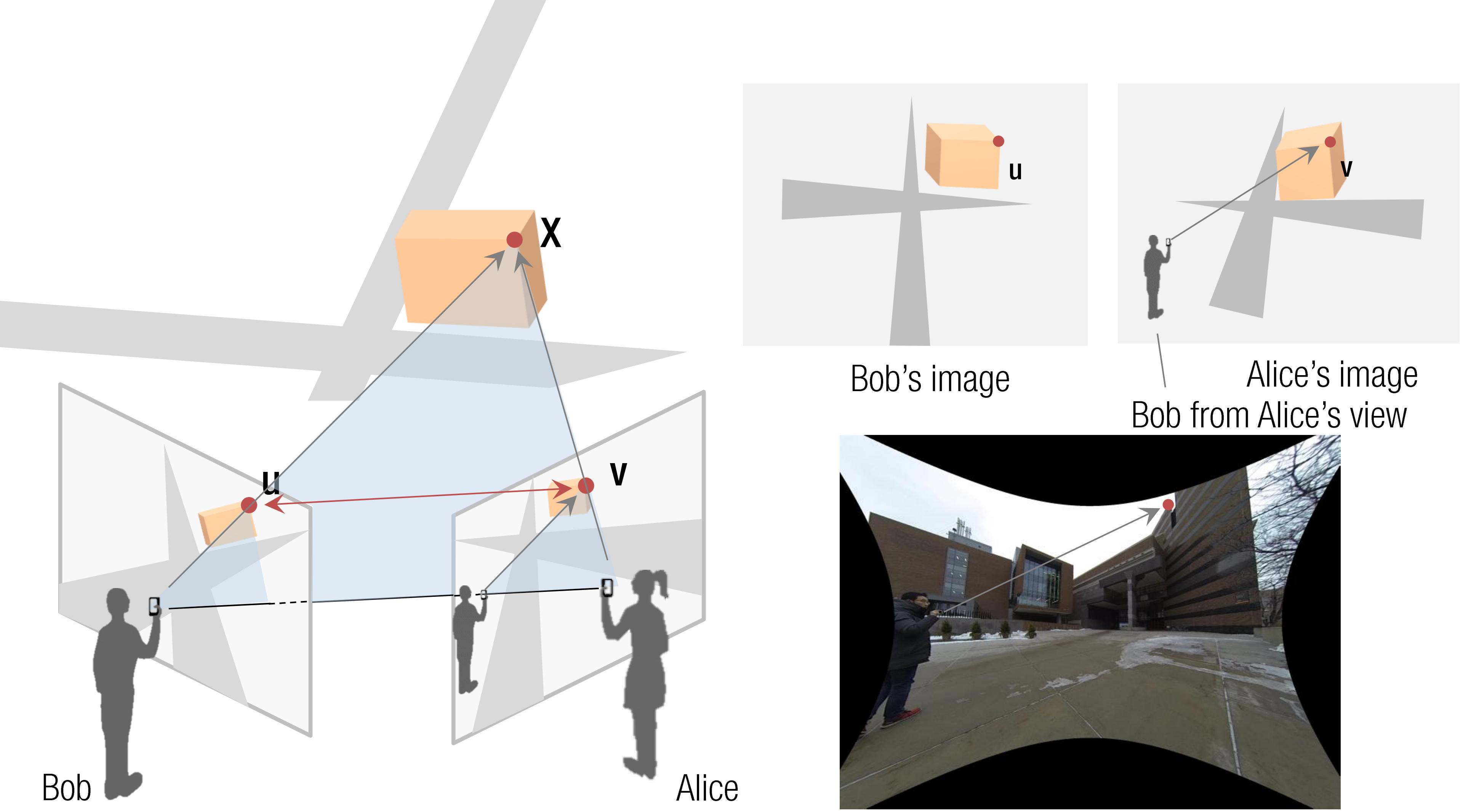
Alice's image
Bob from Alice's view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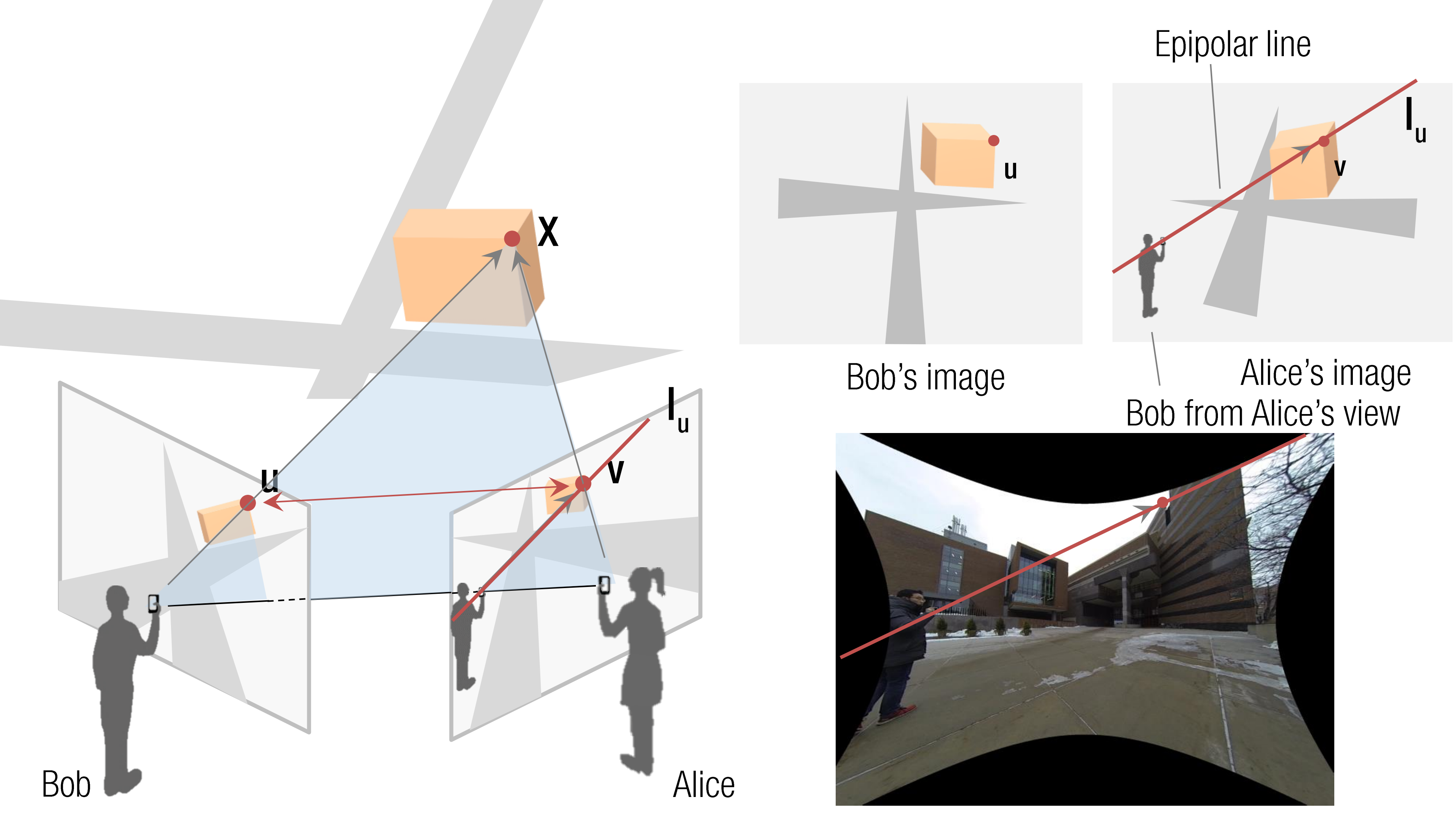
Alice's image
Bob from Alice's view


X
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Epipolar line
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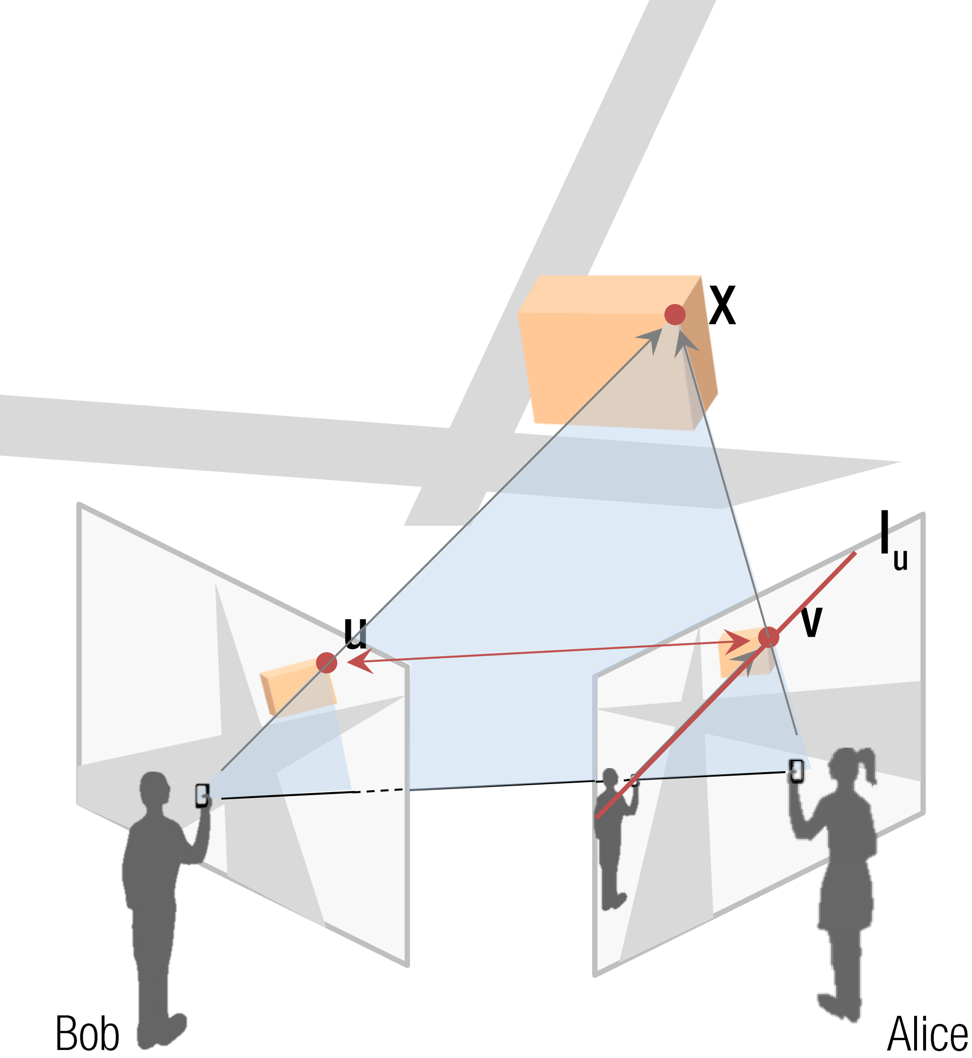
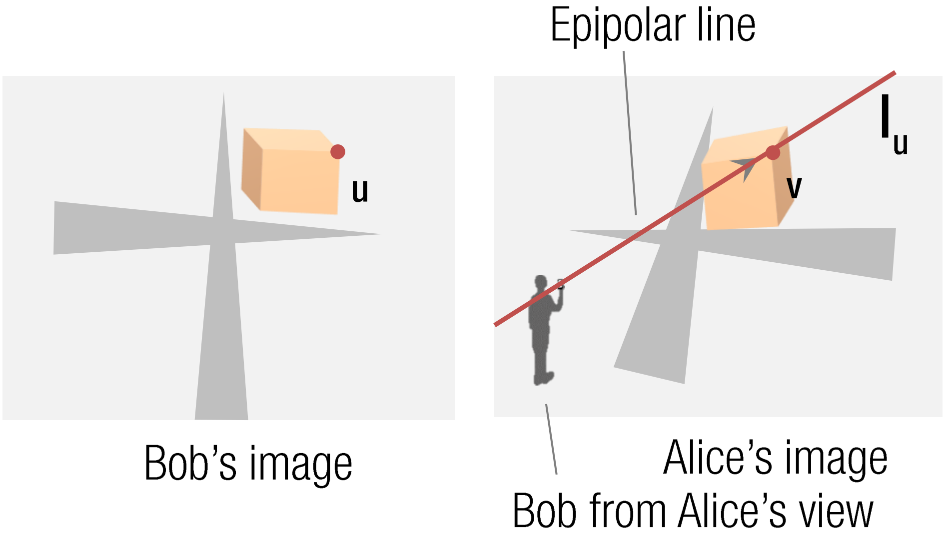
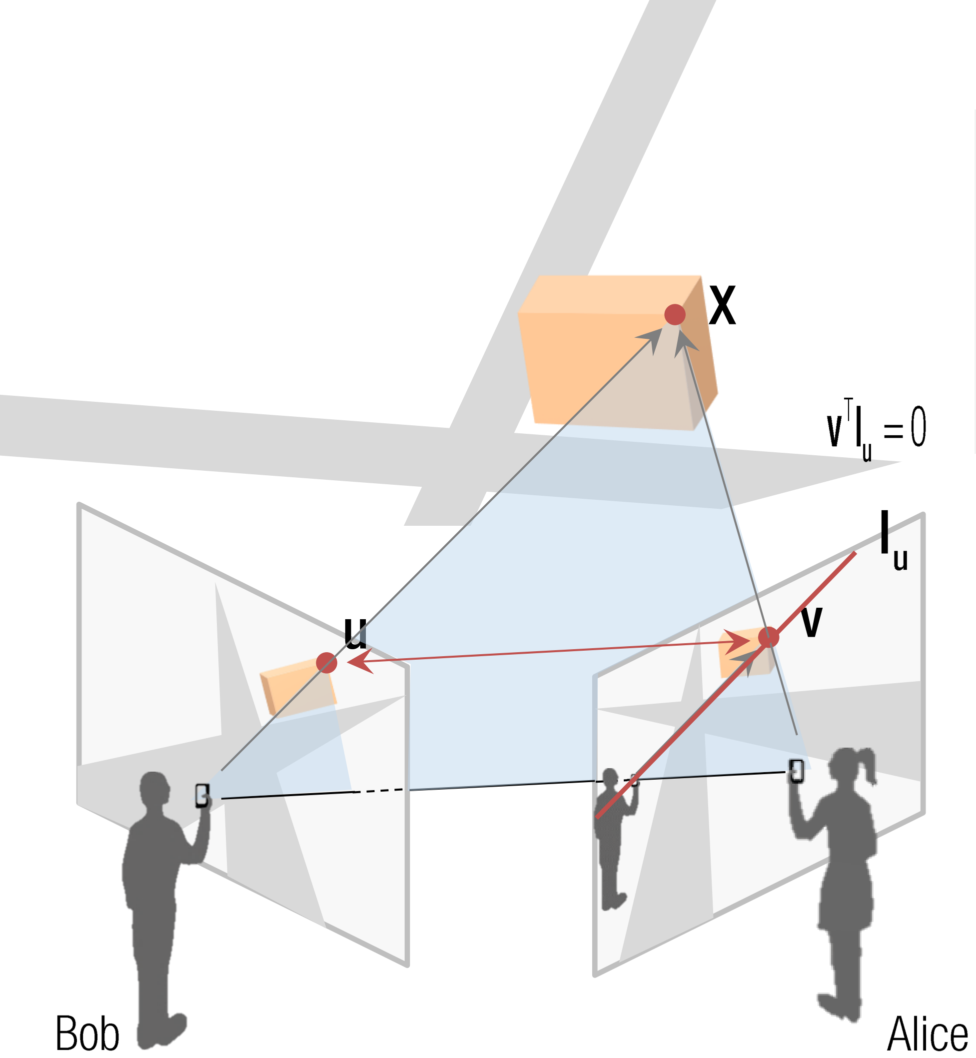
Bob's image
Alice's image
Bob from Alice's view

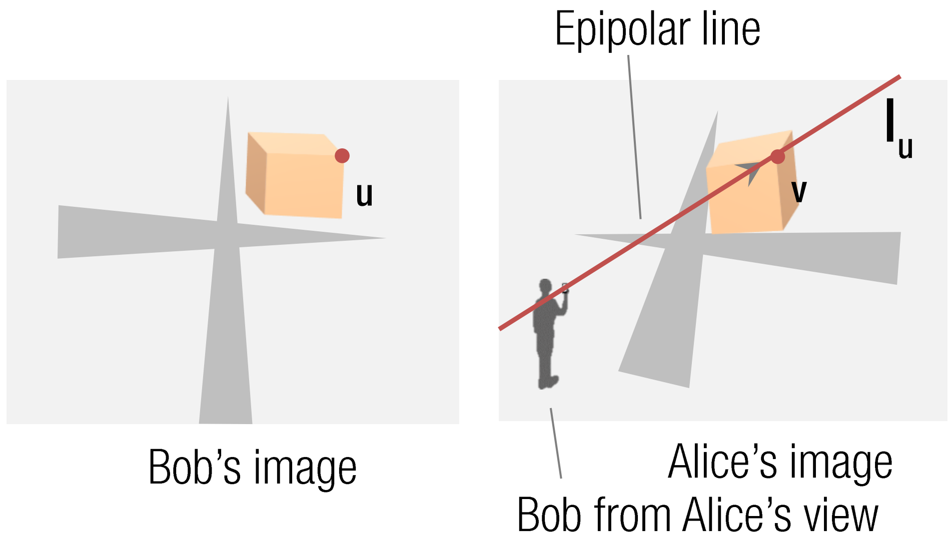




Epipolar constraint between two images:

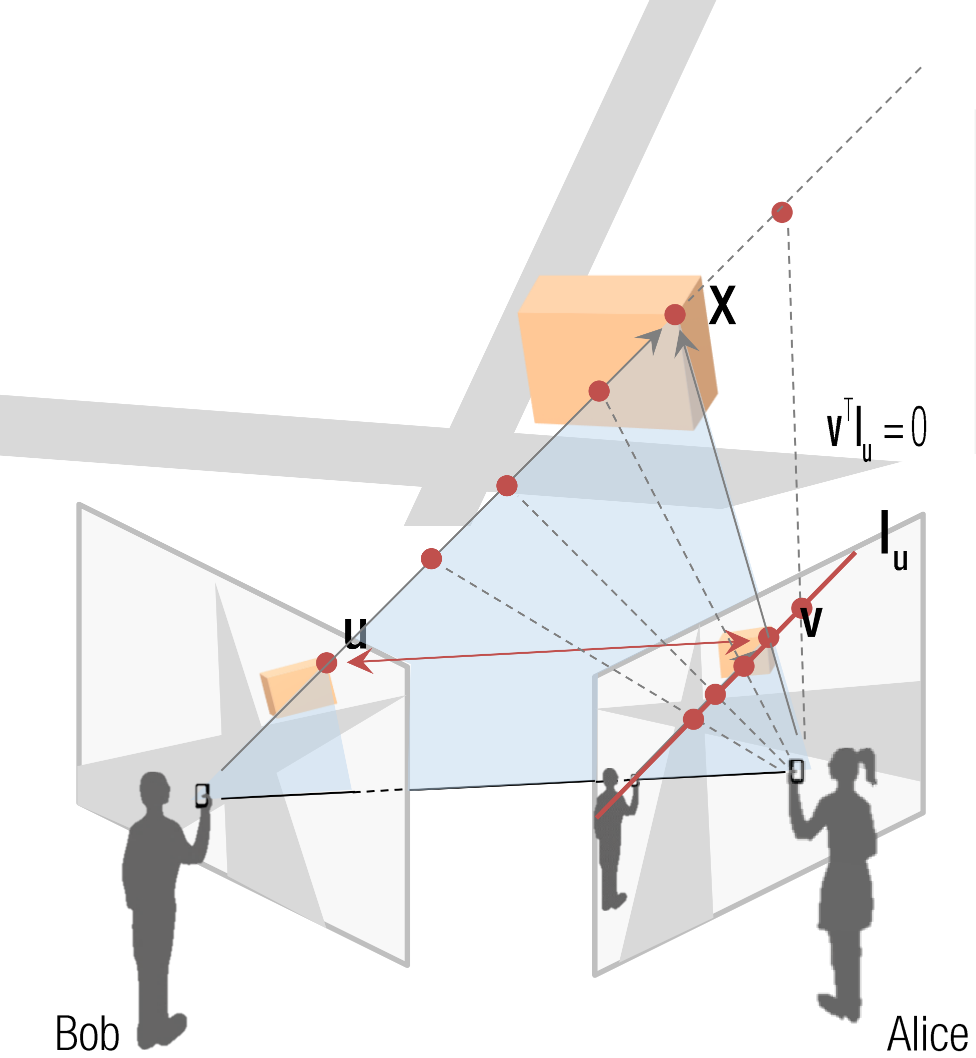
1. A point, **u**, in Bob's image corresponds to an epipolar line l_u in Alice's image.

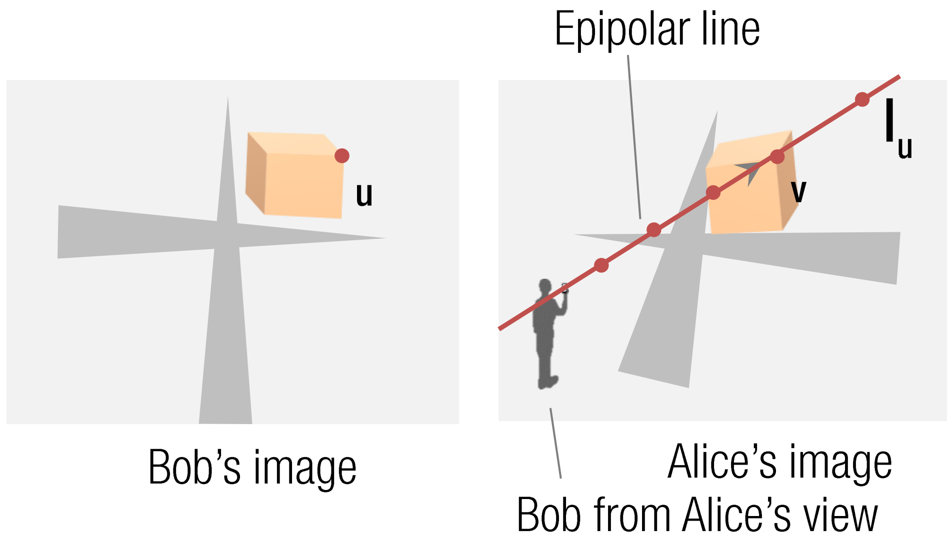




Epipolar constraint between two images:

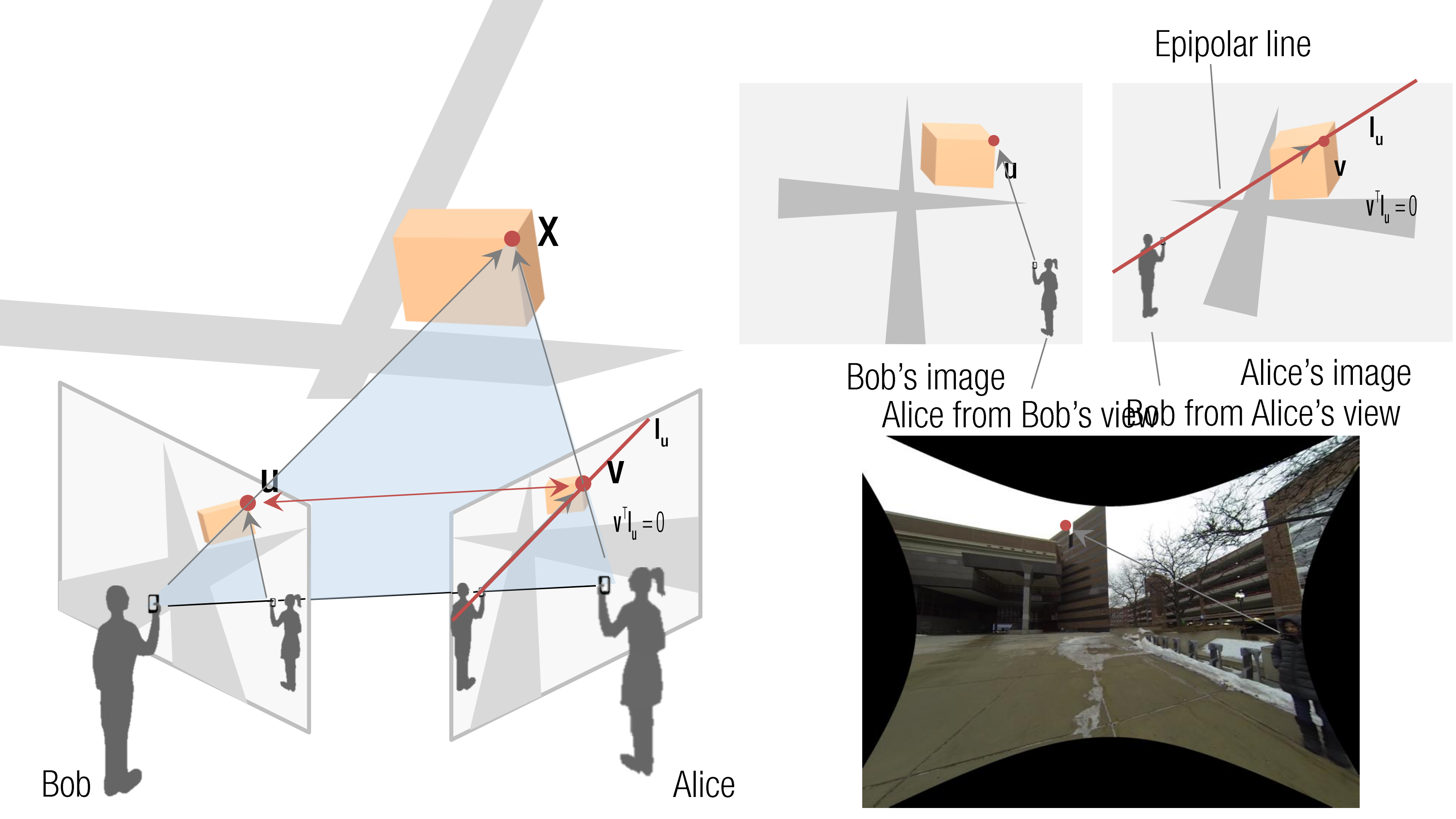
1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$

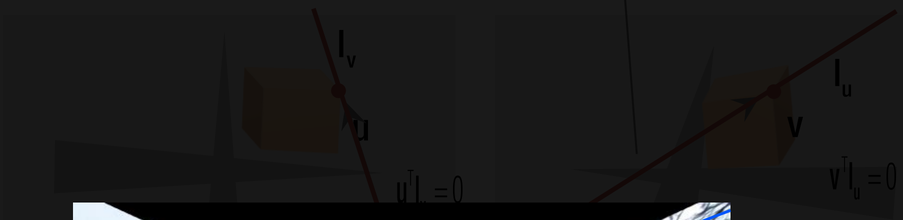


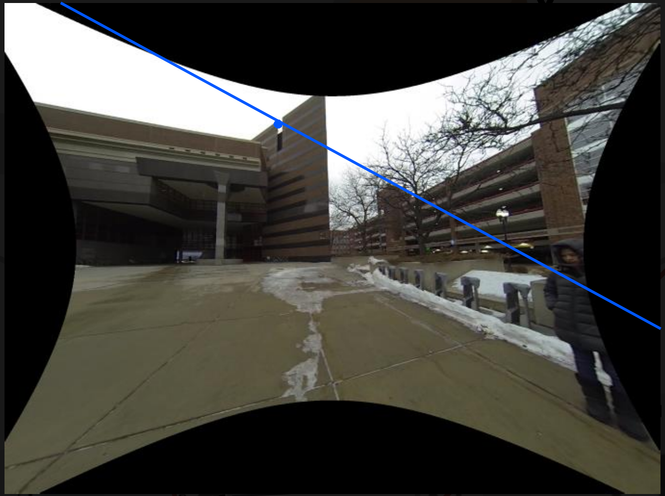


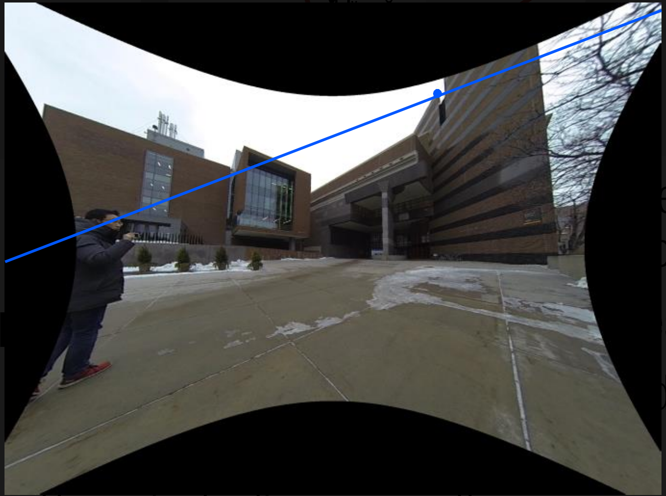
Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.


X
u
v
l_u
$v^T l_u = 0$
Bob
Alice
Epipolar line
u
l_u
v
$v^T l_u = 0$
Bob's image
Alice's image
Alice from Bob's view
Bob from Alice's view







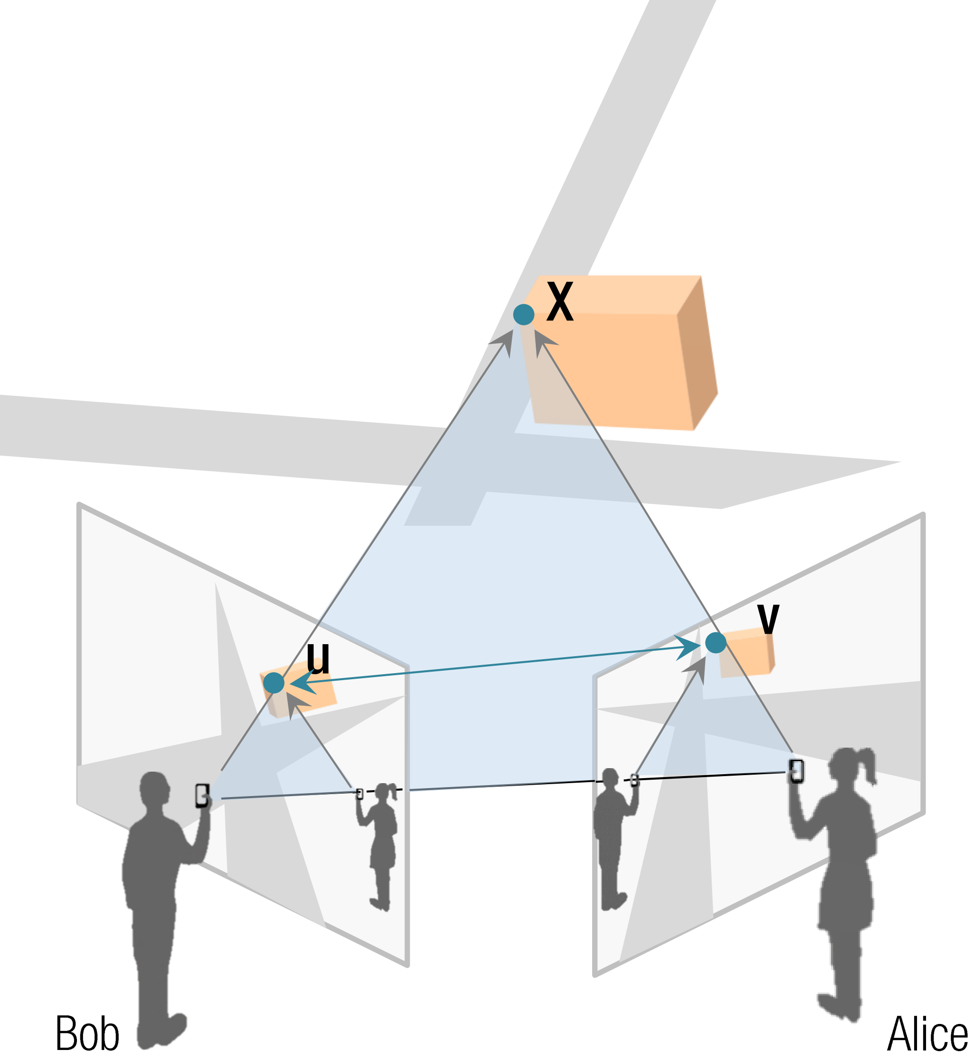
…e's image
…ce's view

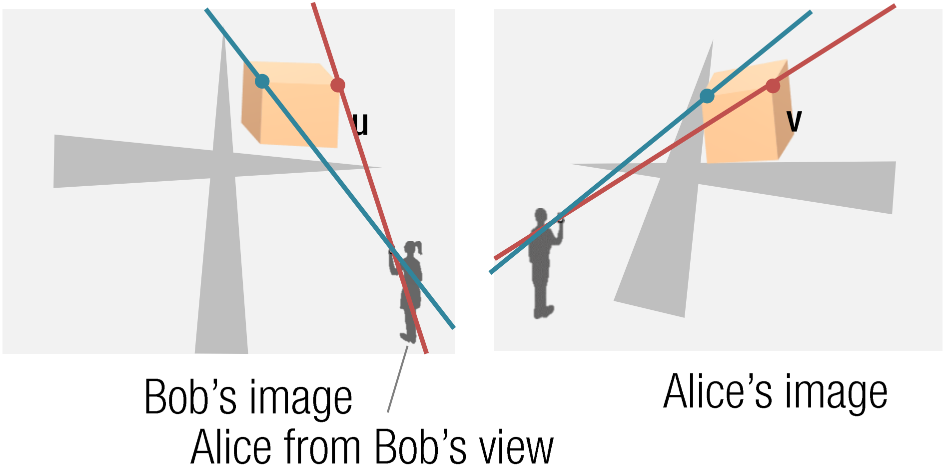
Ep… …s:

1. … …ds to an
2. The epipolar line passes the corresponding point in Alice's image, **v**:
3. Any point along the epipolar line can be a candidate of correspondences.

Bob

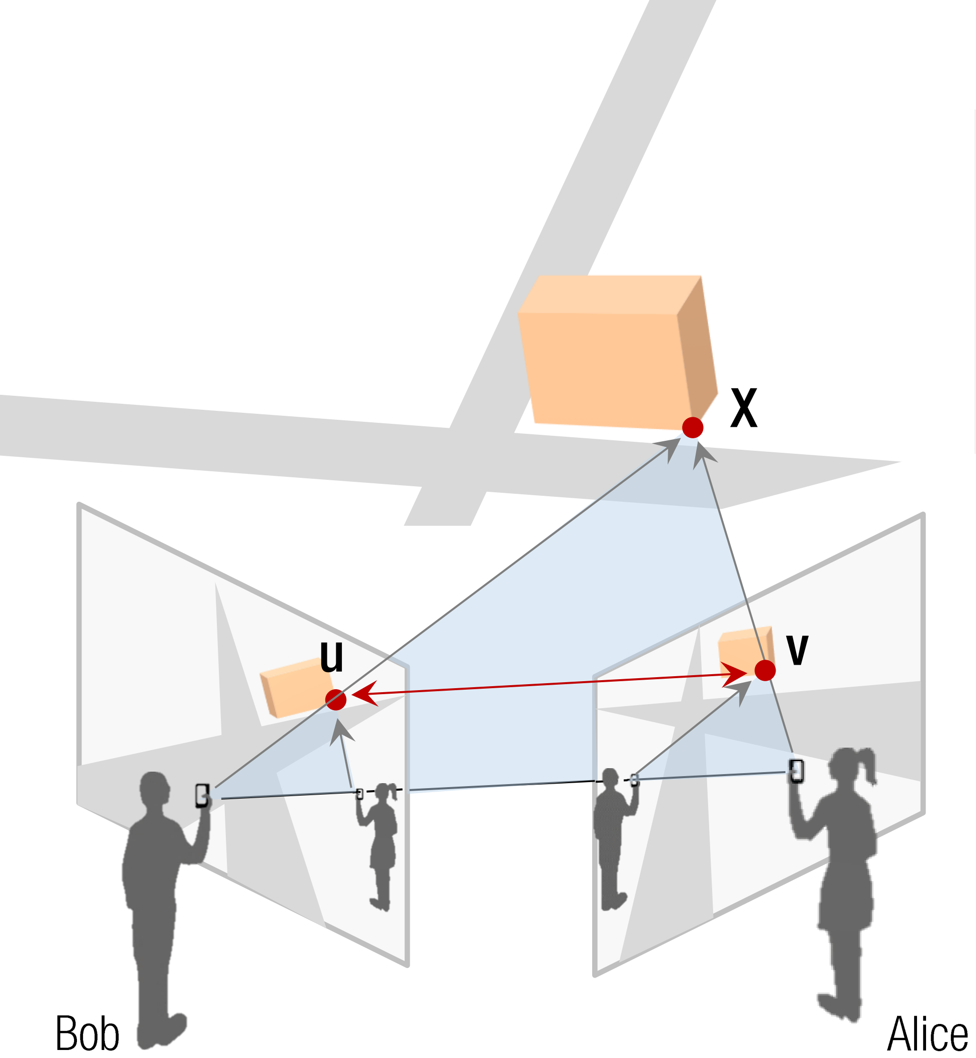
Alice

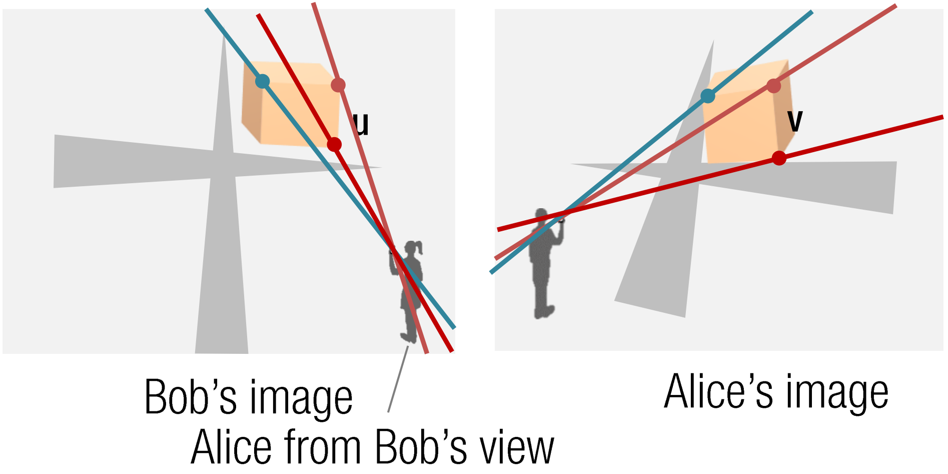




Epipolar constraint between two images:

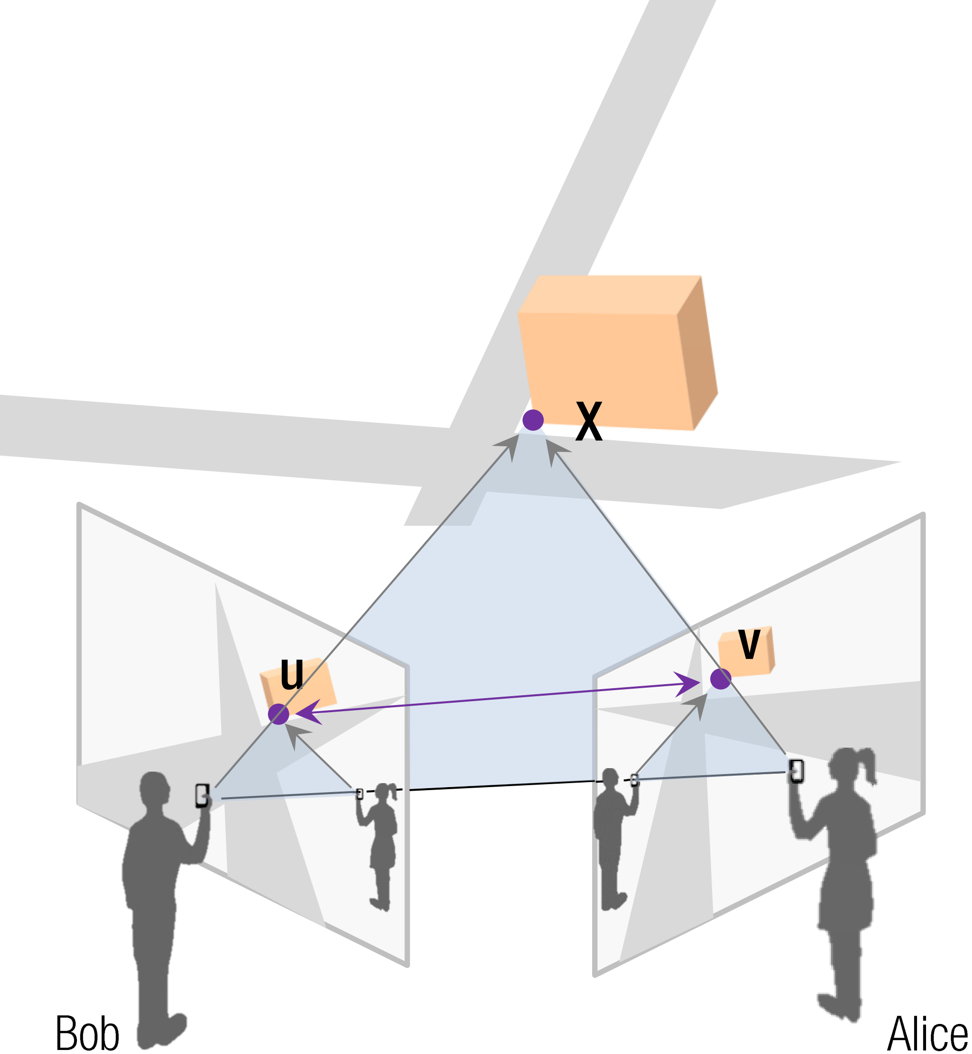
1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

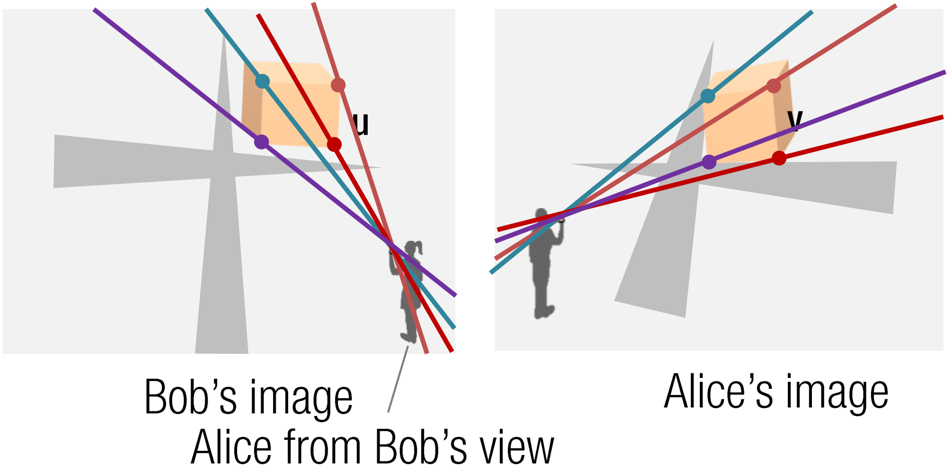




Epipolar constraint between two images:

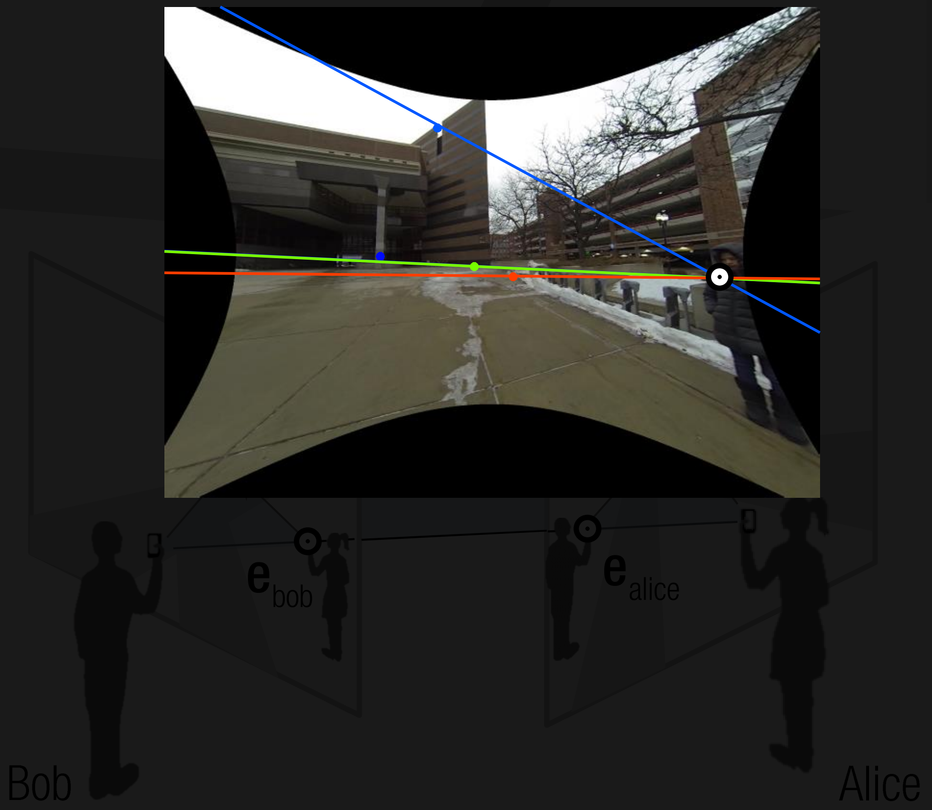
1. A point, **u**, in Bob's image corresponds to an epipolar line l_u in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

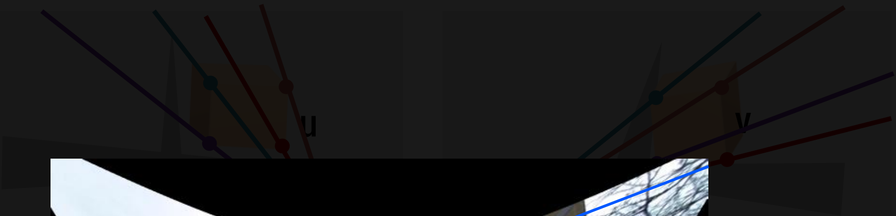


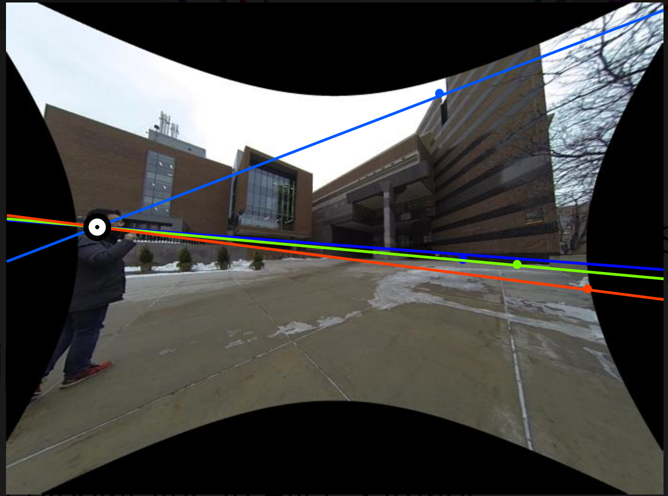


Epipolar constraint between two images:

1. A point, **u**, in Bob's image corresponds to an epipolar line $\mathbf{l}_u$ in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $\mathbf{v}^T\mathbf{l}_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $\mathbf{e}_{bob}^T\mathbf{l}_u = 0$ $\mathbf{e}_{alice}^T\mathbf{l}_v = 0$



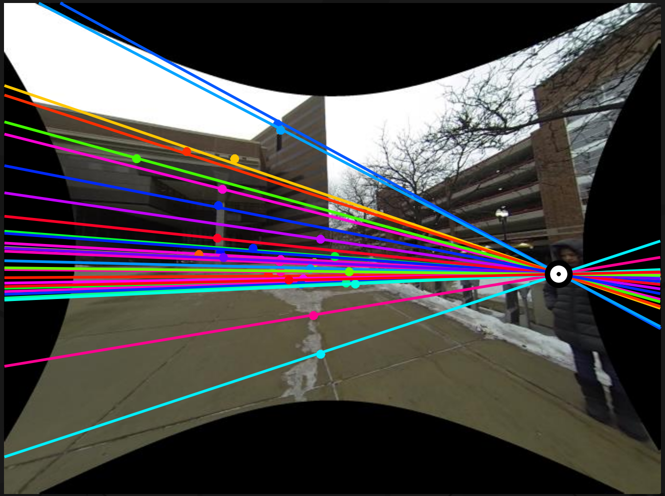


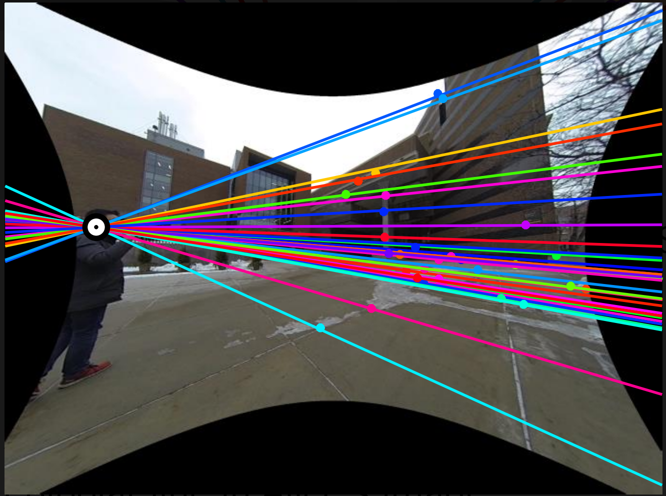


s image
's view

Epi

1. ... to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**: $v^T l_u = 0$ $u^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epiploar lines meet at the epipole: $e_{bob}^T l_u = 0$ $e_{alice}^T l_v = 0$





…s image
…'s view

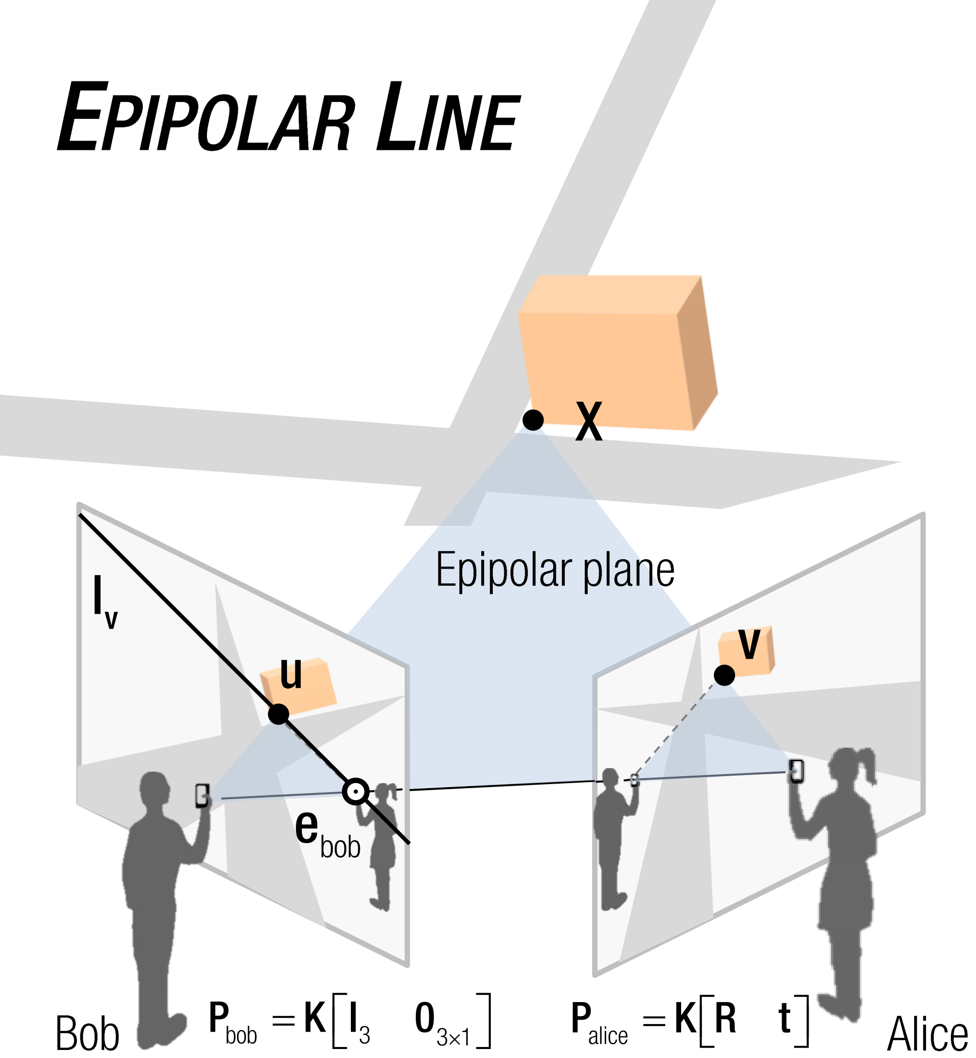
Epi…

1. … to an epipolar line in Alice's image.
2. The epipolar line passes the corresponding point in Alice's image, **v**:

   $\mathbf{v}^T l_u = 0 \quad \mathbf{u}^T l_v = 0$
3. Any point along the epipolar line can be a candidate of correspondences.

   $\mathbf{e}_{bob}^T l_u = 0 \quad \mathbf{e}_{alice}^T l_v = 0$
4. Epiploar lines meet at the epipole:

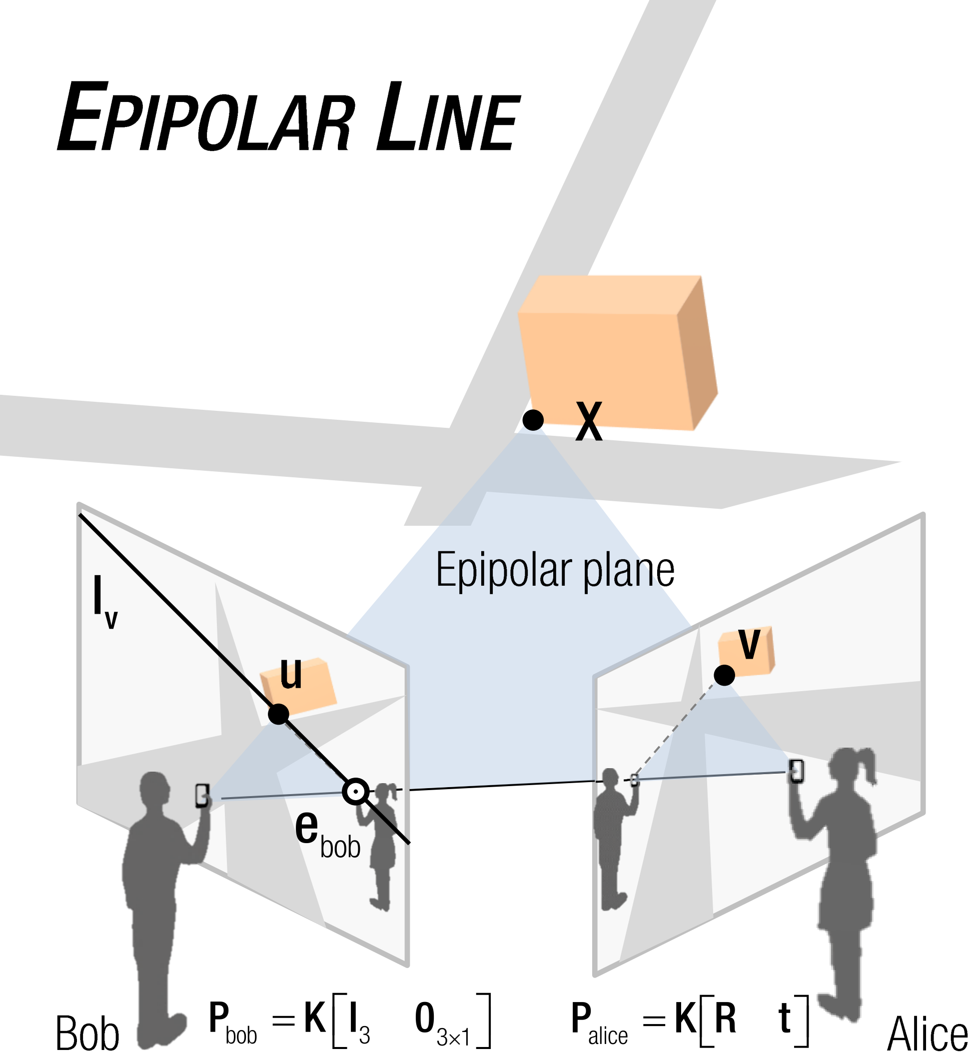
EPIPOLAR LINE



$$l_v = Fv$$

Fundamental matrix

EPIPOLAR LINE



$$l_v = Fv$$

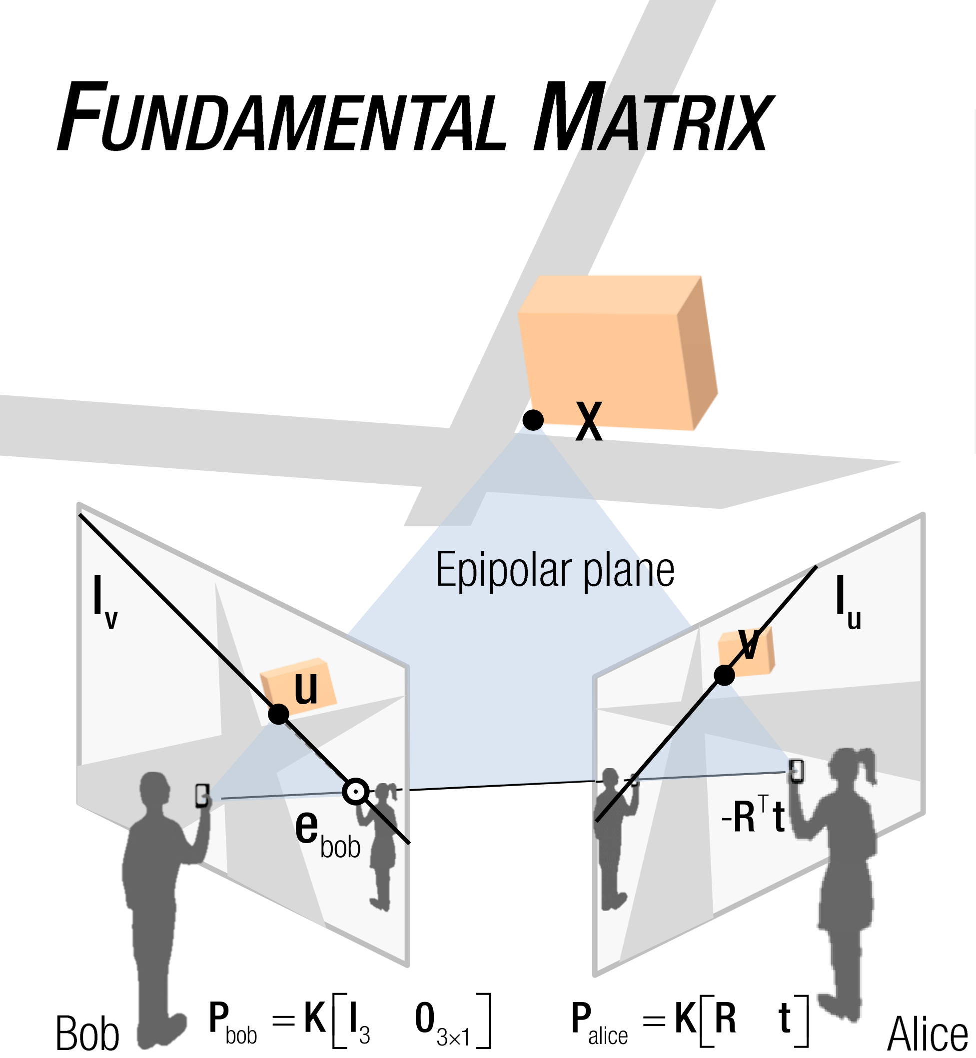
Fundamental matrix

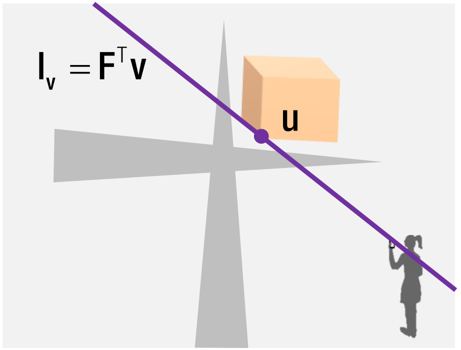
$$u^T Fv = 0$$

where $F = K^{-T}[t]_\times RK^{-1}$

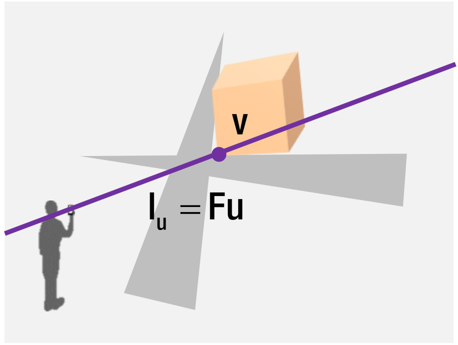
Fundamental matrix

Fundamental Matrix





Bob's image



Alice's image

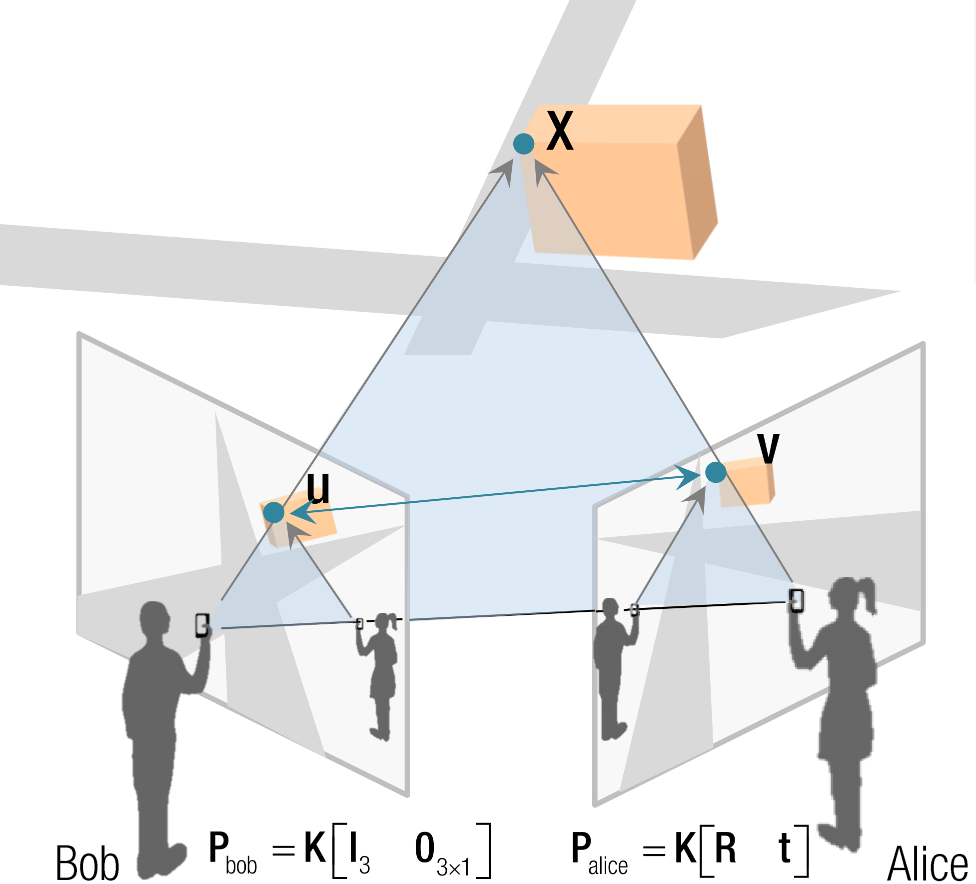
$$\mathbf{v}^T \mathbf{l}_u = \mathbf{v}^T \underline{\mathbf{K}^{-T} [\mathbf{t}]_\times \mathbf{R} \mathbf{K}^{-1}} \mathbf{u} = 0$$

Common for all points

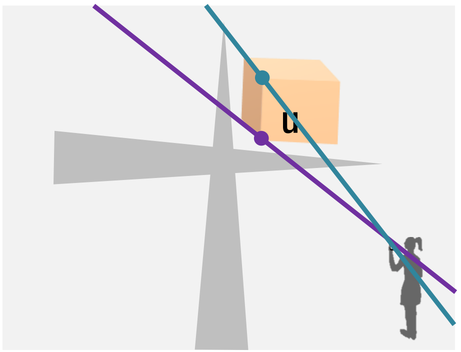
$$= \mathbf{v}^T \mathbf{F} \mathbf{u} = 0$$

$$= \mathbf{v}^T (\mathbf{F}\mathbf{u}) = \mathbf{u}^T (\mathbf{F}^T \mathbf{v}) = 0$$

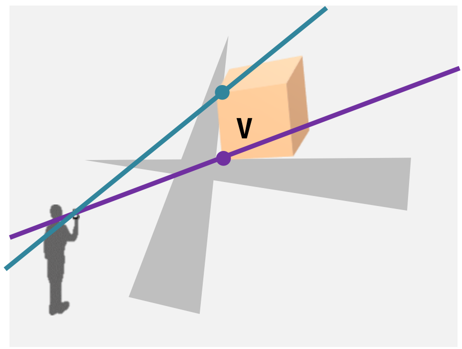
FUNDAMENTAL MATRIX



$$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix} \qquad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$$



Bob's image



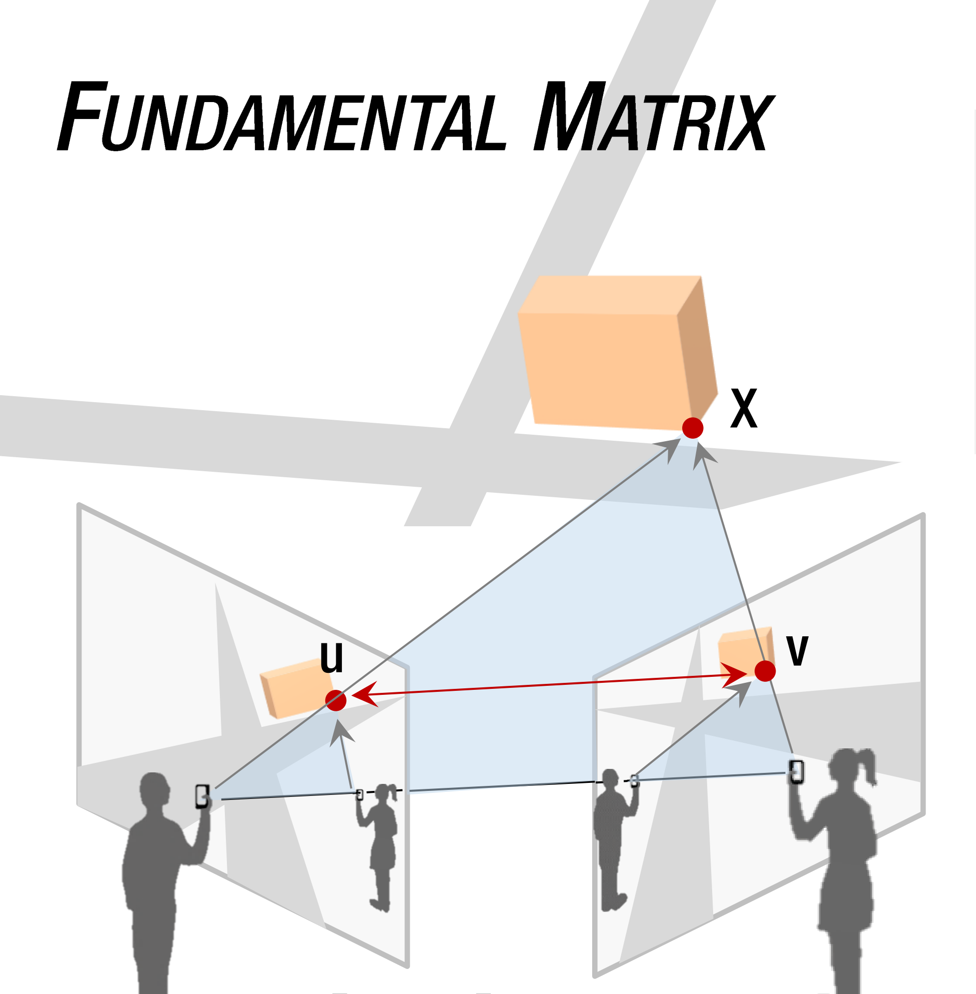
Alice's image

$$\mathbf{v}^T\mathbf{l}_u = \mathbf{v}^T \underbrace{\mathbf{K}^{-T}[\mathbf{t}]_\times \mathbf{R}}_{\text{Common for all points}} \mathbf{K}^{-1}\mathbf{u} = \mathbf{0}$$

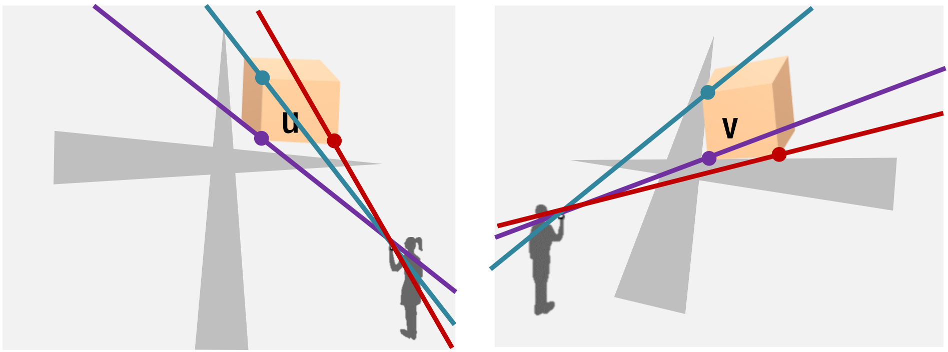
$$= \mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

$$= \mathbf{v}^T(\mathbf{F}\mathbf{u}) = \mathbf{u}^T(\mathbf{F}^T\mathbf{v}) = 0$$

FUNDAMENTAL MATRIX



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $\quad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$



Bob's image

Alice's image

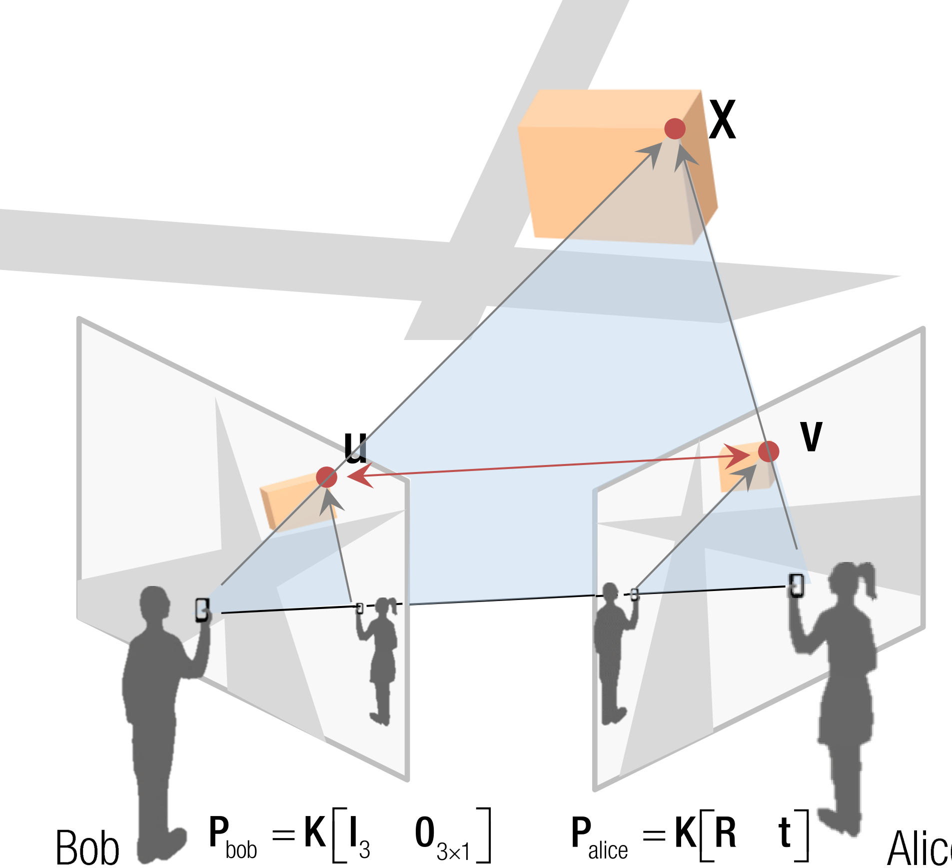
$$v^T l_u = \underline{v^T K^{-T} [t]_\times R K^{-1}} u = 0$$

Common for all points

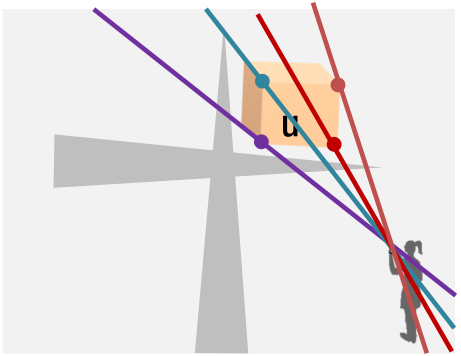
$$= v^T F u = 0$$

$$= v^T (Fu) = u^T (F^T v) = 0$$

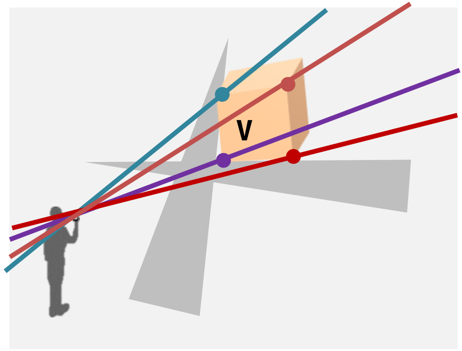
FUNDAMENTAL MATRIX



Bob $\quad P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $\quad P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$ $\quad$ Alice



Bob's image



Alice's image

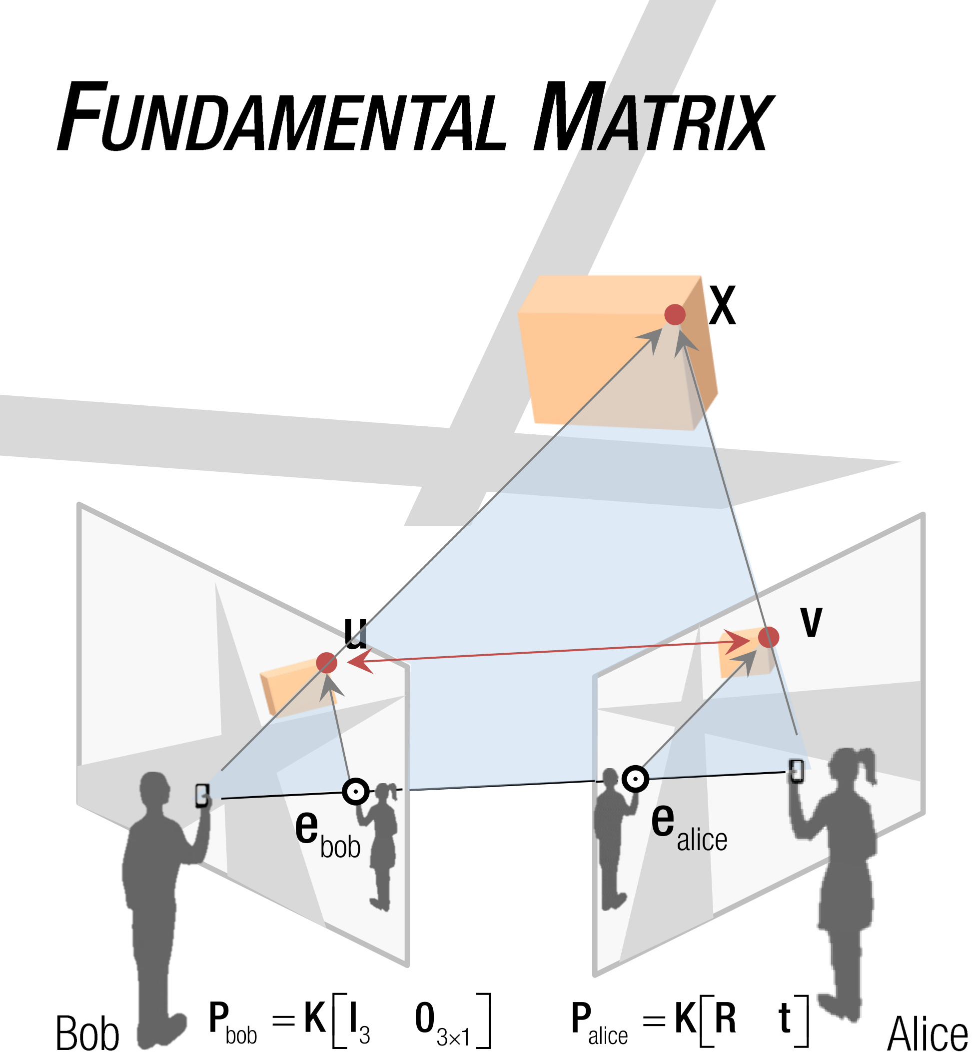
$$\mathbf{v}^{\mathsf{T}}\mathbf{l}_{\mathbf{u}} = \mathbf{v}^{\mathsf{T}}\underline{\mathbf{K}^{-\mathsf{T}}\left[\mathbf{t}\right]_{\times}\mathbf{R}\mathbf{K}^{-1}}\mathbf{u} = \mathbf{0}$$

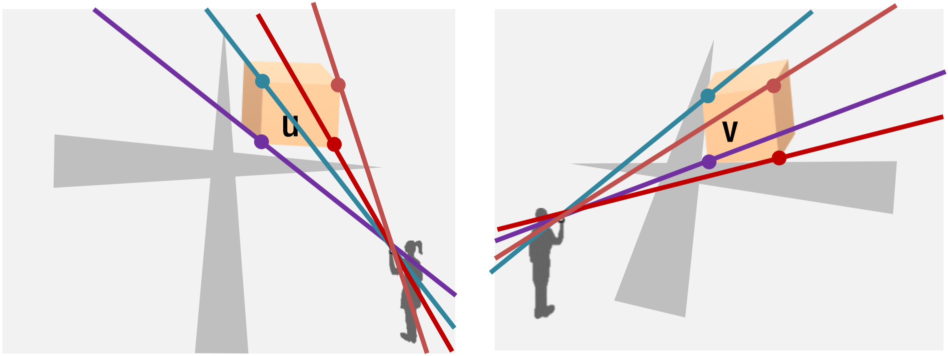
Common for all points

$$= \mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = 0$$

$$= \mathbf{v}^{\mathsf{T}}\left(\mathbf{F}\mathbf{u}\right) = \mathbf{u}^{\mathsf{T}}\left(\mathbf{F}^{\mathsf{T}}\mathbf{v}\right) = 0$$

Fundamental Matrix





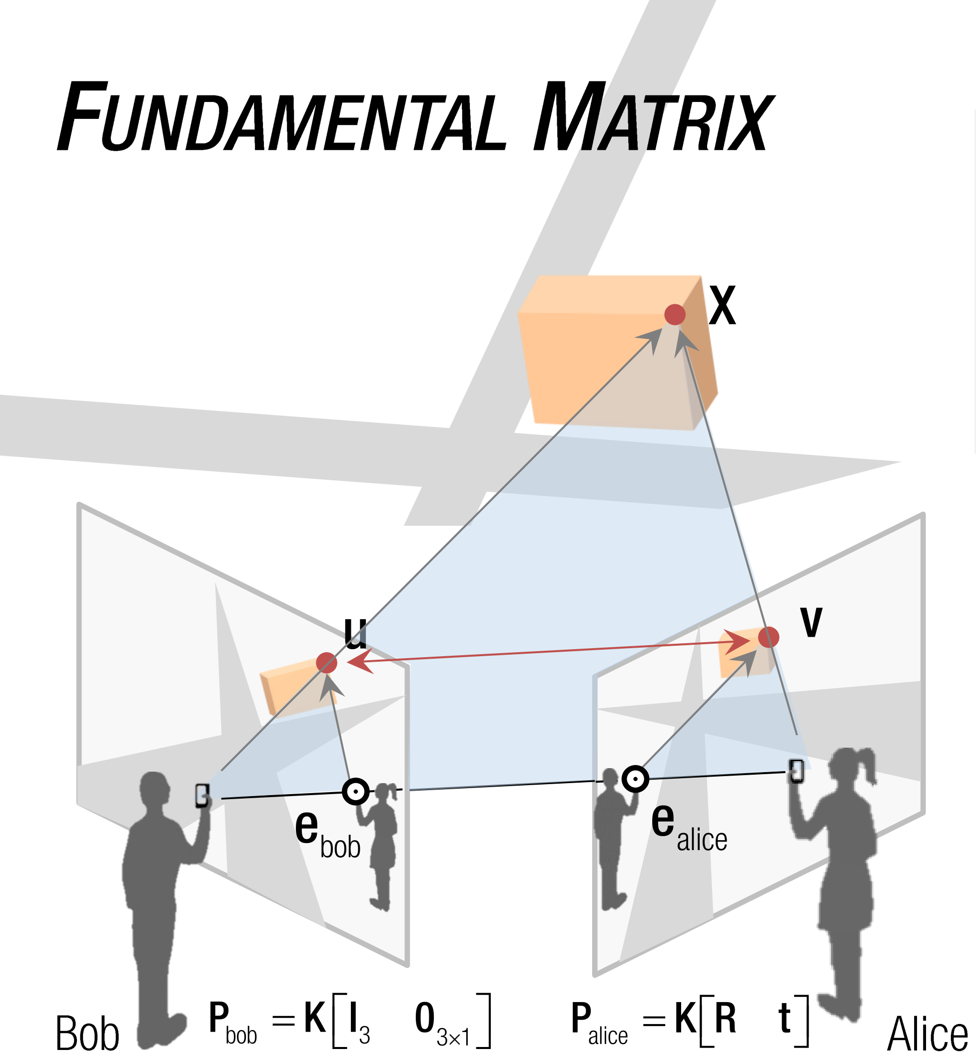
Bob's image

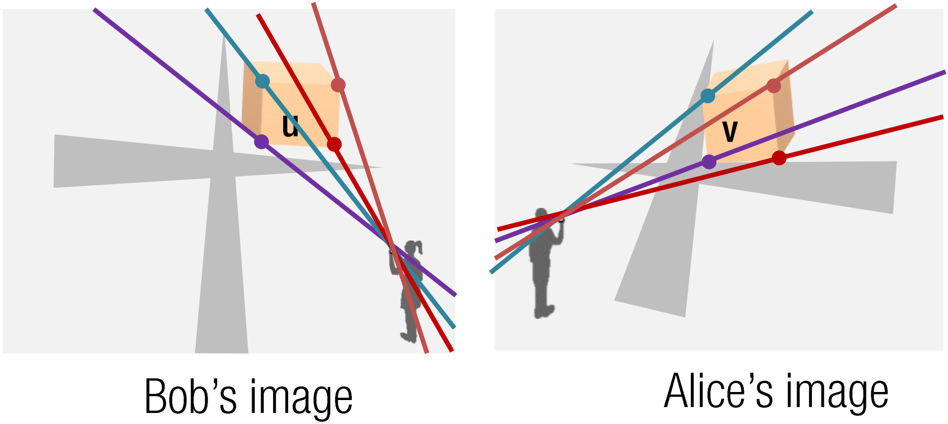
Alice's image

Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.

FUNDAMENTAL MATRIX

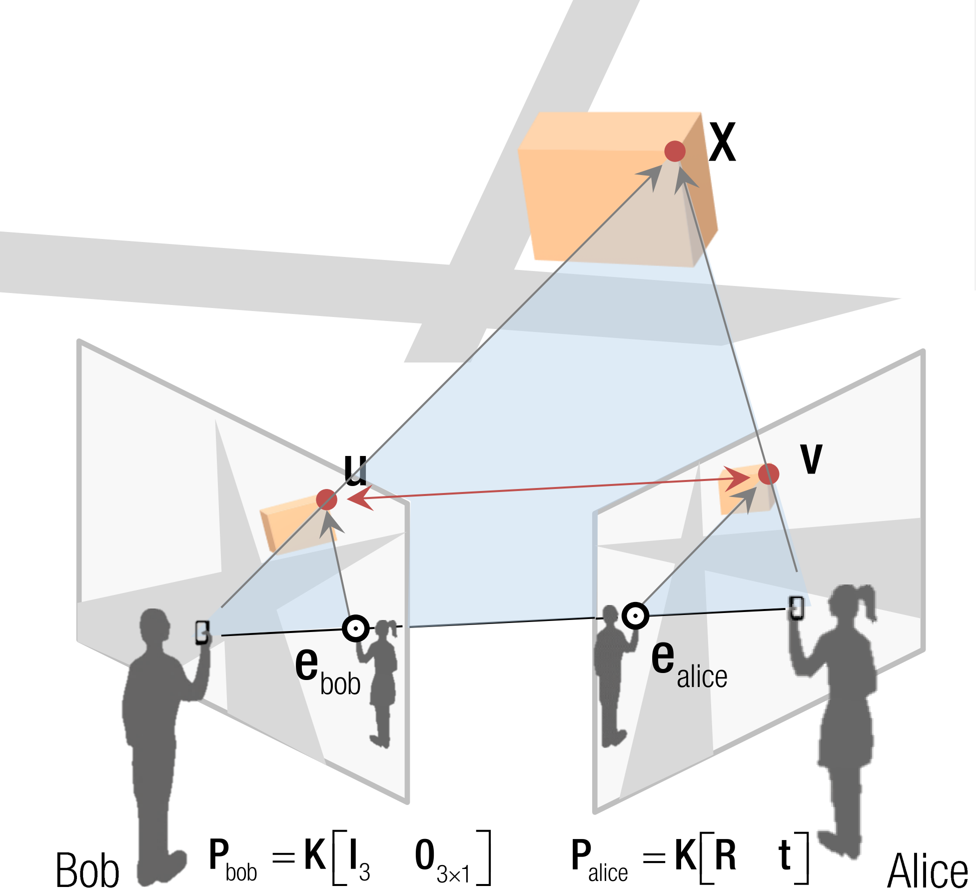




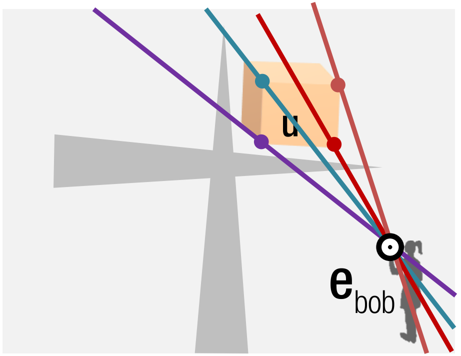
Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$

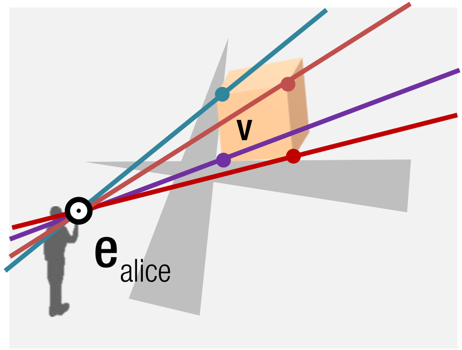
Fundamental Matrix



$P_{bob} = K\begin{bmatrix} I_3 & 0_{3\times1} \end{bmatrix}$ $P_{alice} = K\begin{bmatrix} R & t \end{bmatrix}$



Bob's image



Alice's image

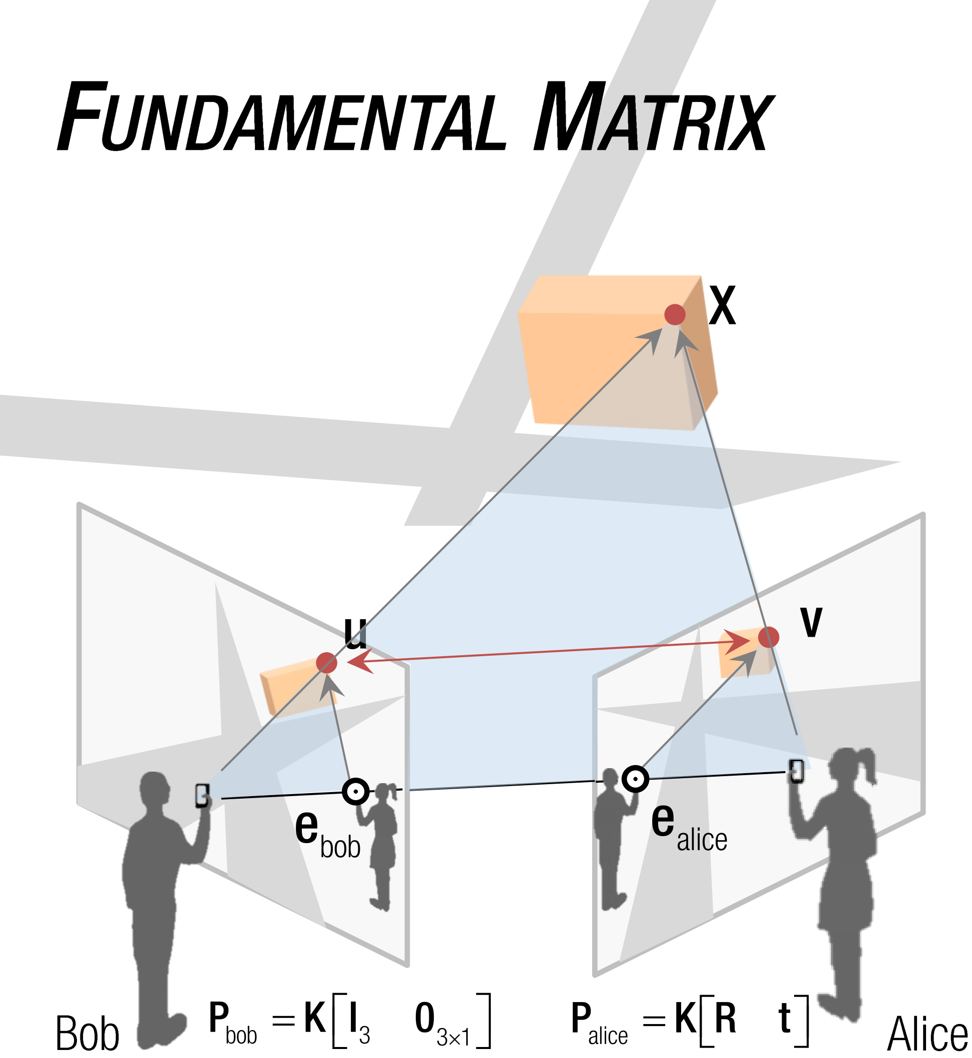
Properties of Fundamental Matrix

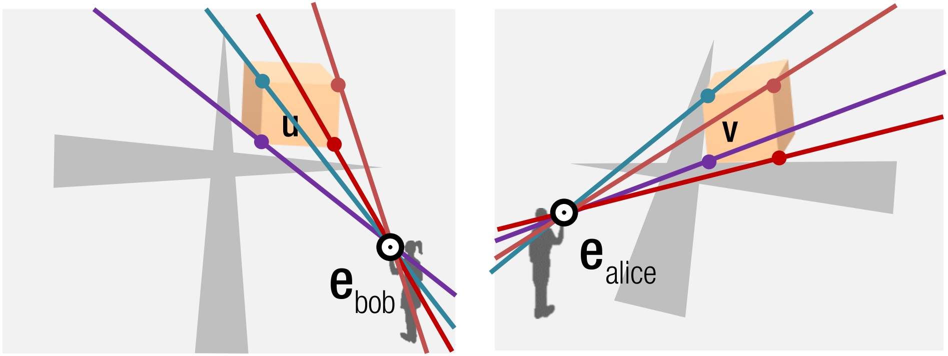
- Transpose: if $\mathbf{F}$ is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l_u} = \mathbf{Fu} \quad \mathbf{l_v} = \mathbf{F}^T\mathbf{v}$
- Epipole: $\mathbf{Fe}_{bob} = \mathbf{0} \quad \mathbf{F}^T\mathbf{e}_{alice} = \mathbf{0}$

$$\because \mathbf{v}_i^T\mathbf{Fe}_{bob} = 0, \quad \mathbf{u}_i^T\mathbf{F}^T\mathbf{e}_{alice} = 0, \quad \forall i$$

$$\longrightarrow \mathbf{e}_{bob} = \text{null}(\mathbf{F}), \quad \mathbf{e}_{alice} = \text{null}(\mathbf{F}^T)$$

FUNDAMENTAL MATRIX





Bob's image Alice's image

Properties of Fundamental Matrix

- Transpose: if **F** is for $\mathbf{P}_{bob}$, $\mathbf{P}_{alice}$, then $\mathbf{F}^T$ is for $\mathbf{P}_{alice}$, $\mathbf{P}_{bob}$.
- Epipolar line: $\mathbf{l}_u = \mathbf{F}\mathbf{u} \quad \mathbf{l}_v = \mathbf{F}^T\mathbf{v}$
- Epipole: $\mathbf{F}\mathbf{e}_{bob} = \mathbf{0} \quad \mathbf{F}^T\mathbf{e}_{alice} = \mathbf{0}$
- rank(**F**)=2:

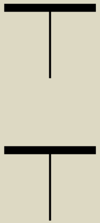
DoF 9 (3x3 matrix)-1 (scale)-1 (rank)=7

CAMERA MOTION

Camera Motion

Image 2

Image 1



Image

Image

Forward motion

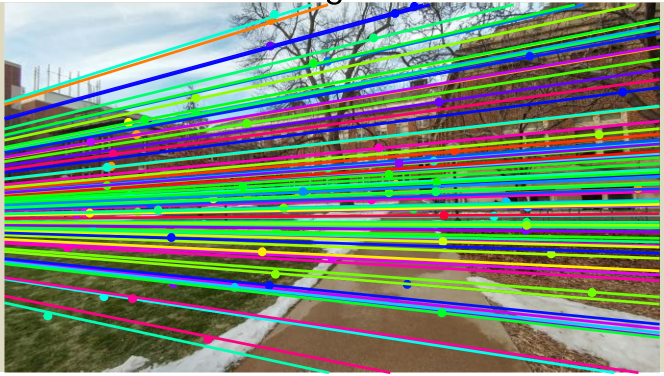


Image 2

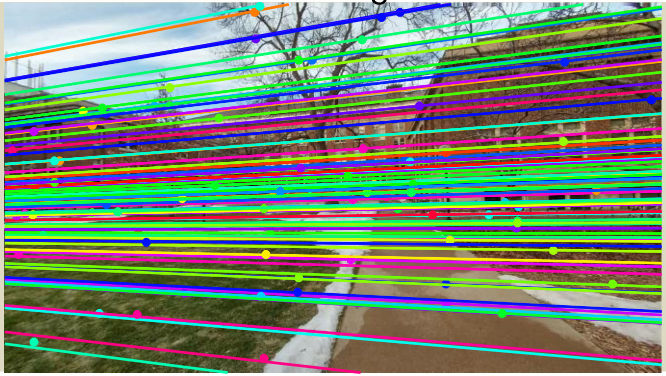
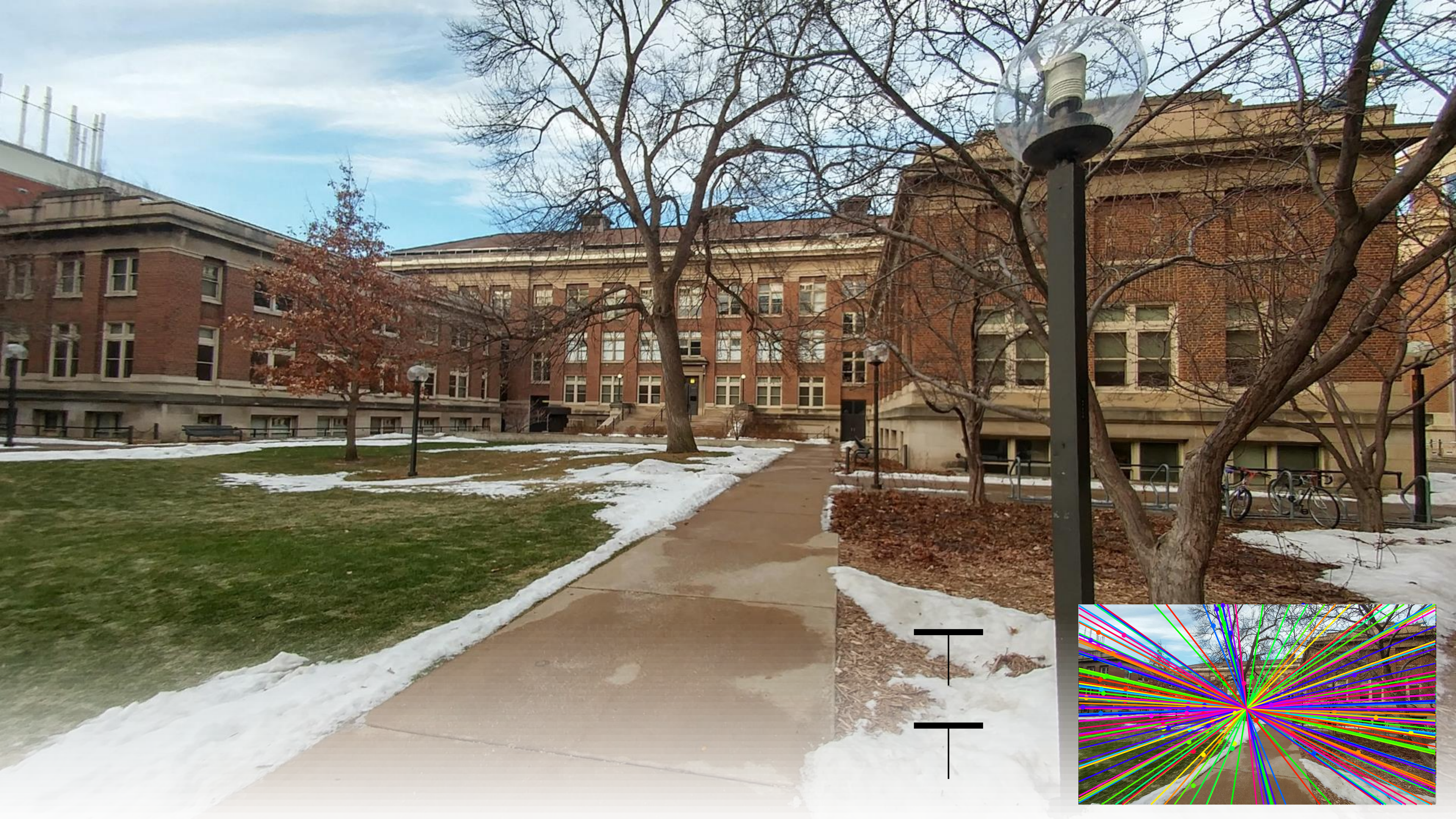
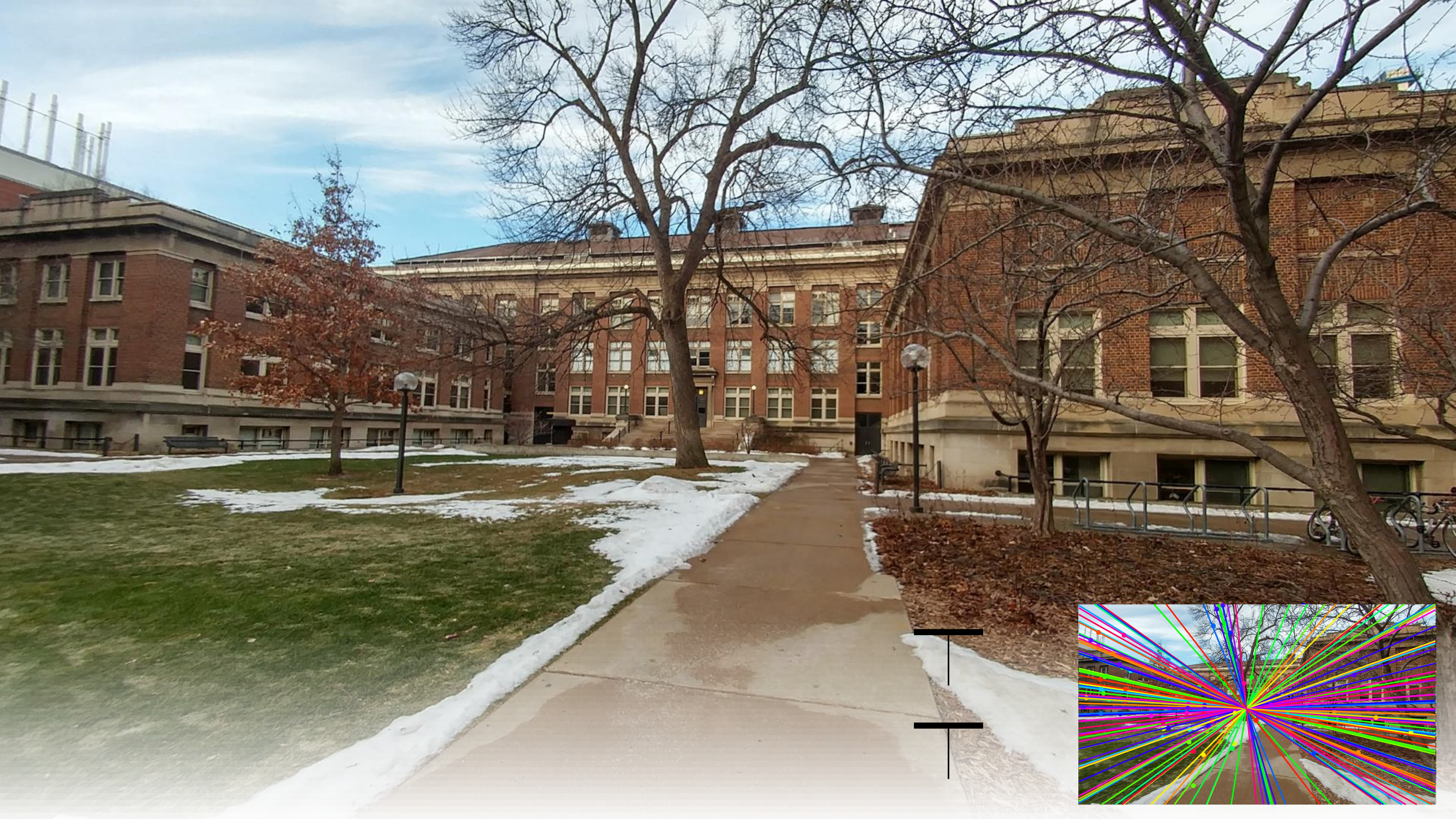


Image 1

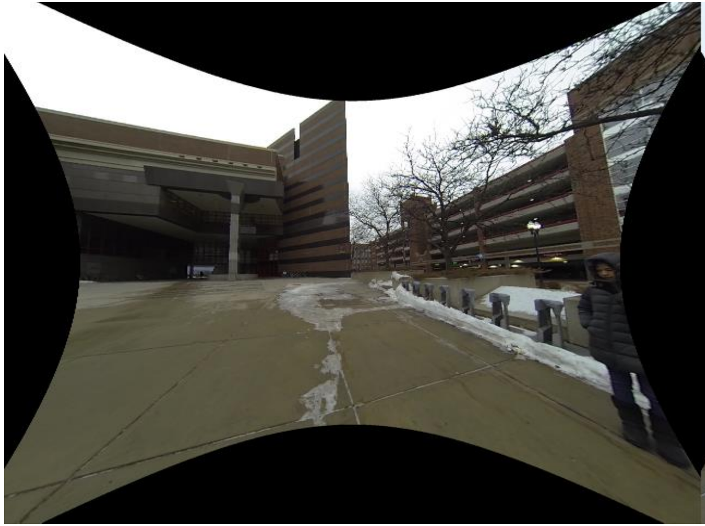
Image 2 Image 1

Lateral motion

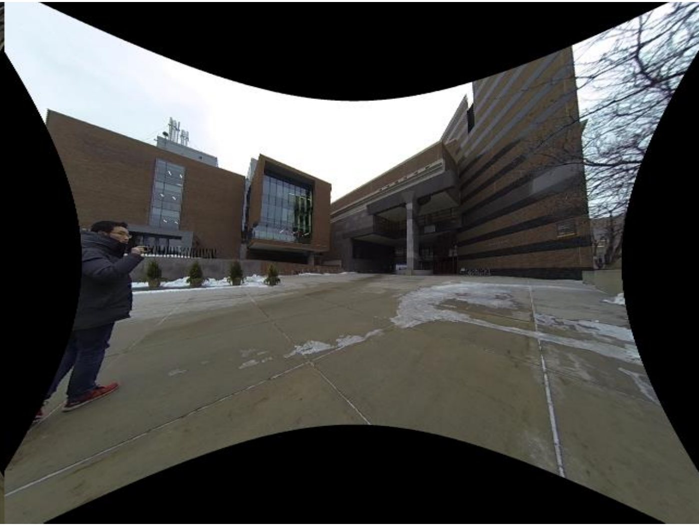




2D CORRESPONDENCE

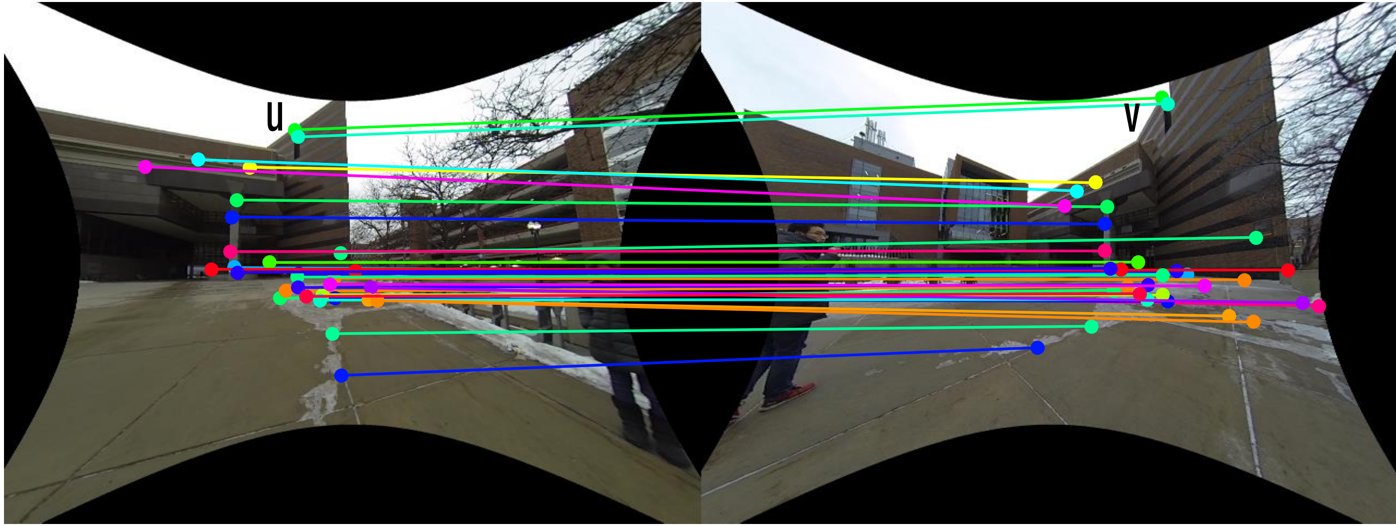


Bob’s image



Alice’s image

2D Correspondence

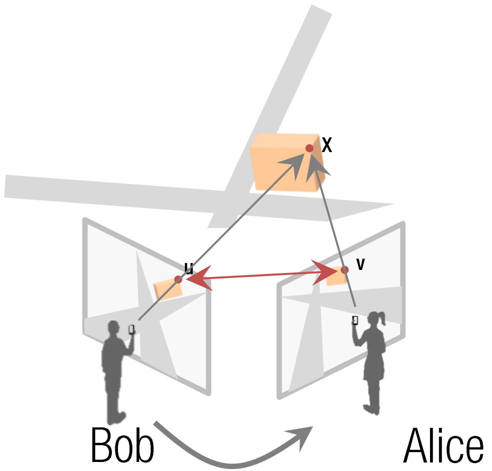


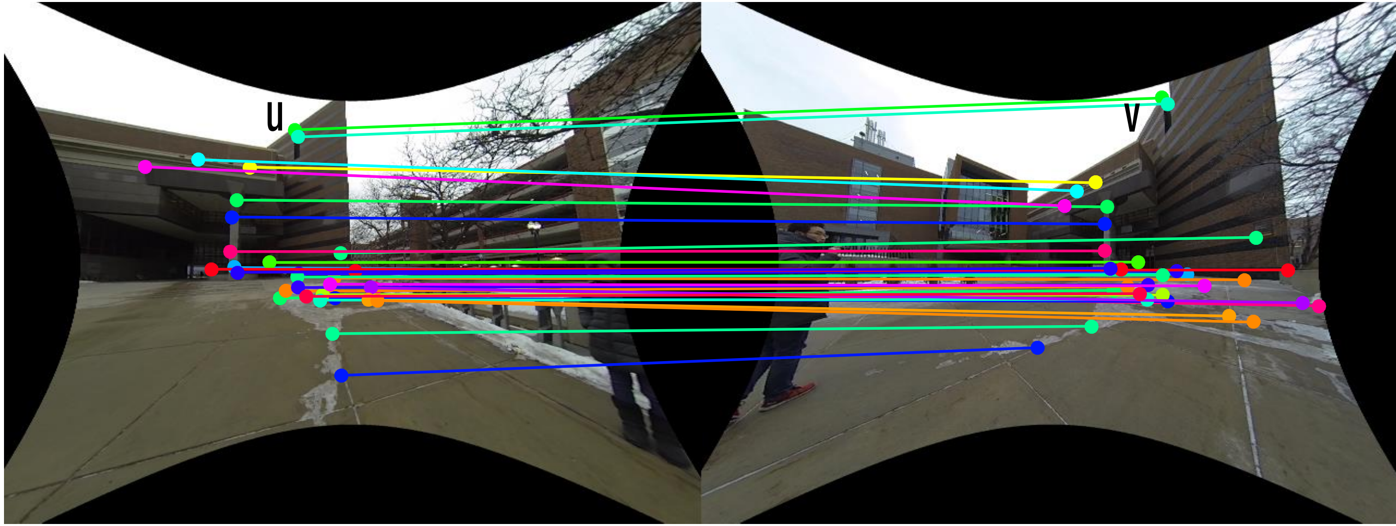
Bob's image

Alice's image

$$\mathbf{v}^{\mathsf{T}}\mathbf{F}\mathbf{u} = 0$$

2D Correspondence





Bob's image

Alice's image

$$\mathbf{F} = \mathbf{F}(\mathbf{R}, \mathbf{t})$$

$$= \mathbf{K}^{-T}[\mathbf{t}]_{\times}\mathbf{R}\mathbf{K}^{-1}$$

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$$

How to compute fundamental matrix?

8 Point Algorithm (Longuet-Higgins, Nature 1981)

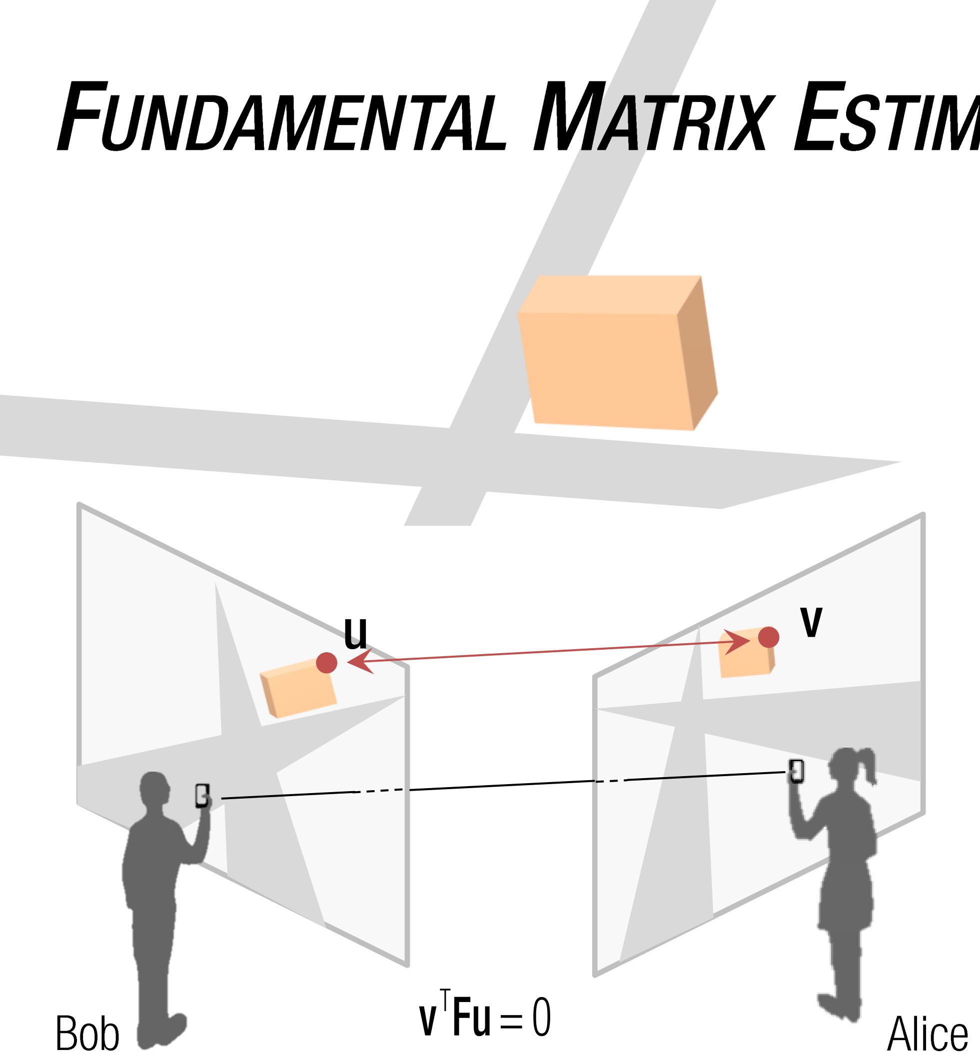
A computer algorithm for reconstructing a scene from two projections

H. C. Longuet-Higgins

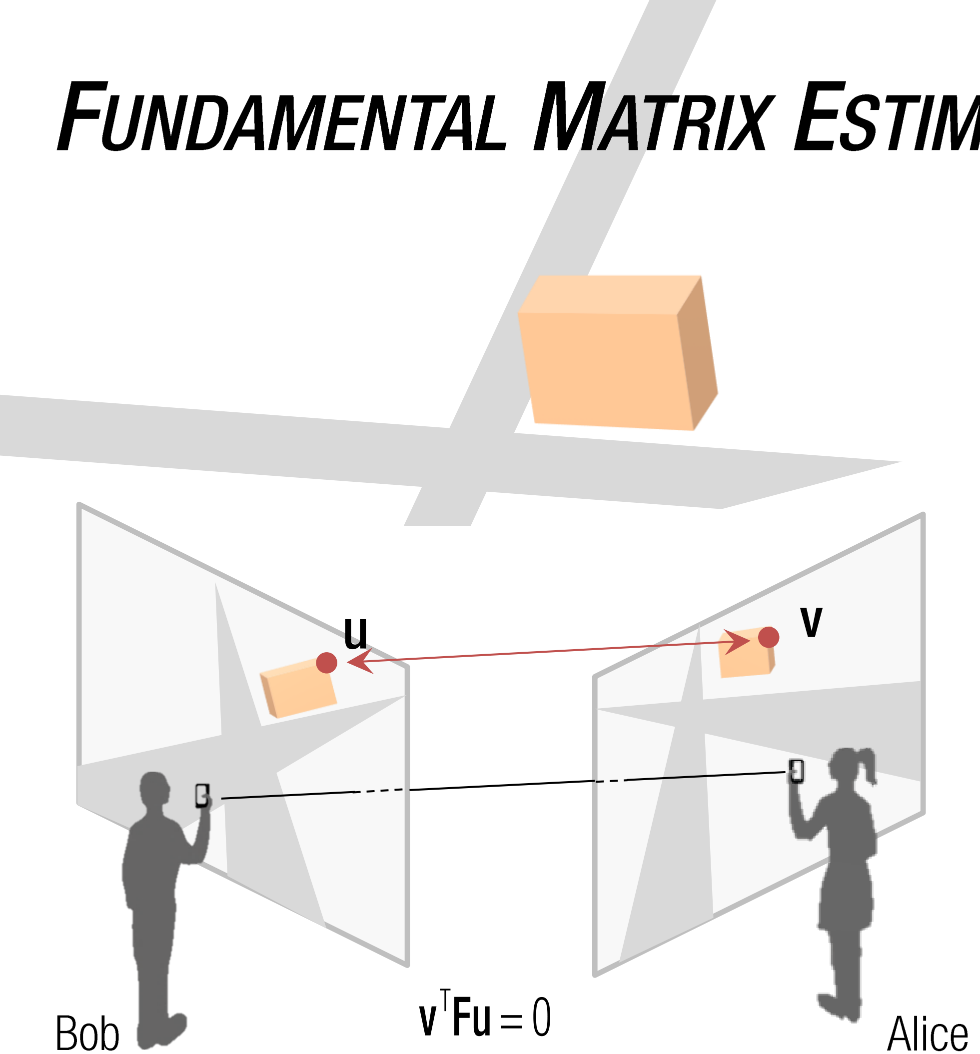
Laboratory of Experimental Psychology, University of Sussex, Brighton BN1 9QG, UK

A simple algorithm for computing the three-dimensional ture of a scene from a correlated pair of perspective proj is described here, when the spatial relationship between t projections is unknown. This problem is relevant not o photographic surveying[1] but also to binocular vision[2], wh non-visual information available to the observer abo

FUNDAMENTAL MATRIX ESTIMATION



Fundamental Matrix Estimation

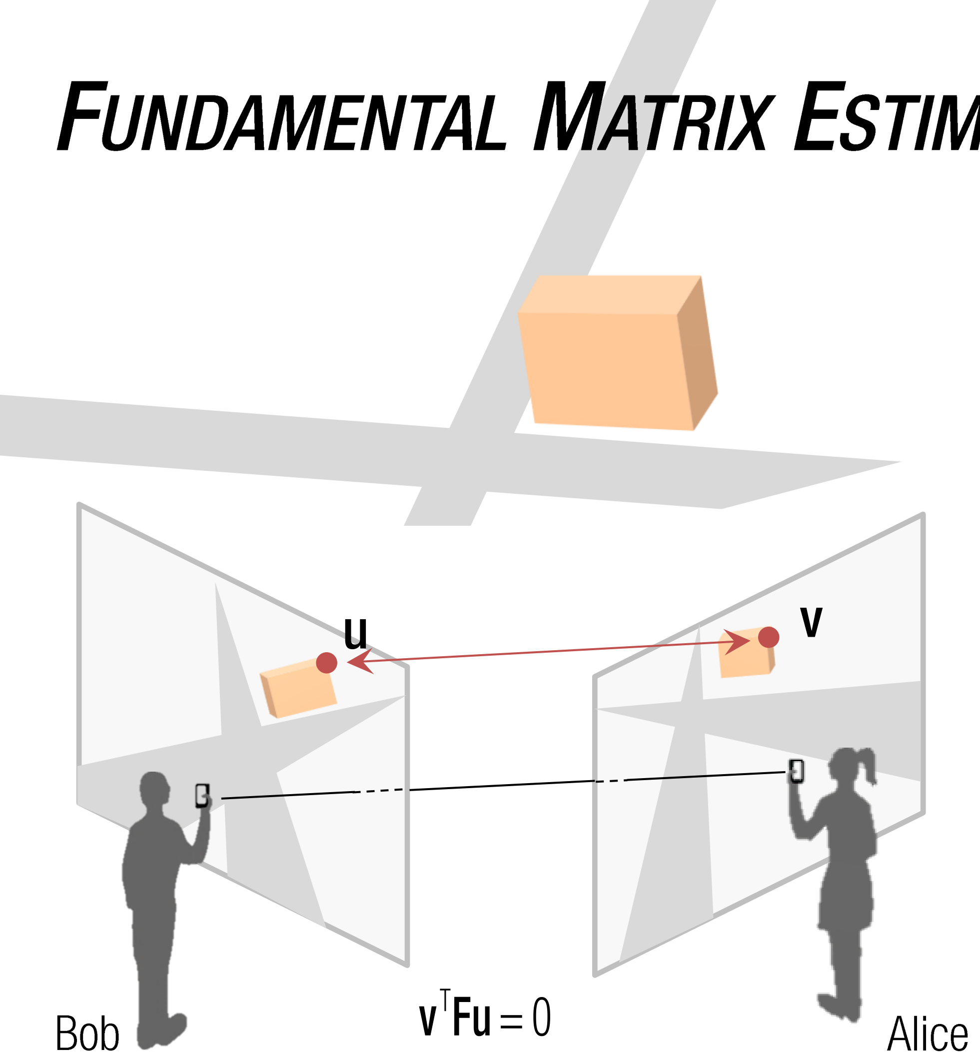


$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:

FUNDAMENTAL MATRIX ESTIMATION



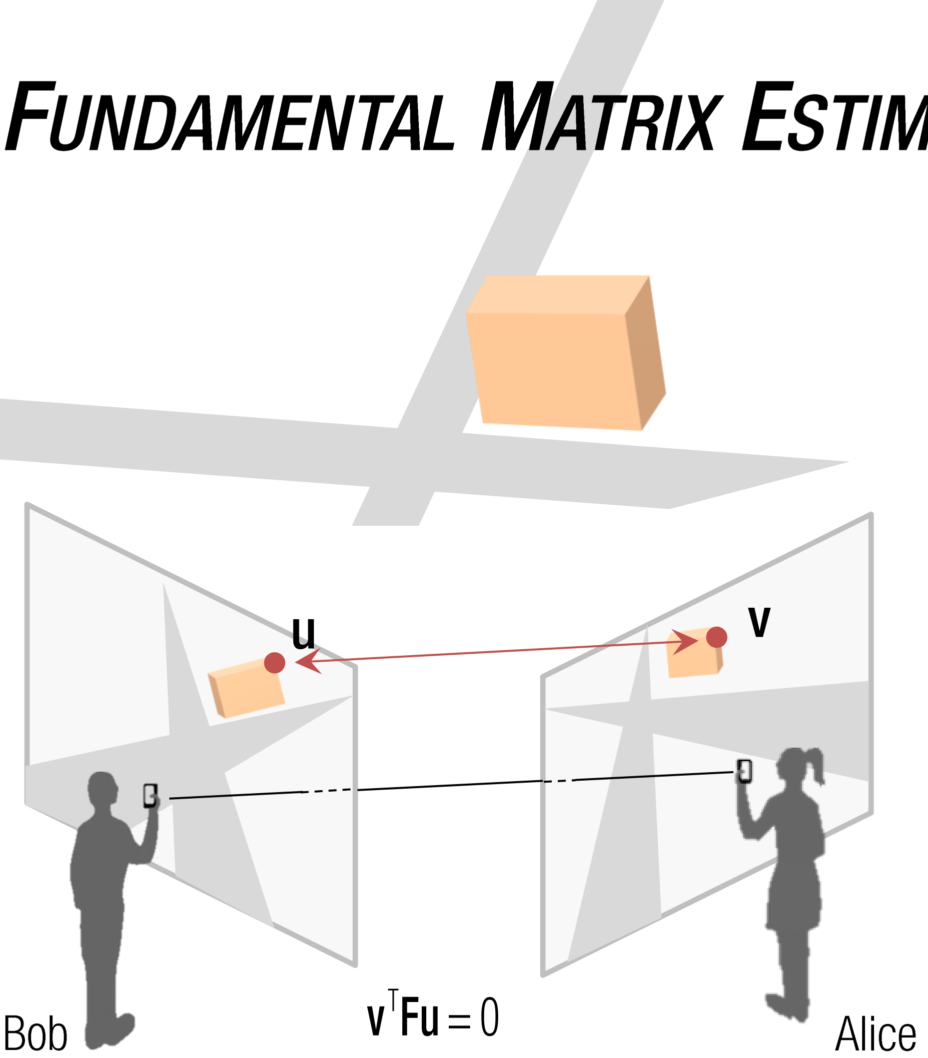
$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Degree of freedom of fundamental matrix:
7 = 9 (3x3 matrix) – 1(scale) – 1 (rank 2)

We will estimate fundamental matrix with 8 parameter by ignoring rank constraint and then project onto rank 2 matrix:

Fundamental Matrix Estimation

$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

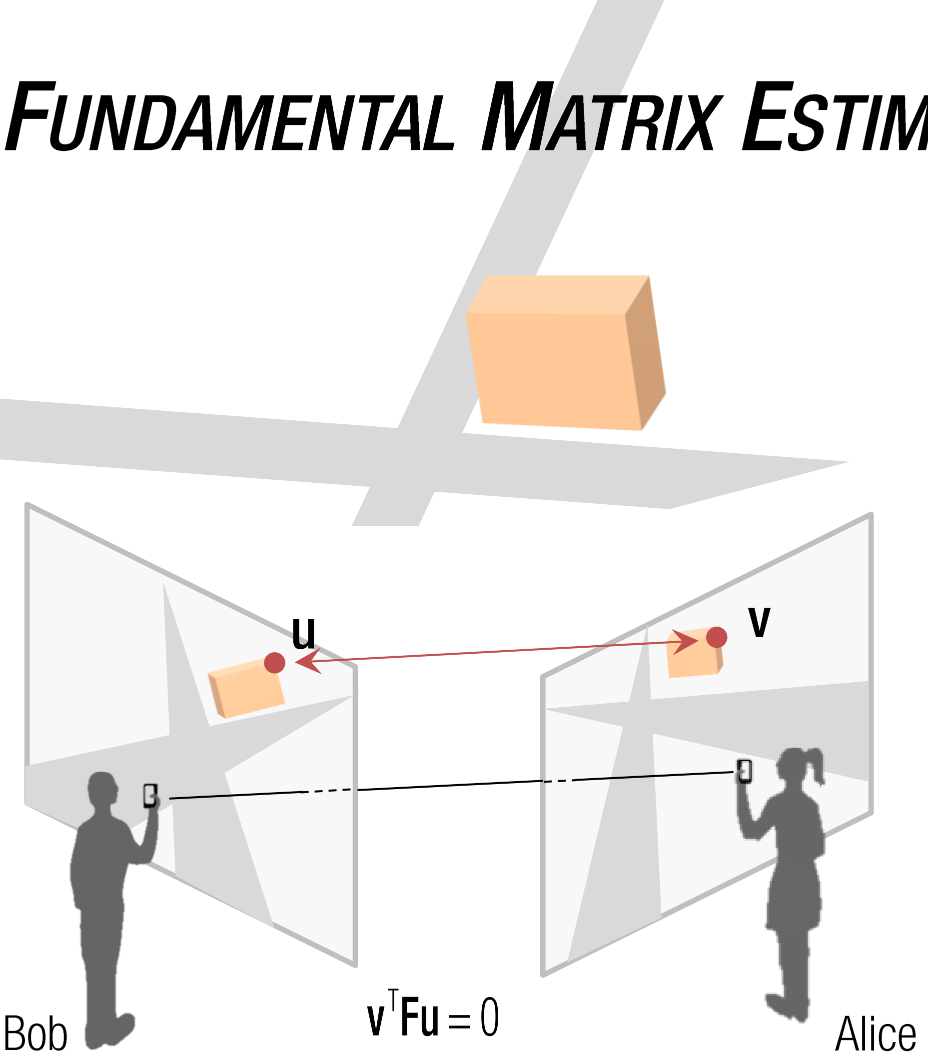


$$\mathbf{v}^{T}\mathbf{F}\mathbf{u} = 0$$

FUNDAMENTAL MATRIX ESTIMATION

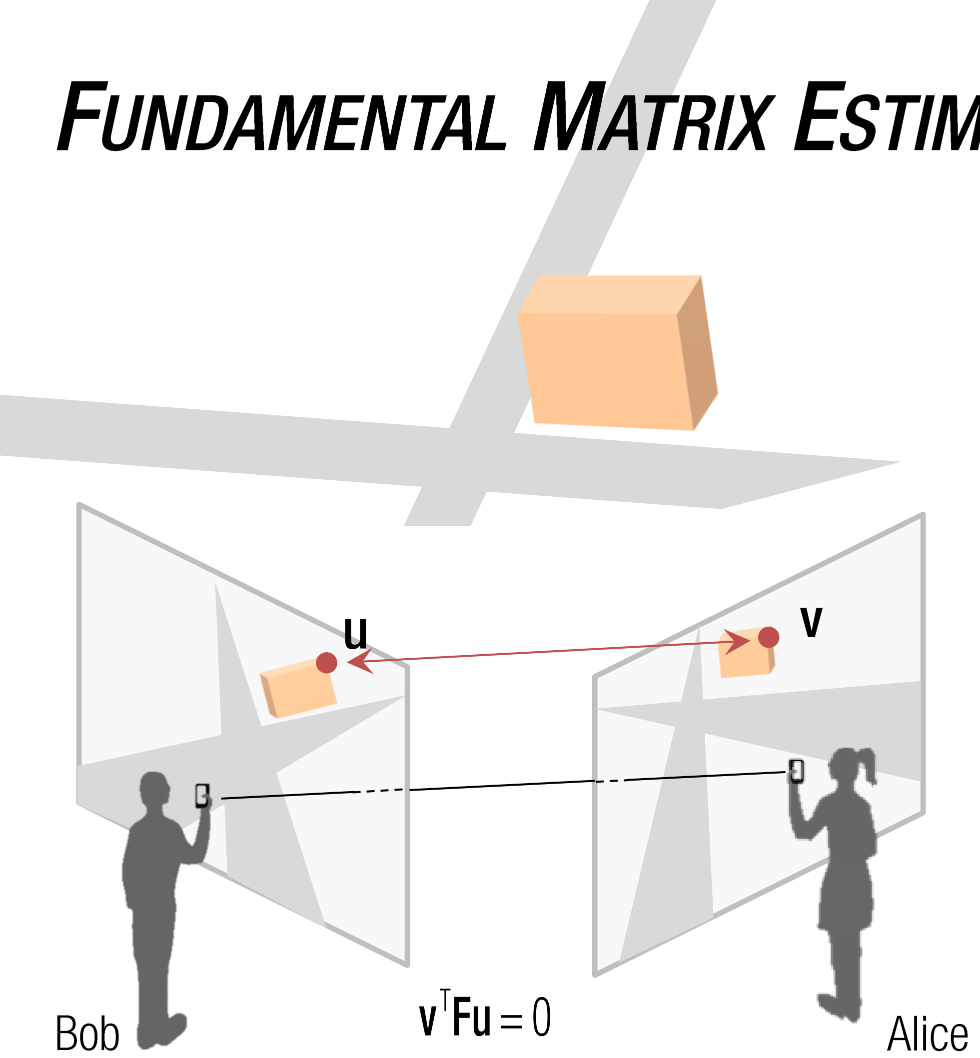
$$\mathbf{v}^{\mathrm{T}}\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$



$\mathbf{v}^{\mathrm{T}}\mathbf{F}\mathbf{u} = 0$

FUNDAMENTAL MATRIX ESTIMATION



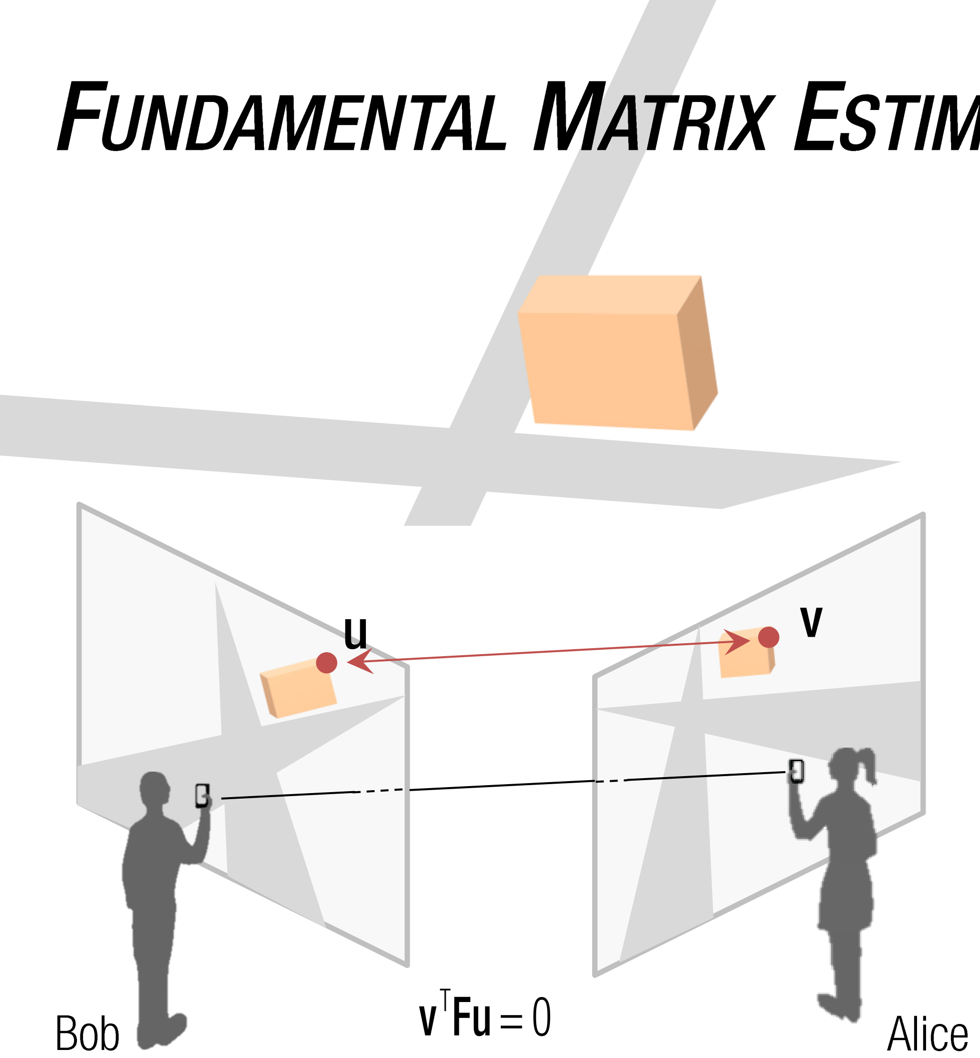
$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

Linear in **F**.

Fundamental Matrix Estimation



$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

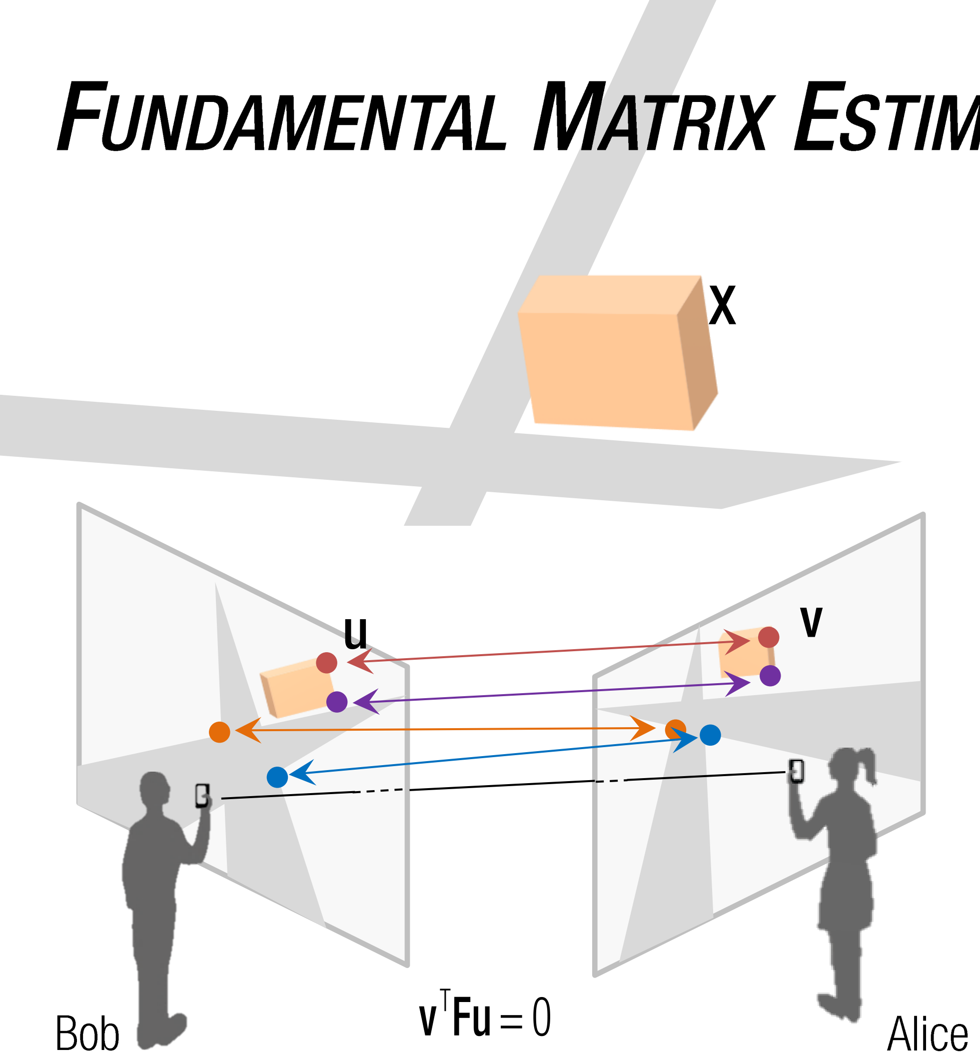
Linear in **F**.

$$\longrightarrow \begin{bmatrix} u^x v^x & u^y v^x & v^x & u^x v^y & u^y v^y & v^y & u^x & u^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

# of unknowns: 9

# of equations per correspondence: 1

Fundamental Matrix Estimation



$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

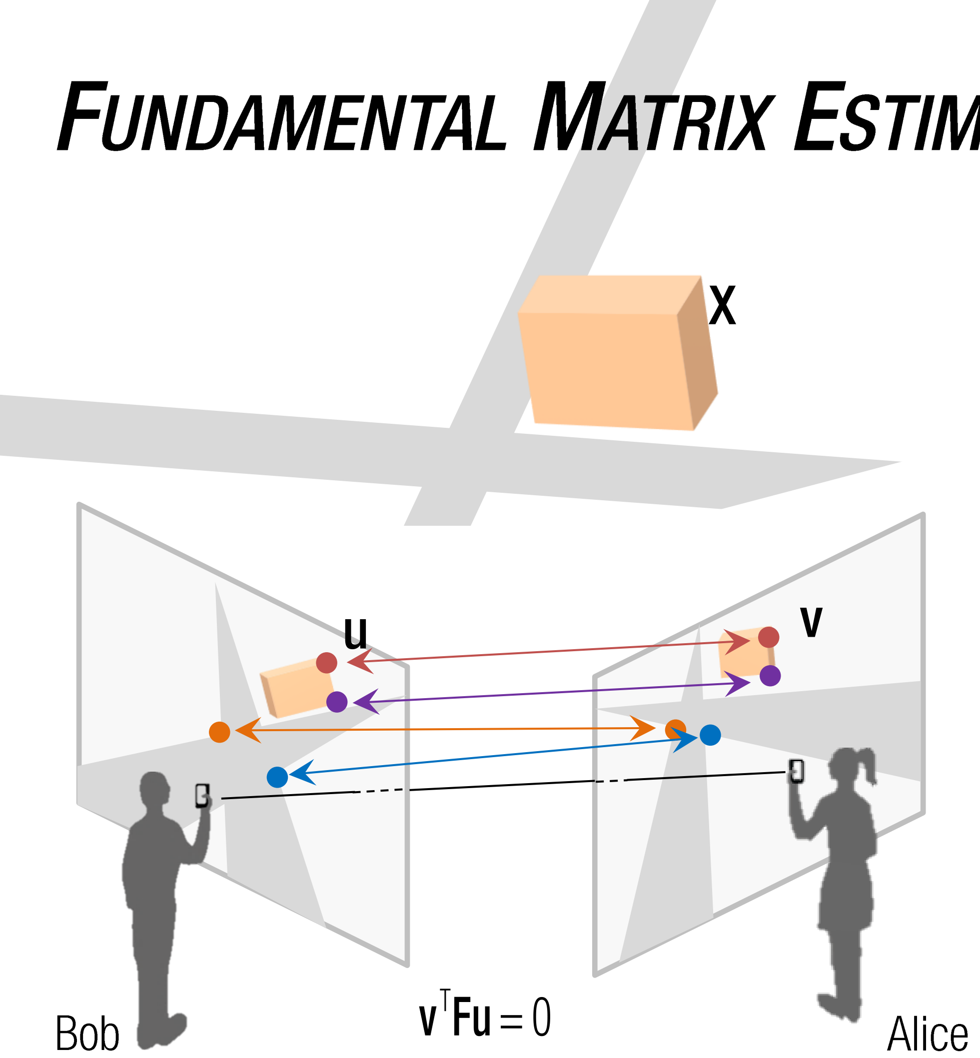
$$= 0$$

Linear in **F**.

$$\rightarrow \begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = \mathbf{0}_{m\times 1}$$

What is minimum m?

Fundamental Matrix Estimation



$\mathbf{v}^T\mathbf{F}\mathbf{u} = 0$

$$\mathbf{v}^T\mathbf{F}\mathbf{u} = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix}$$

$$= f_{11}u^x v^x + f_{12}u^y v^x + f_{13}v^x + f_{21}u^x v^y + f_{22}u^y v^y + f_{23}v^y + f_{31}u^x + f_{32}u^y + f_{33}$$

$$= 0$$

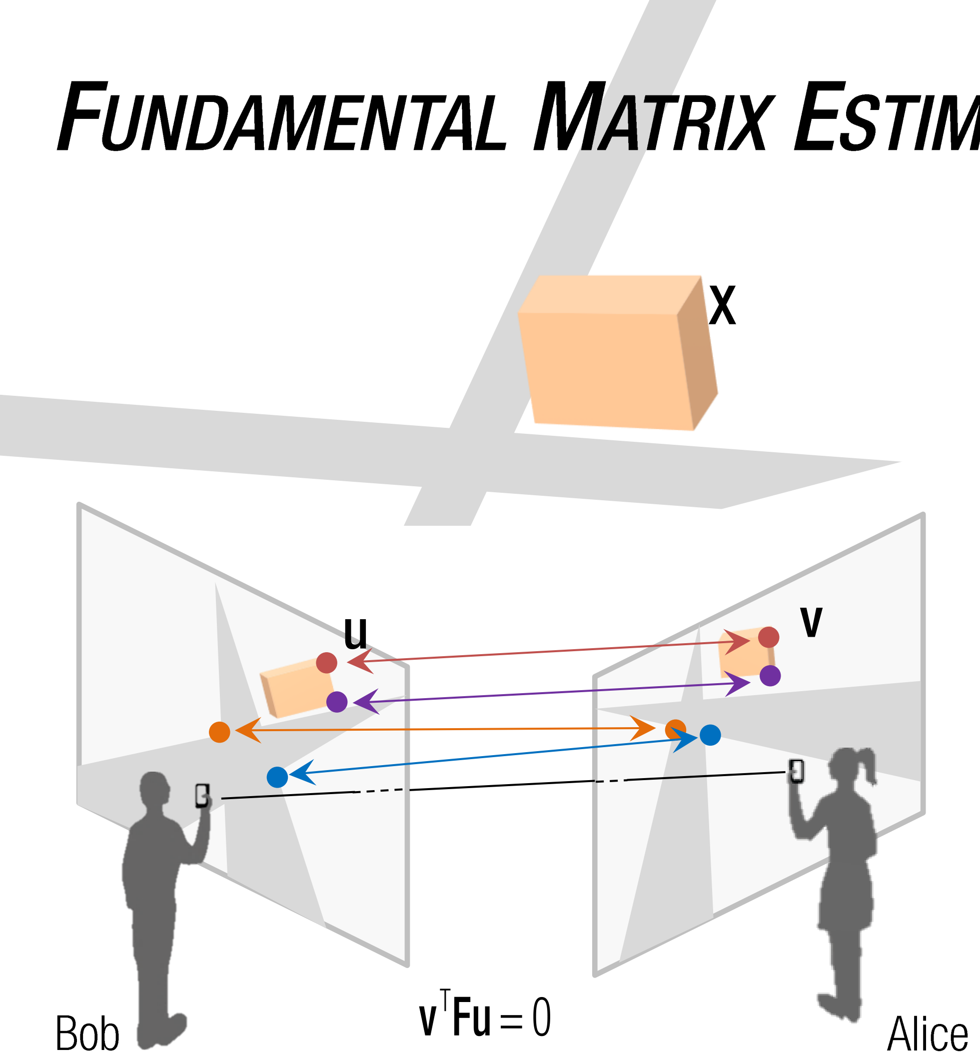
Linear in **F**.

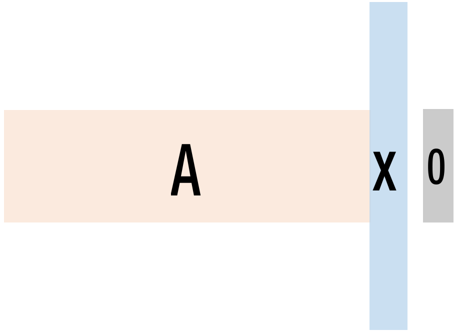
$$\begin{bmatrix} u_1^x v_1^x & u_1^y v_1^x & v_1^x & u_1^x v_1^y & u_1^y v_1^y & v_1^y & u_1^x & u_1^y & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^x v_m^x & u_m^y v_m^x & v_m^x & u_m^x v_m^y & u_m^y v_m^y & v_m^y & u_m^x & u_m^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0_{m \times 1}$$

$\mathbf{A}\mathbf{X} = \mathbf{0}$

What is minimum m?

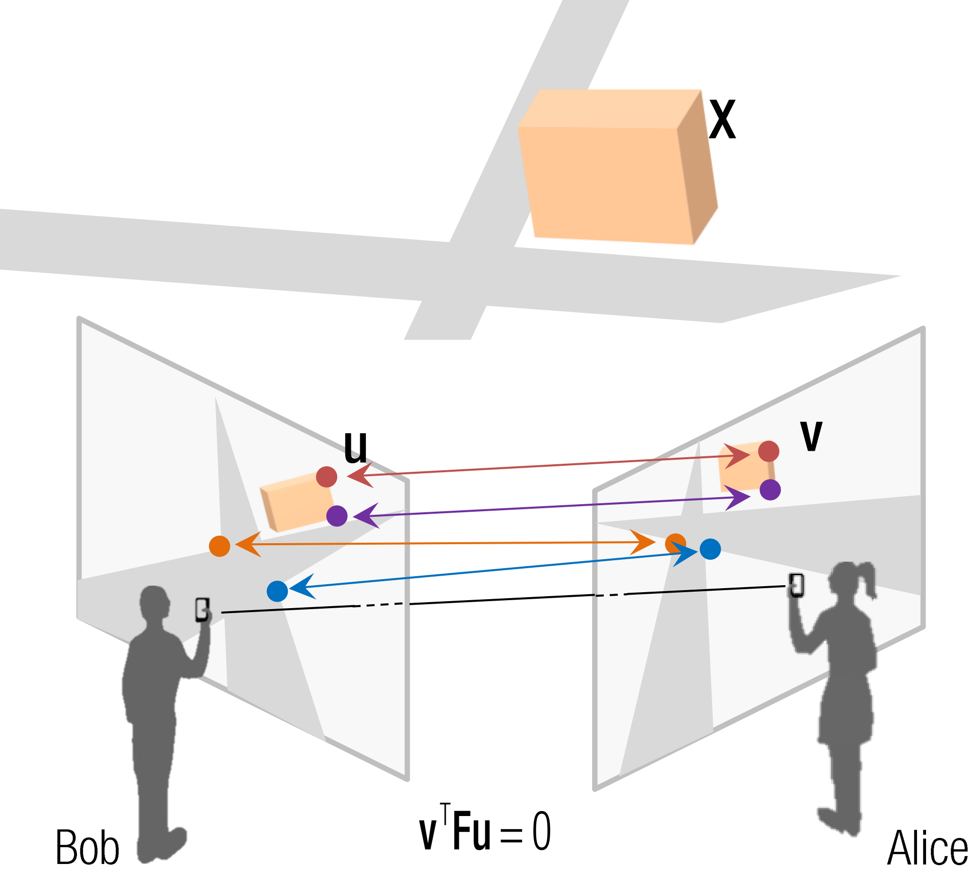
FUNDAMENTAL MATRIX ESTIMATION

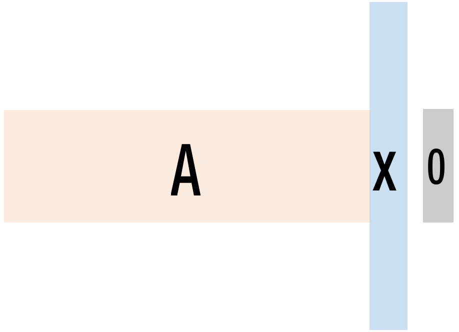




The solution is not necessarily satisfy rank 2 con

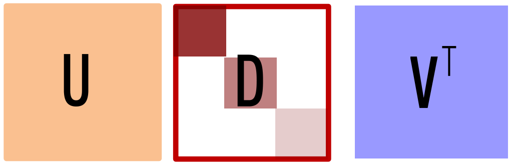
Fundamental Matrix Estimation



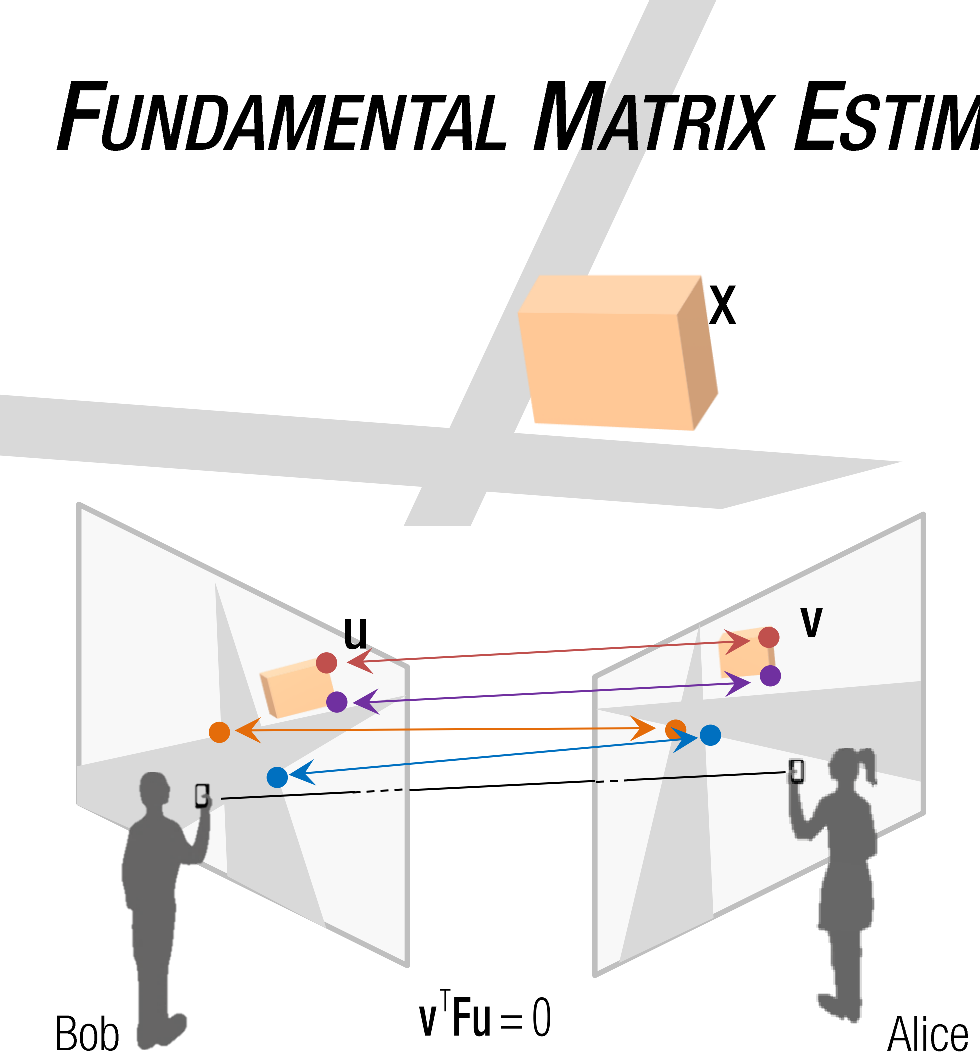


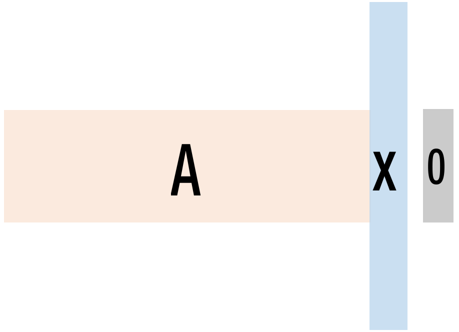
The solution is not necessarily satisfy rank 2 con

$$\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} =$$

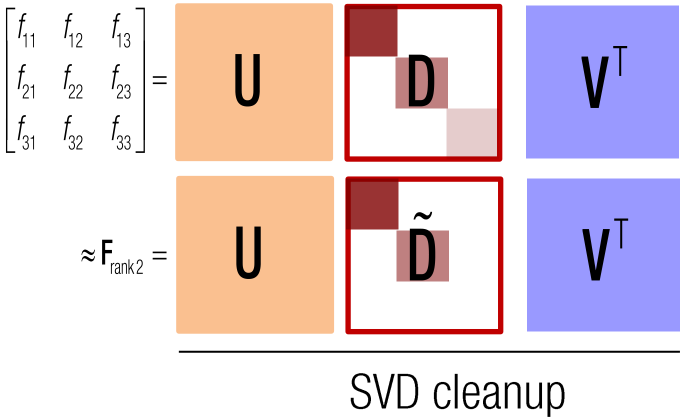


Fundamental Matrix Estimation





The solution is not necessarily satisfy rank 2 con



Camera Pose From F