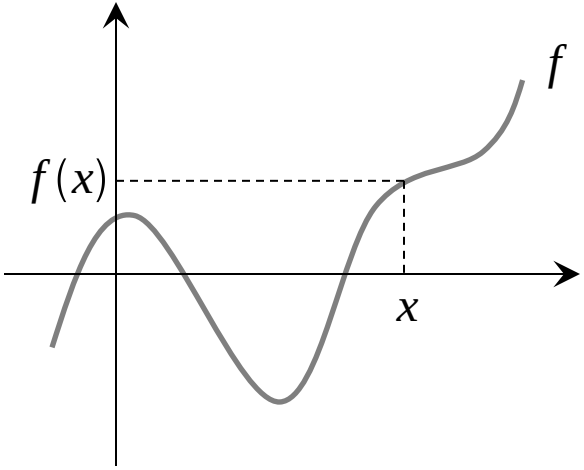
Image Filtering

Hyun Soo Park


OLO PAPAYA 6 FOR $2

$I(x_1)$

Pixel value at x



$I(x_1)$

Pixel value at *x*

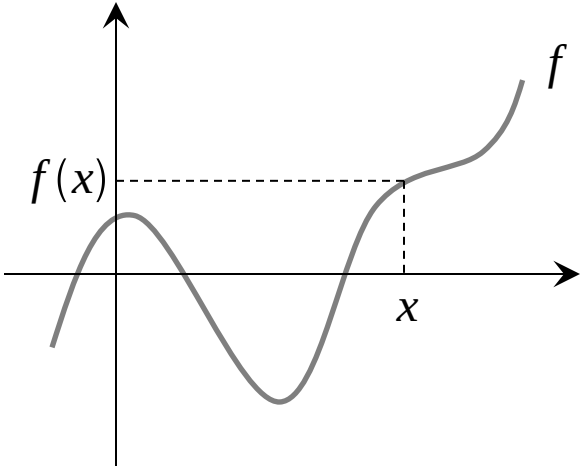
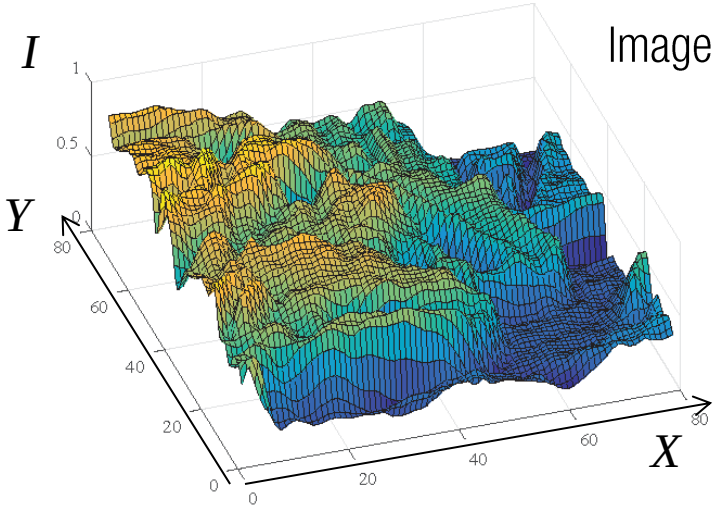
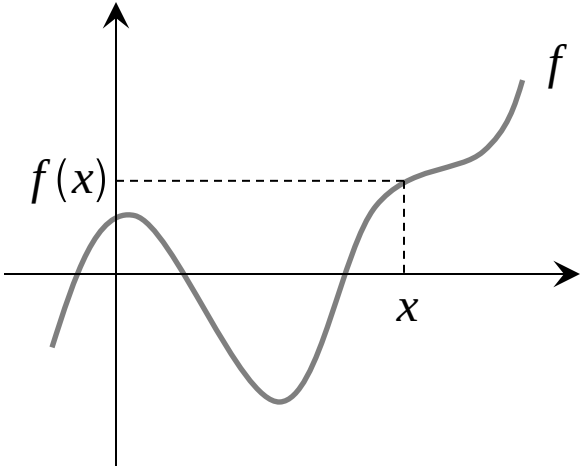


Image as a 2D function



$I(x_1) = (212, 200, 221)$

Pixel value at *x*



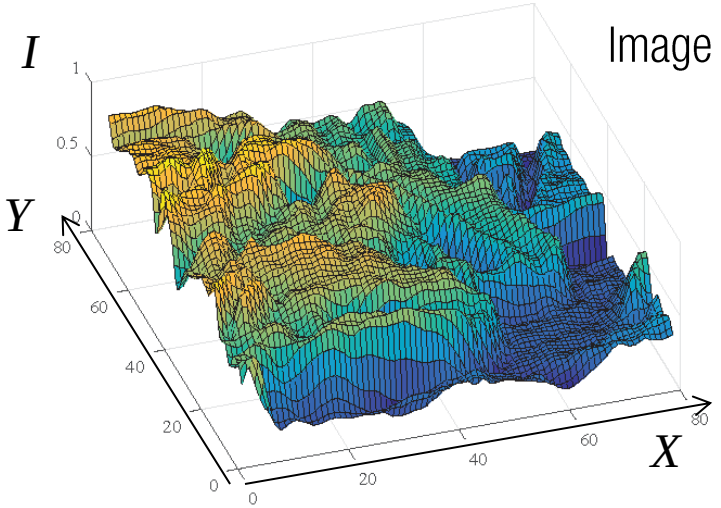
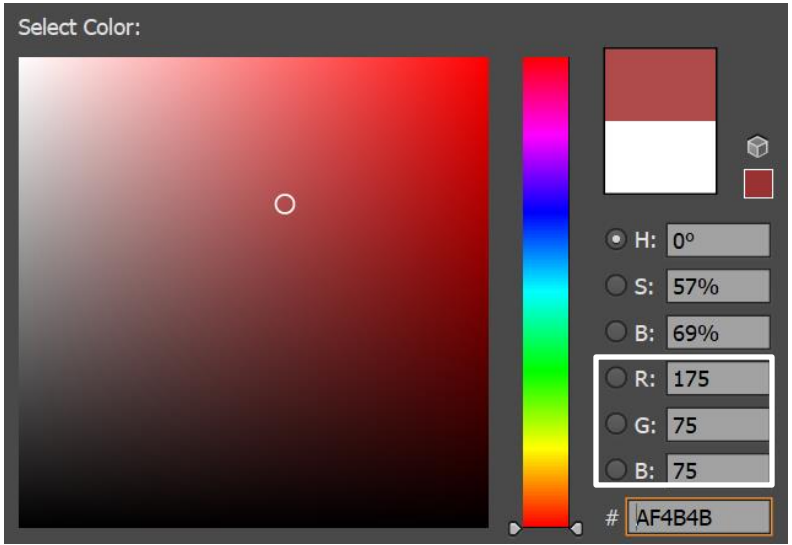


Image as a 2D function

$I(x_1) = (212, 200, 221)$

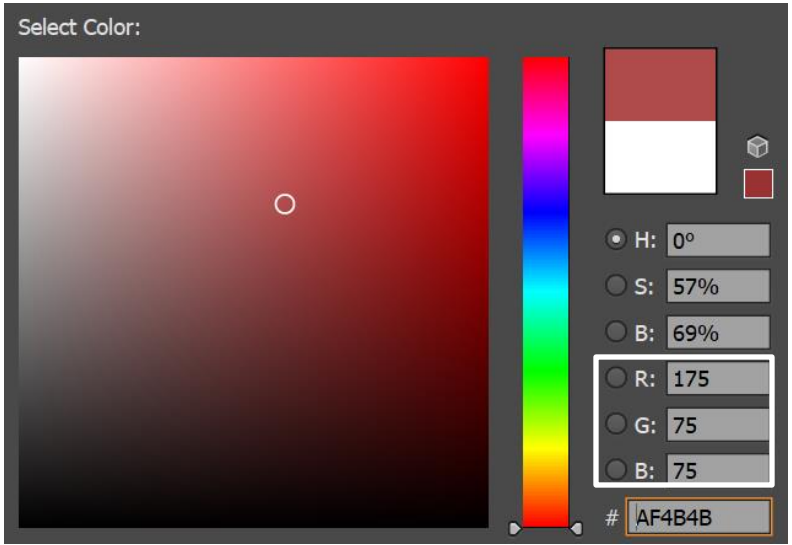
Pixel value at *x*



$I(x_1) = (212, 200, 221)$

Pixel value at *x*

$I(x_2) = (212, 20, 51)$



$I(x_1) = (212, 200, 221)$

Pixel value at x

$I(x_2) = (212, 20, 51)$

$I(x) \in 2^8 \times 2^8 \times 2^8$

8-bit image

IMAGE FILTERING

$$f(I) = I_f$$

IMAGE FILTERING

$$I(x_1) = (212,200,221)$$

$$f(I) = I_f$$

$$f(I(x_1)) = f(212,200,221)$$
$$= (242,152,135)$$

IMAGE FILTERING (PIXEL-WISE FILTERING)

$$I_f = I + 100$$

$$I_f = I - 100$$

$$I_f = 255 - I$$

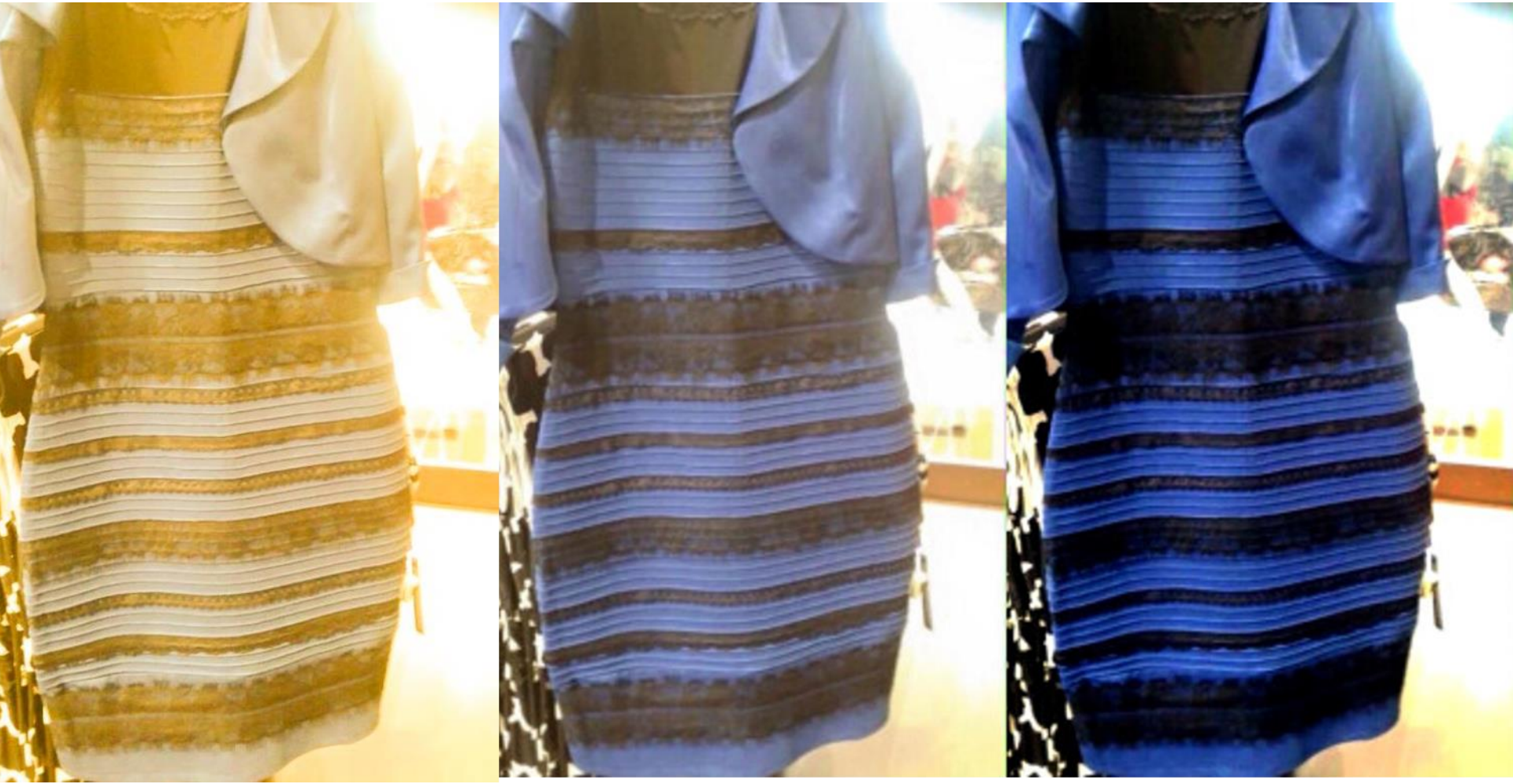
$$I_r > 100 \,\&\, I_g < 100 \,\&\, I_b < 100$$

IMAGE FILTERING (PIXEL-WISE FILTERING)

$$I_r > 100 \,\&\, I_g < 100 \,\&\, I_b < 100$$

Blue/black

White/gold



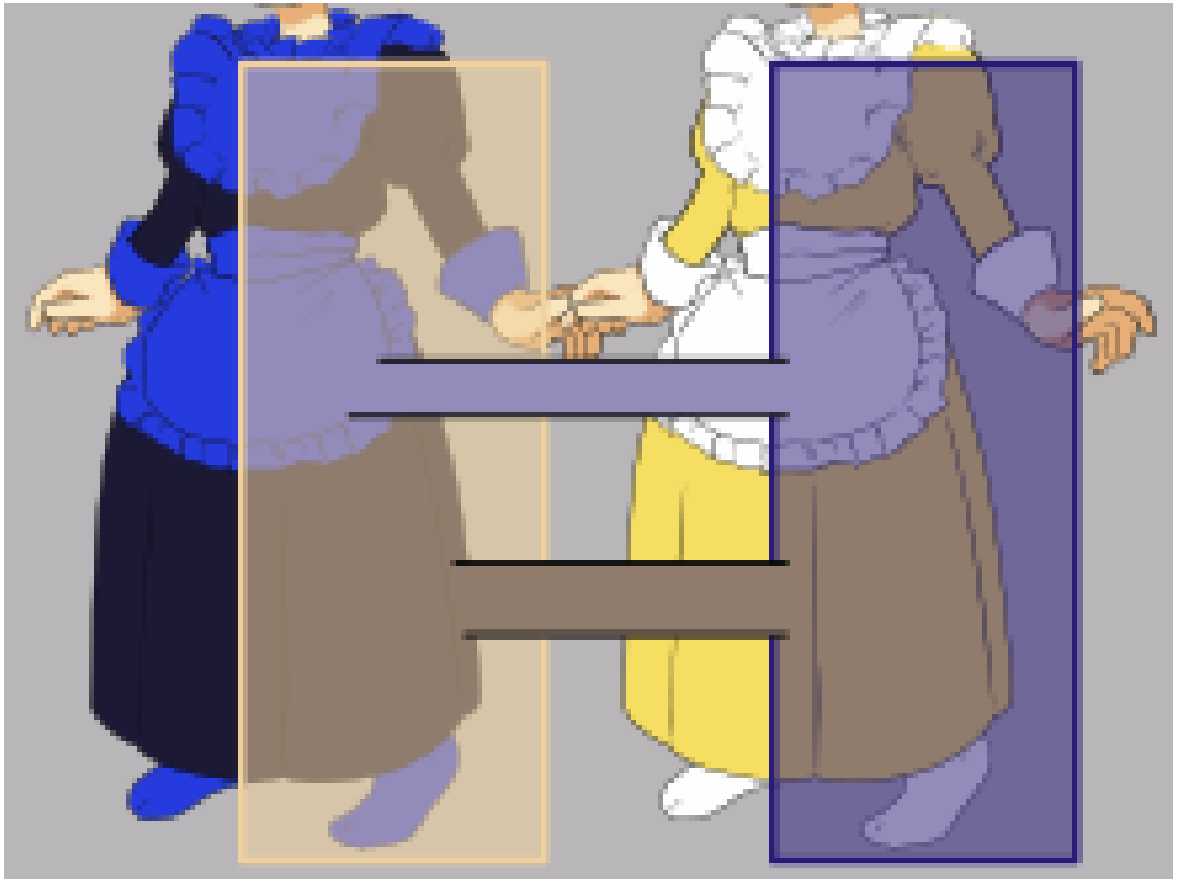
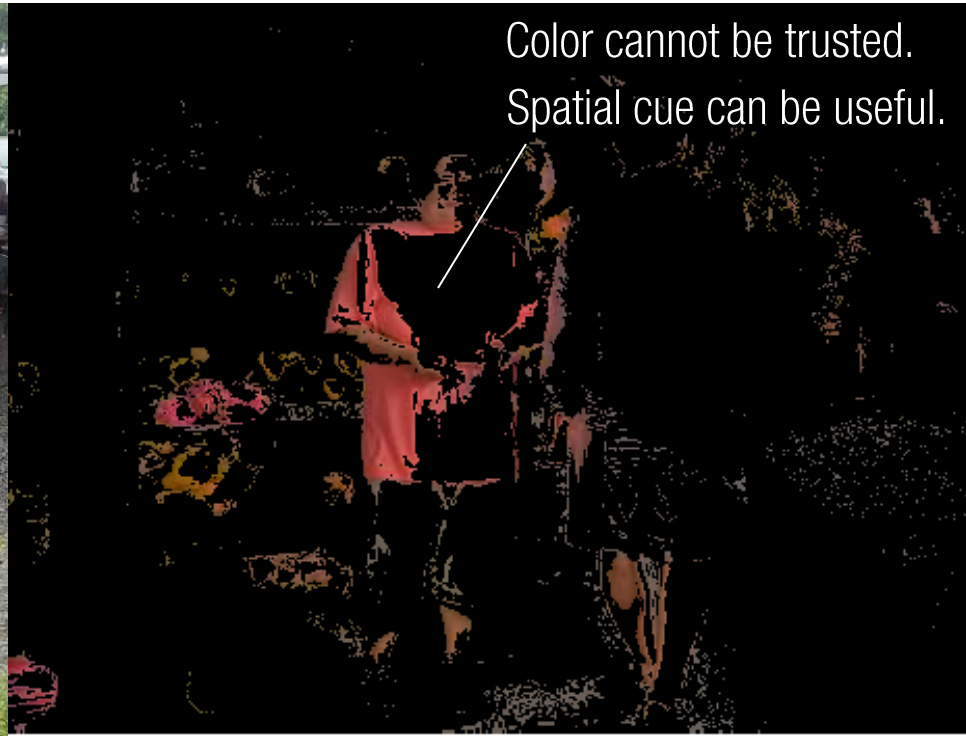


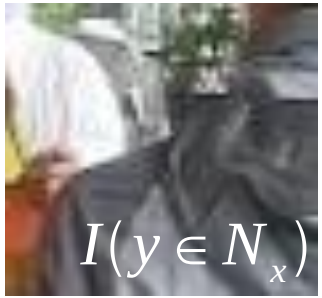
IMAGE FILTERING (PIXEL-WISE FILTERING)



$$I_r > 100 \,\&\, I_g < 100 \,\&\, I_b < 100$$

IMAGE SPATIAL FILTERING

$$I_f(x) = f(I(y \in N_x))$$



Local patch around x

N_x : Neighboring pixels of x

IMAGE SPATIAL FILTERING

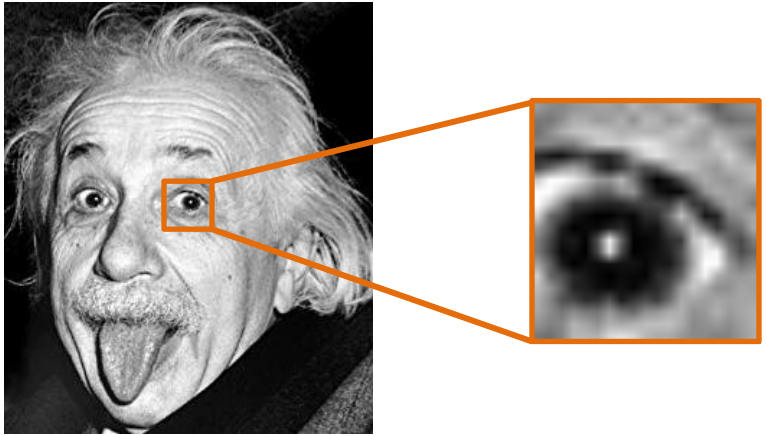
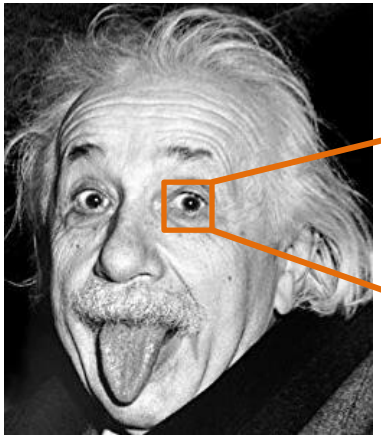


IMAGE SPATIAL FILTERING



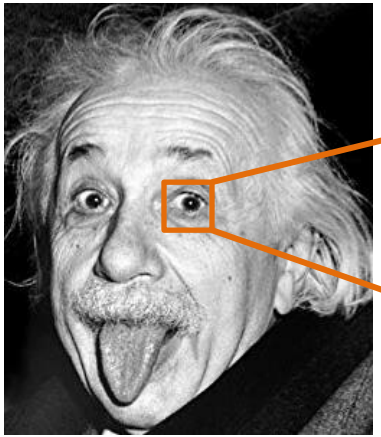
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

I

Image

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

I

Image

$\otimes$

a	b	c
d	e	f
g	h	i

z

Filter

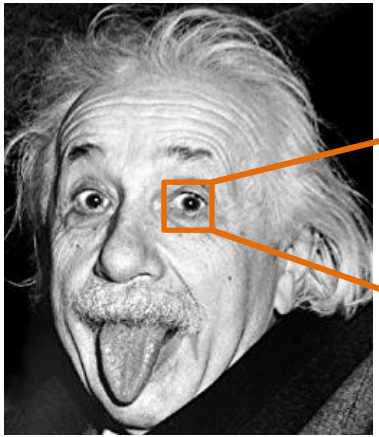
$=$

I_f

Filtered image

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

$=$

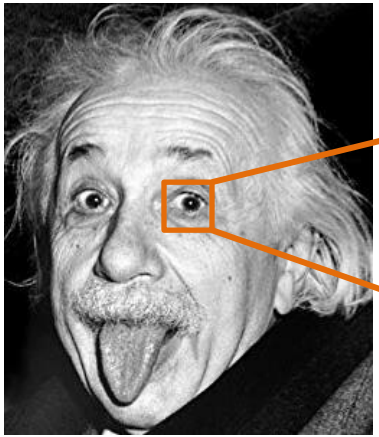
	y			

Image Filter Filtered image

$$y = 2a + b + 4c + d + 2e + 2f + 3g + 3h + 5i$$

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

$=$

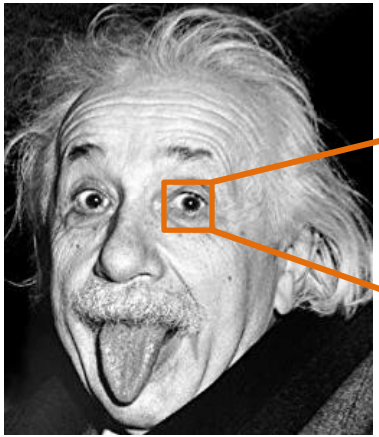
		y		

Image Filter Filtered image

$$y = a + 4b + 4c + 2d + 2e + 3f + 3g + 5h + 8i$$

IMAGE SPATIAL FILTERING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

$=$

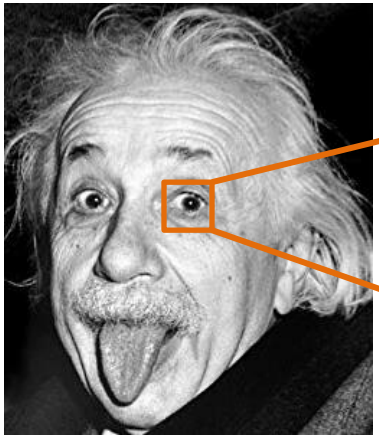
			y	

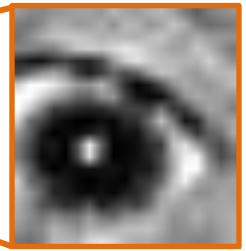
Image Filter Filtered image

$$y = 5a + 8b + 9c + 2d + 6e + 7f + 2g + h + 3i$$

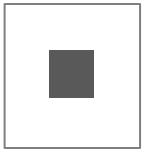
IDENTITY

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



$=$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

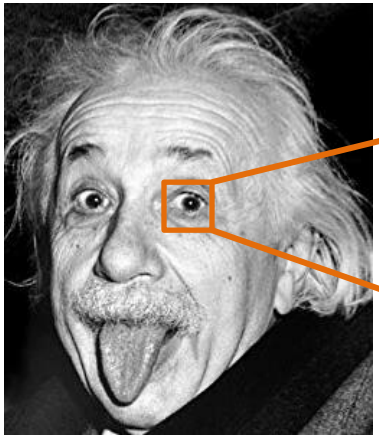
$\otimes$

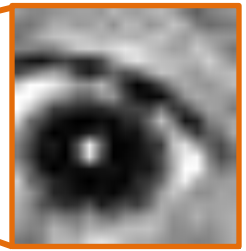
0	0	0
0	1	0
0	0	0

$=$

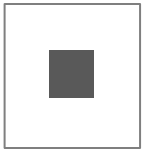
IDENTITY

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



$=$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

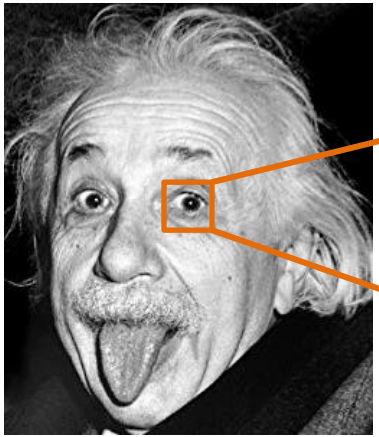
0	0	0
0	1	0
0	0	0

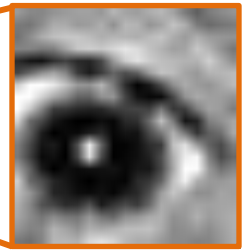
$=$

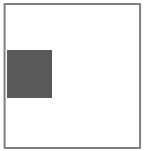
	2	2	3	
	3	5	8	
	2	2	6	

SHIFTING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l)z(k,l)$$







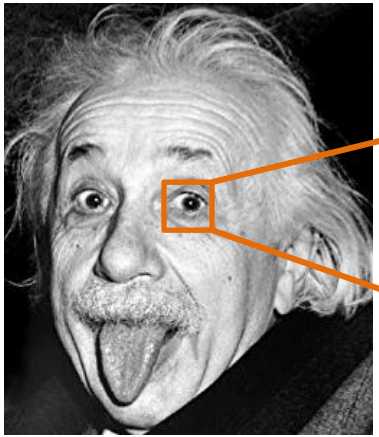
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

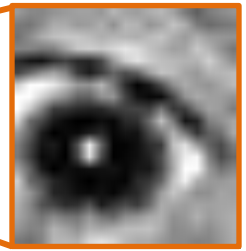
⊗

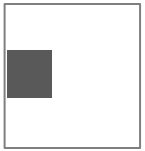
0	0	0
1	0	0
0	0	0

SHIFTING

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l)z(k,l)$$







2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

⊗

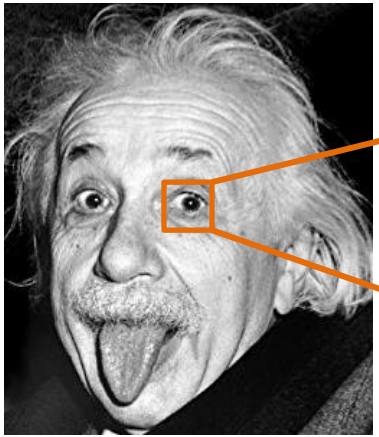
0	0	0
1	0	0
0	0	0

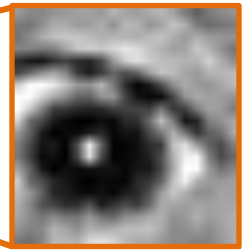
=

	1	2	2	
	3	3	5	
	5	2	2	

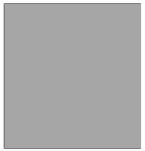
Box Filter (Moving Averaging)

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$



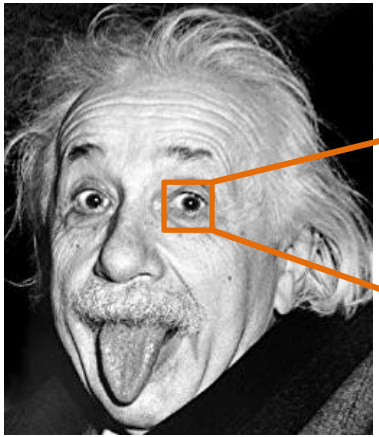
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

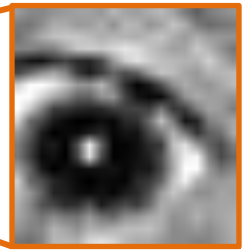
$\otimes$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

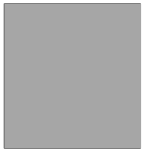
Box Filter (Moving Averaging)

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$

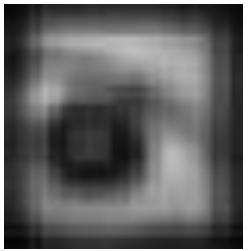




$\otimes$



$=$



2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

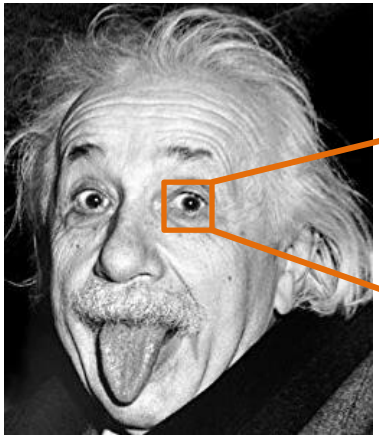
$\otimes$ $\frac{1}{9}$

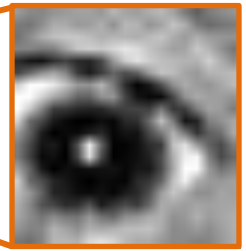
1	1	1
1	1	1
1	1	1

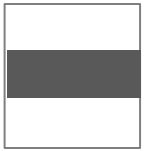
Average

Directional Blur

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$







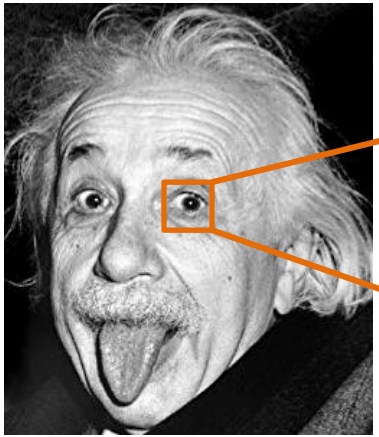
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

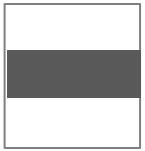
$\otimes$ $\frac{1}{3}$

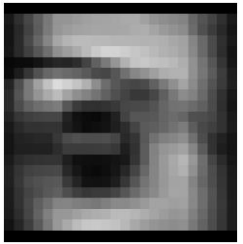
0	0	0
1	1	1
0	0	0

Directional Blur

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$







2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

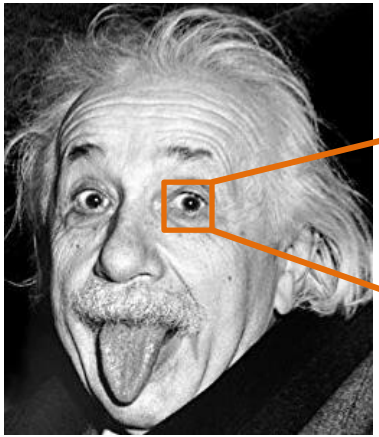
$\otimes \frac{1}{3}$

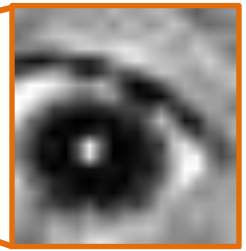
0	0	0
1	1	1
0	0	0

Horizontal blur

Directional Blur

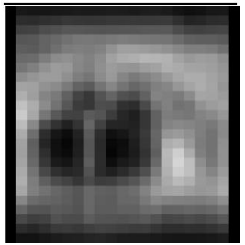
$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$





$\otimes$

$=$



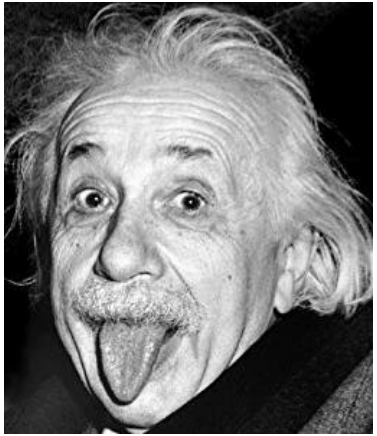
2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$ $\frac{1}{3}$

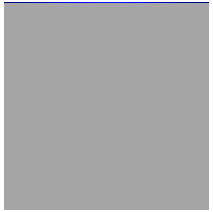
0	1	0
0	1	0
0	1	0

Vertical blur

Box Filter (Moving Averaging)



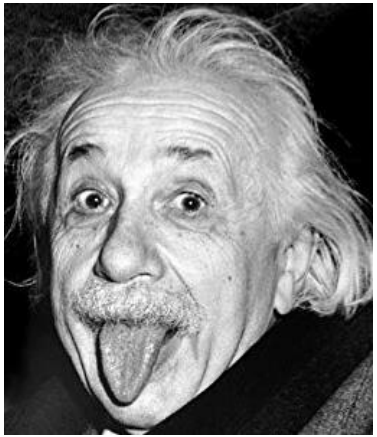
$\otimes$



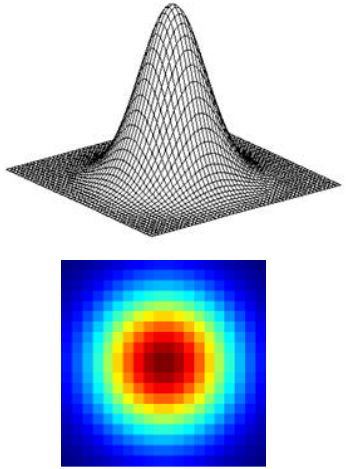
$=$

Pixel averaging

Gaussian Blurring (Moving Weighted Average)



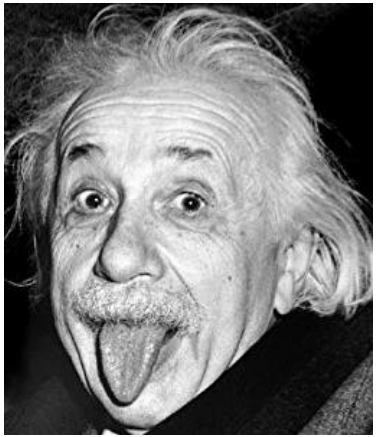
$\otimes$



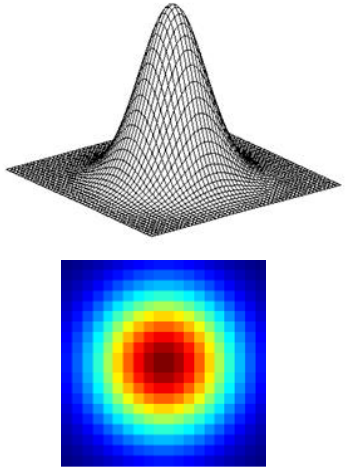
$=$

$$z(k,l) = \frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

Gaussian Blurring (Moving Weighted Average)



$\otimes$

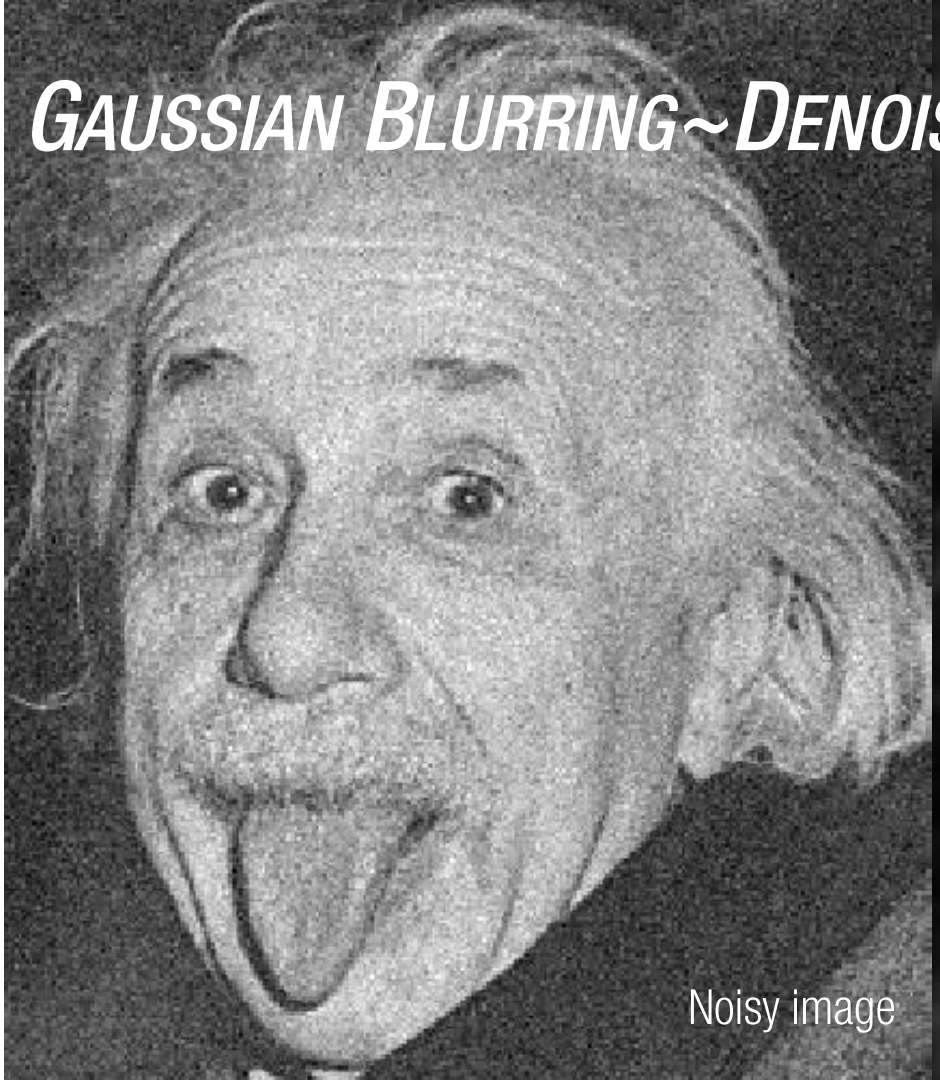


$=$

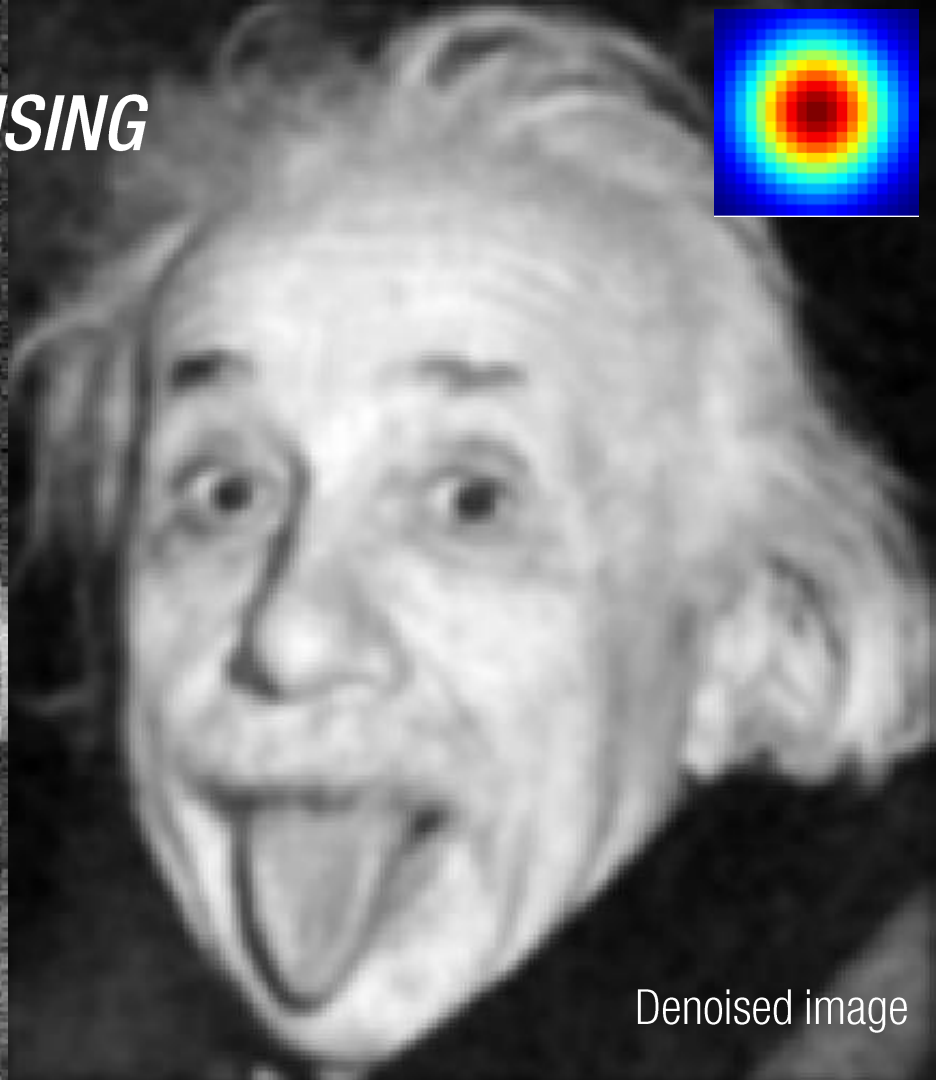
$$z(k,l) = \frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

Gaussian blur

Gaussian Blurring~Denoising



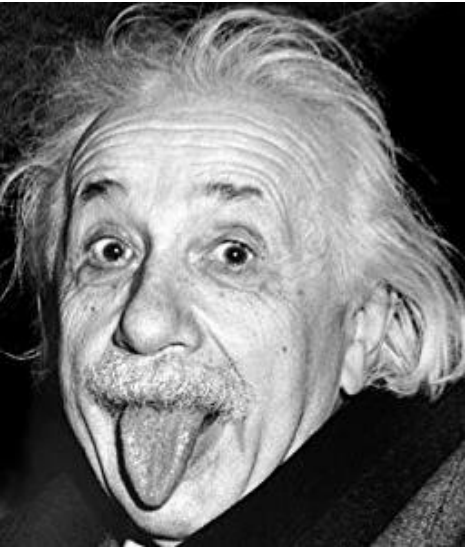
Noisy image

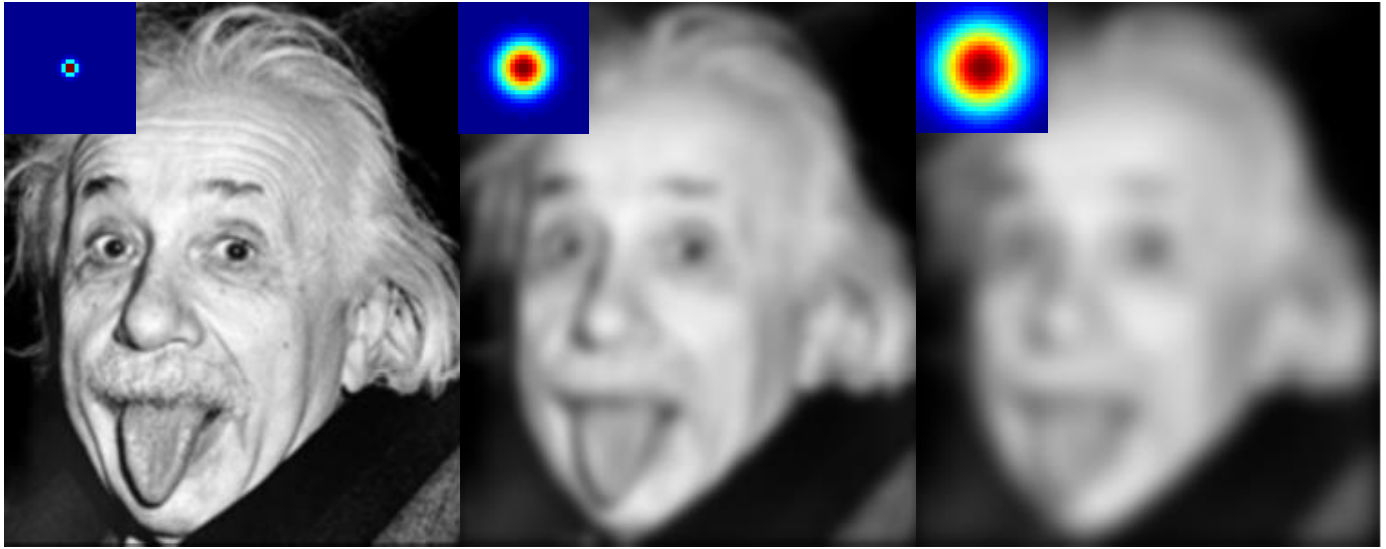


Denoised image

GAUSSIAN BLURRING

$$z(k,l) = \frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

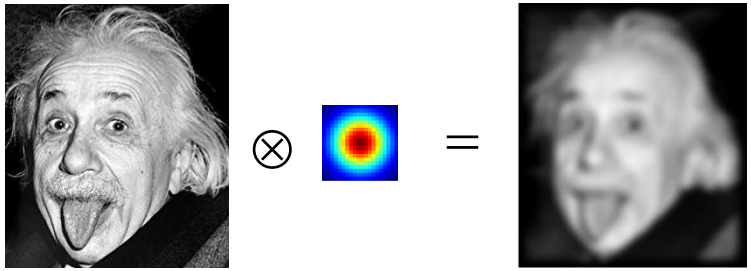




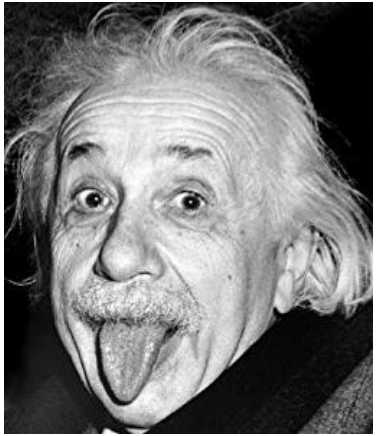
$\sigma = 1$ $\sigma = 4$ $\sigma = 7$

IMAGE SPATIAL FILTERING

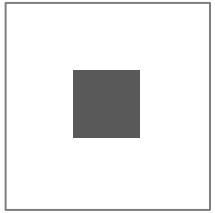
```
function im_f = Filtering(im, filter)

% filtered image initialization
im_f = zeros(size(im));
center_k = floor(size(filter,1)/2)+1;
center_l = floor(size(filter,2)/2)+1;
for i = 1 : size(im,1)
    for j = 1 : size(im,2)
        % filtering
        v = 0;
        for k = 1 : size(filter,1)
            for l = 1 : size(filter,2)
                i1 = i + k - center_k;
                j1 = j + l - center_l;
                if i1 <= 0 || i1 > size(im_f,1) || j1 <= 0 || j1 > size(im_f,2)
                    continue;
                end
                v = v + im(i1,j1)*filter(k,l);
            end
        end
        im_f(i,j) = v;
    end
end
```



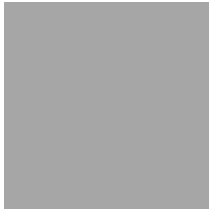
$$I_f(i,j) = \sum_{k,l} I(i+k, j+l) z(k,l)$$



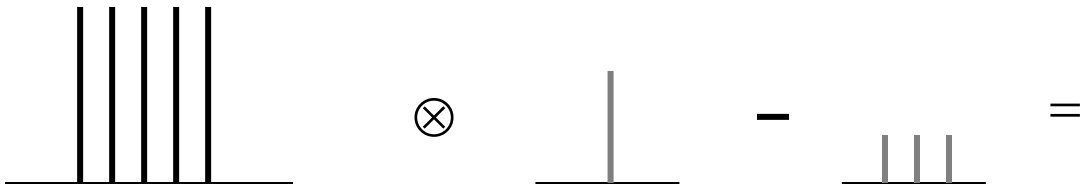
$\otimes$

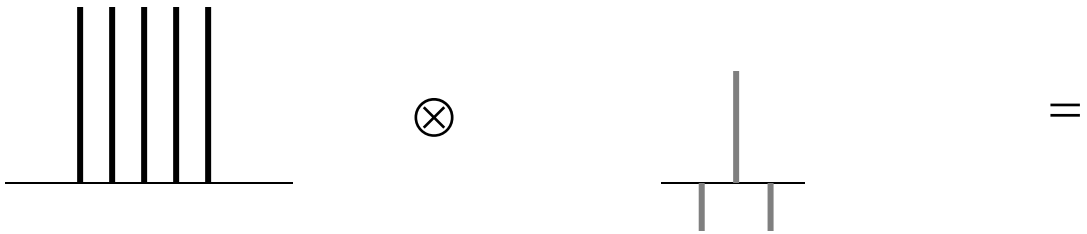


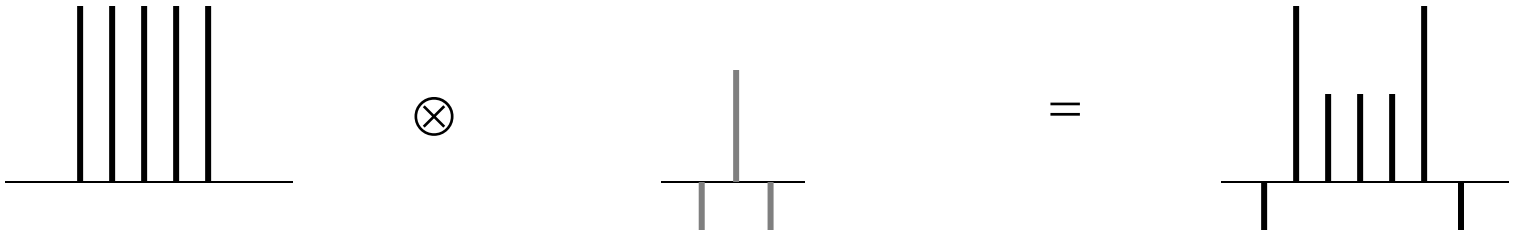
$-$



$=$







High response at edge

IMAGE SHARPENING

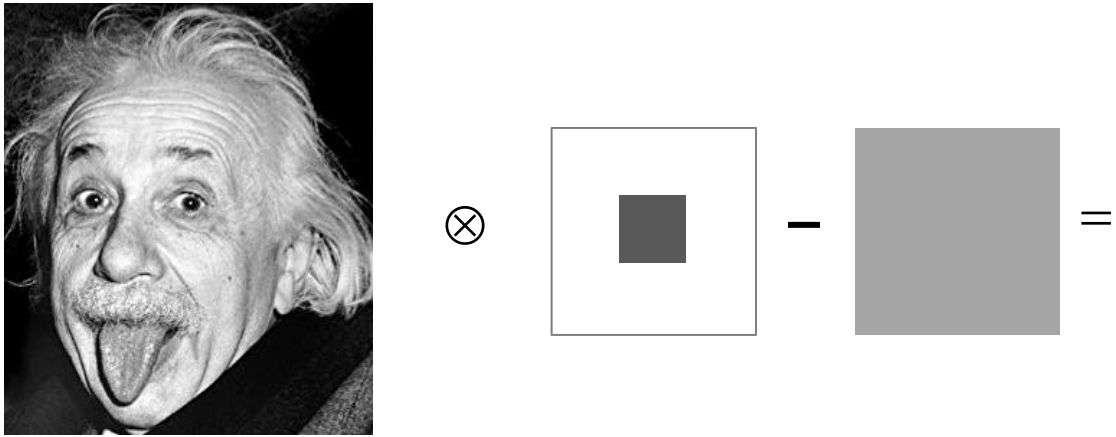
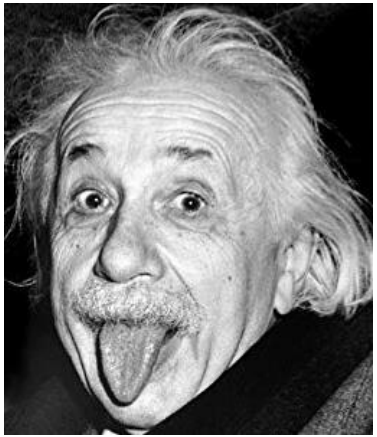
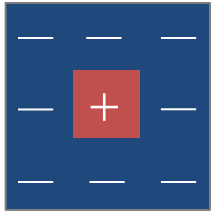


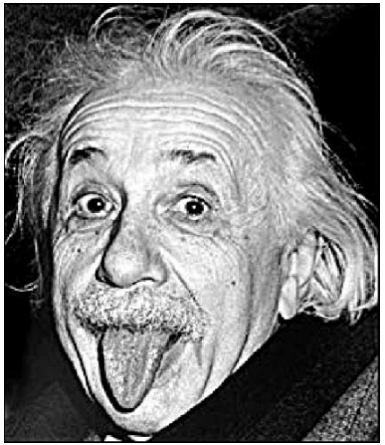
IMAGE SHARPENING



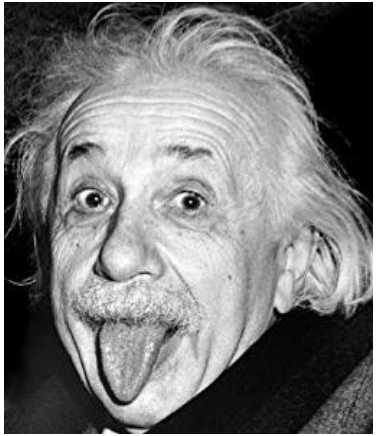
⊗



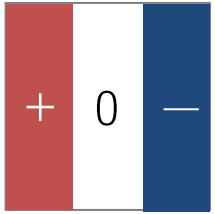
=



Sharpening

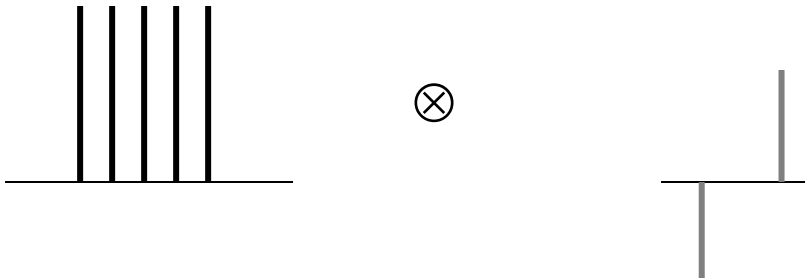


$\otimes$

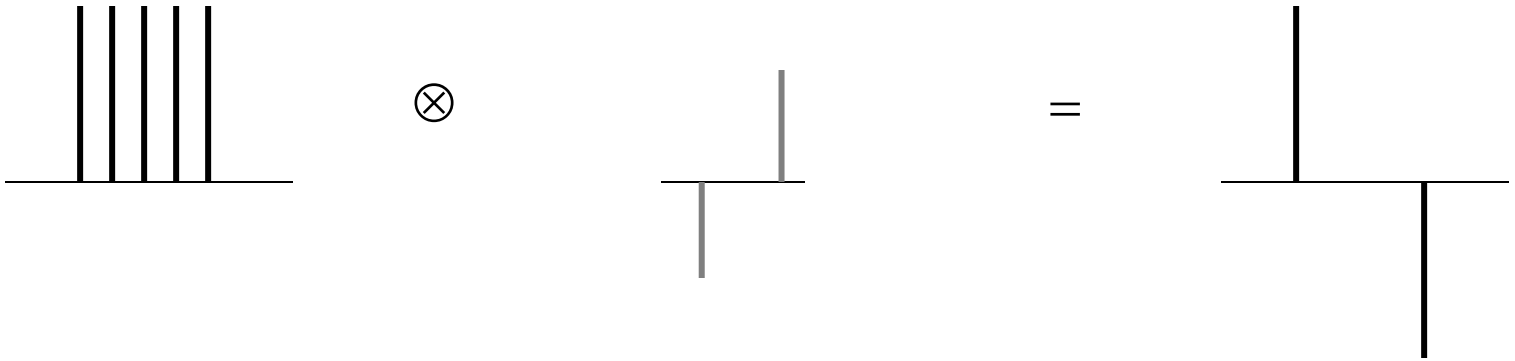


$=$

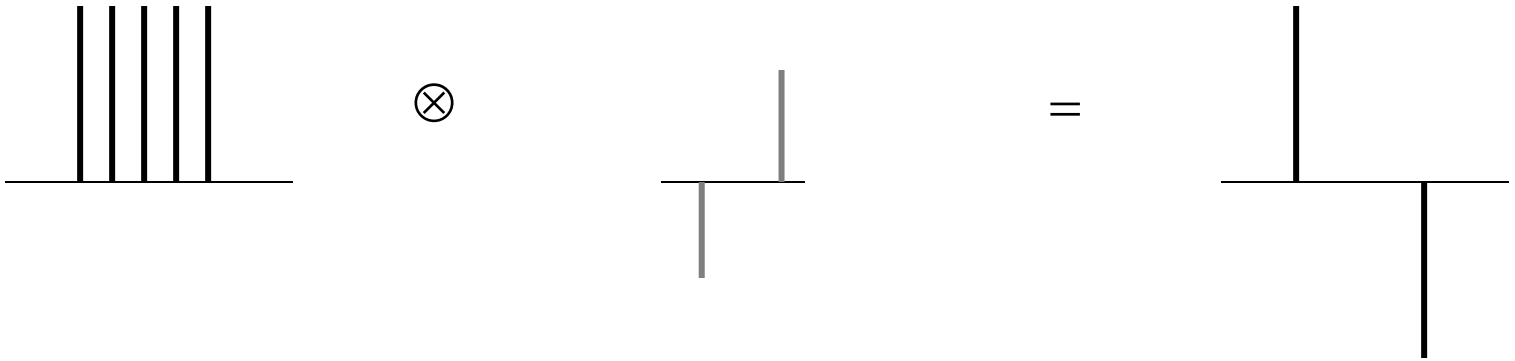
DIFFERENTIATION



DIFFERENTIATION

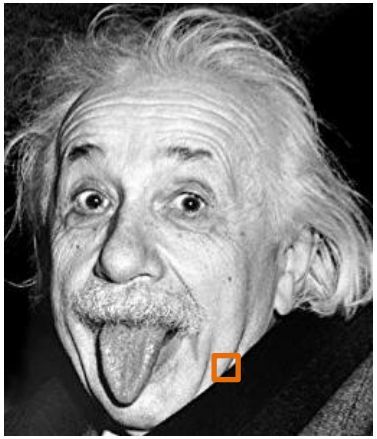


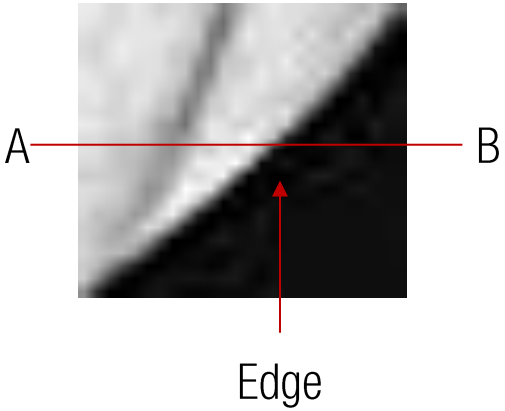
DIFFERENTIATION

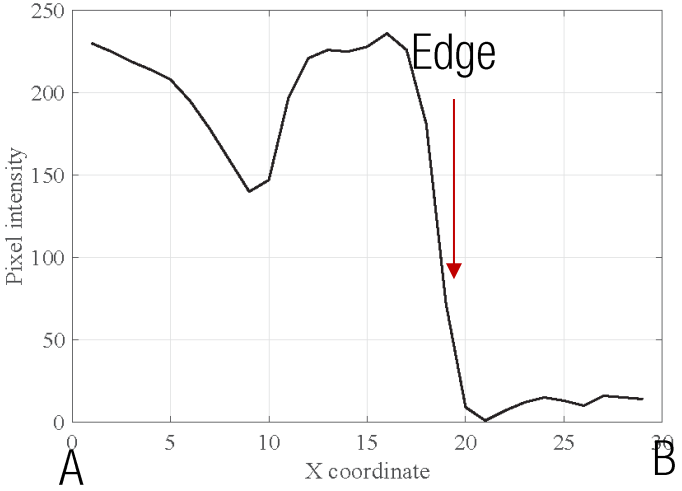


$$\frac{df}{du} = \lim_{h \to 0} \frac{f(u+h) - f(u-h)}{2h} \longrightarrow I \otimes z = \frac{\partial I}{\partial u}$$

DIFFERENTIATION







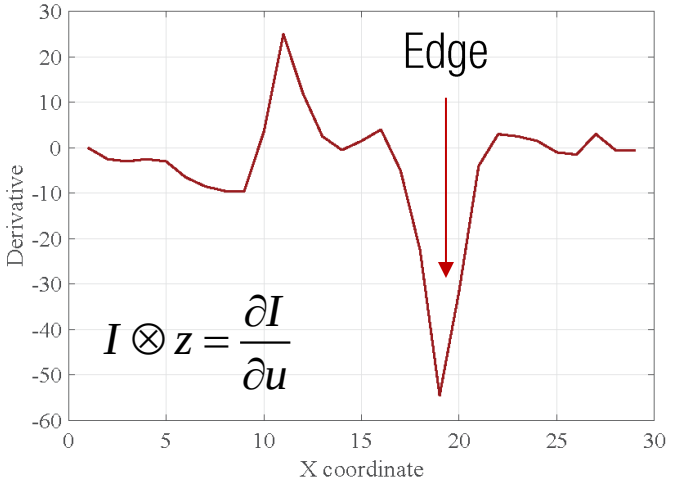
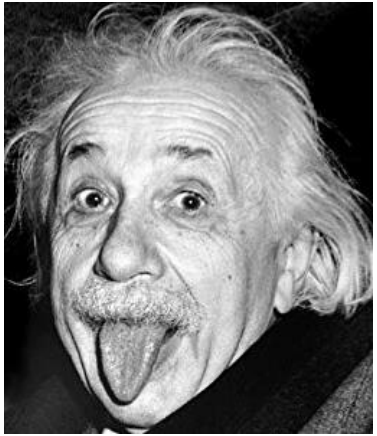
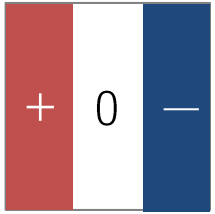


IMAGE DIFFERENTIATION

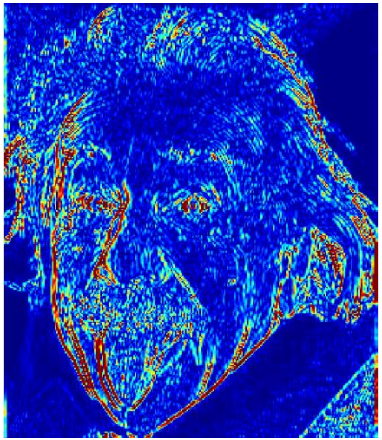


$\otimes$



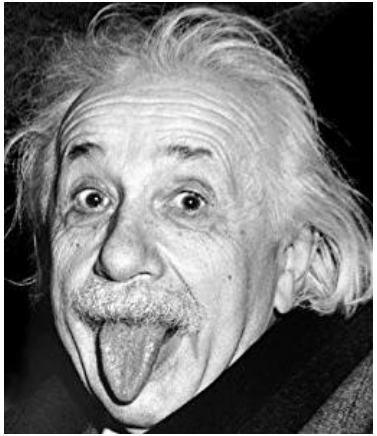
$$I \otimes z = \frac{\partial I}{\partial u}$$

$=$

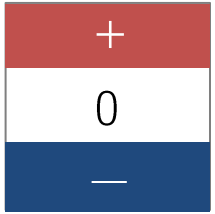


Differentiation

IMAGE DIFFERENTIATION

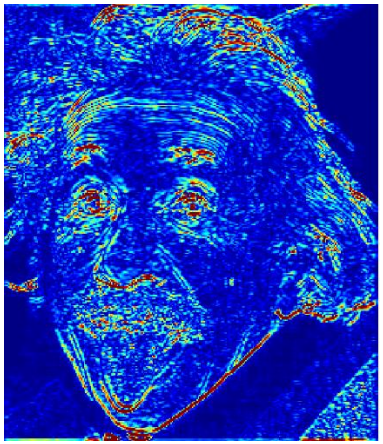


$\otimes$



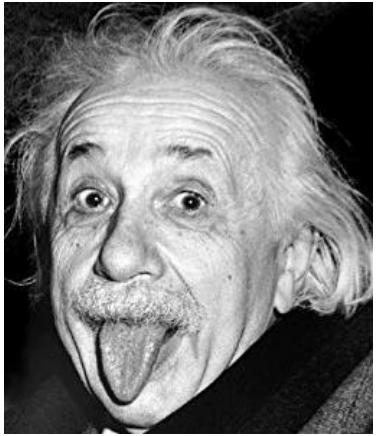
$$I \otimes z = \frac{\partial I}{\partial v}$$

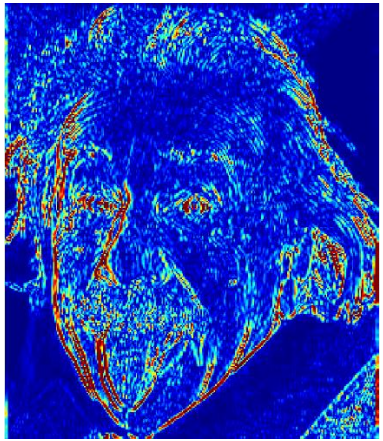
$=$

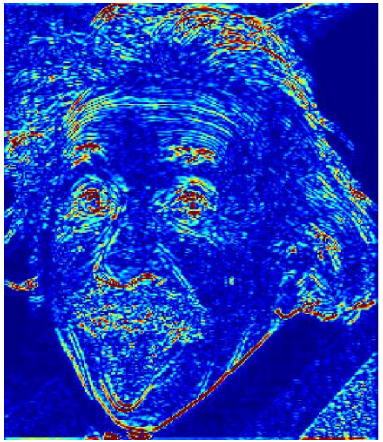


Differentiation

EDGE RESPONSE







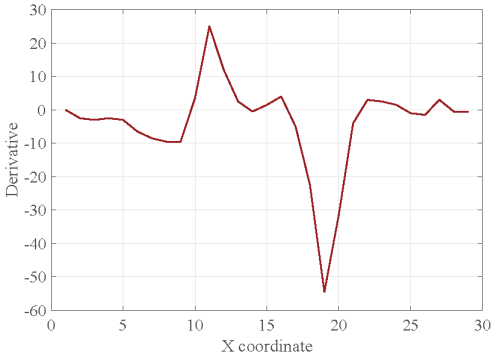


Image differentiation can be used for edge detection but it is VERY noisy.

Nonlinear Filter