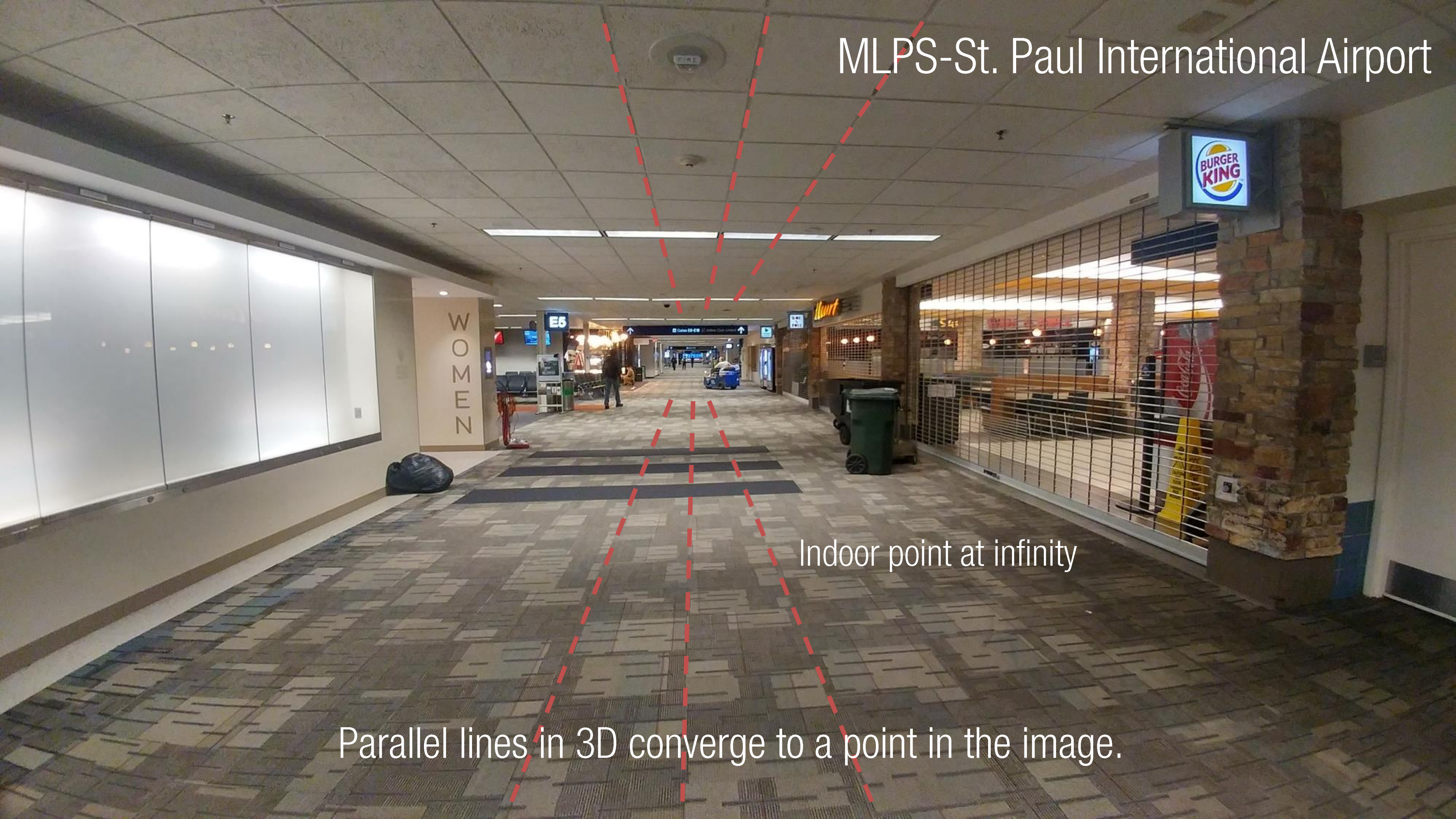
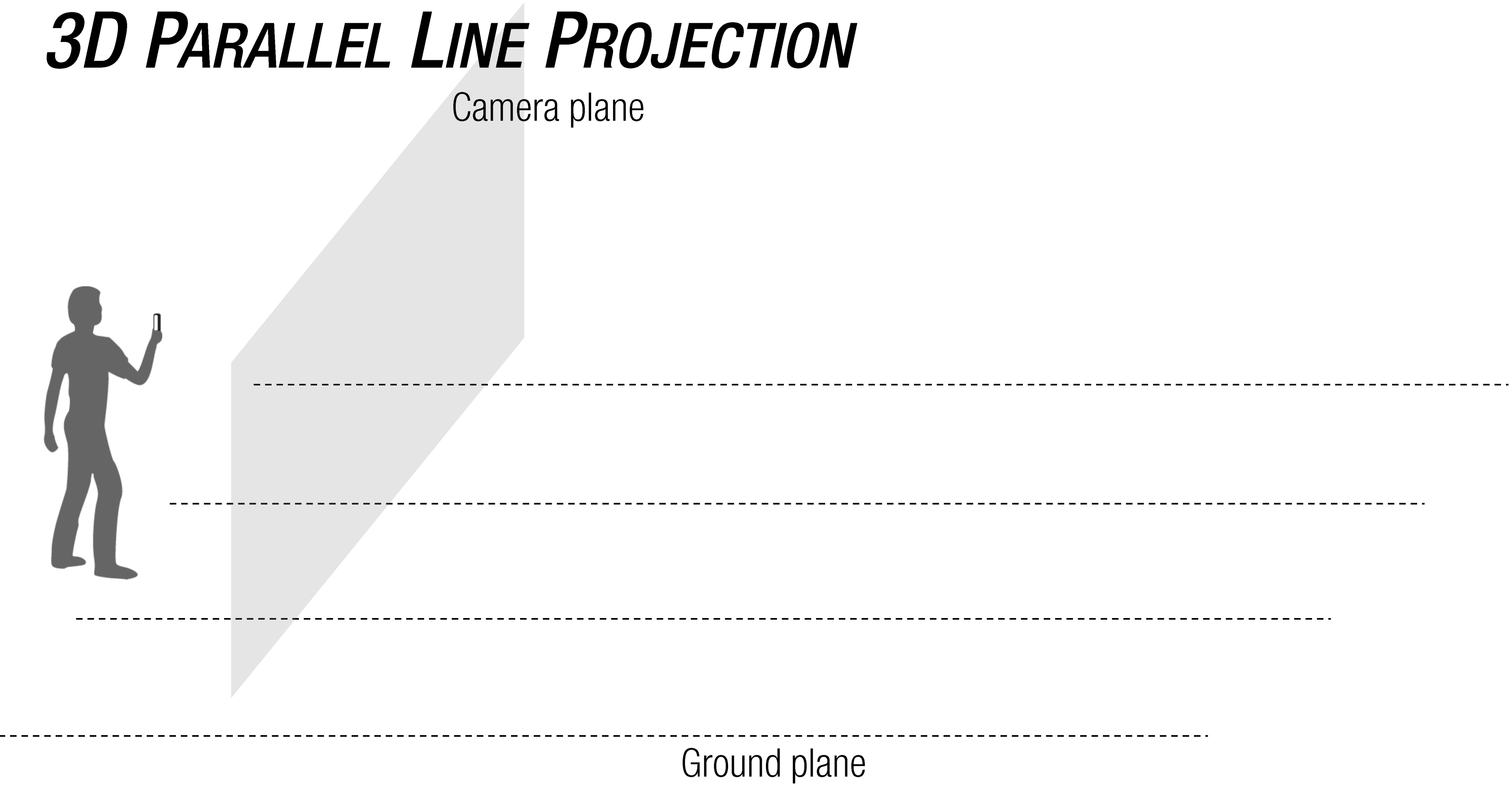
Projective Line

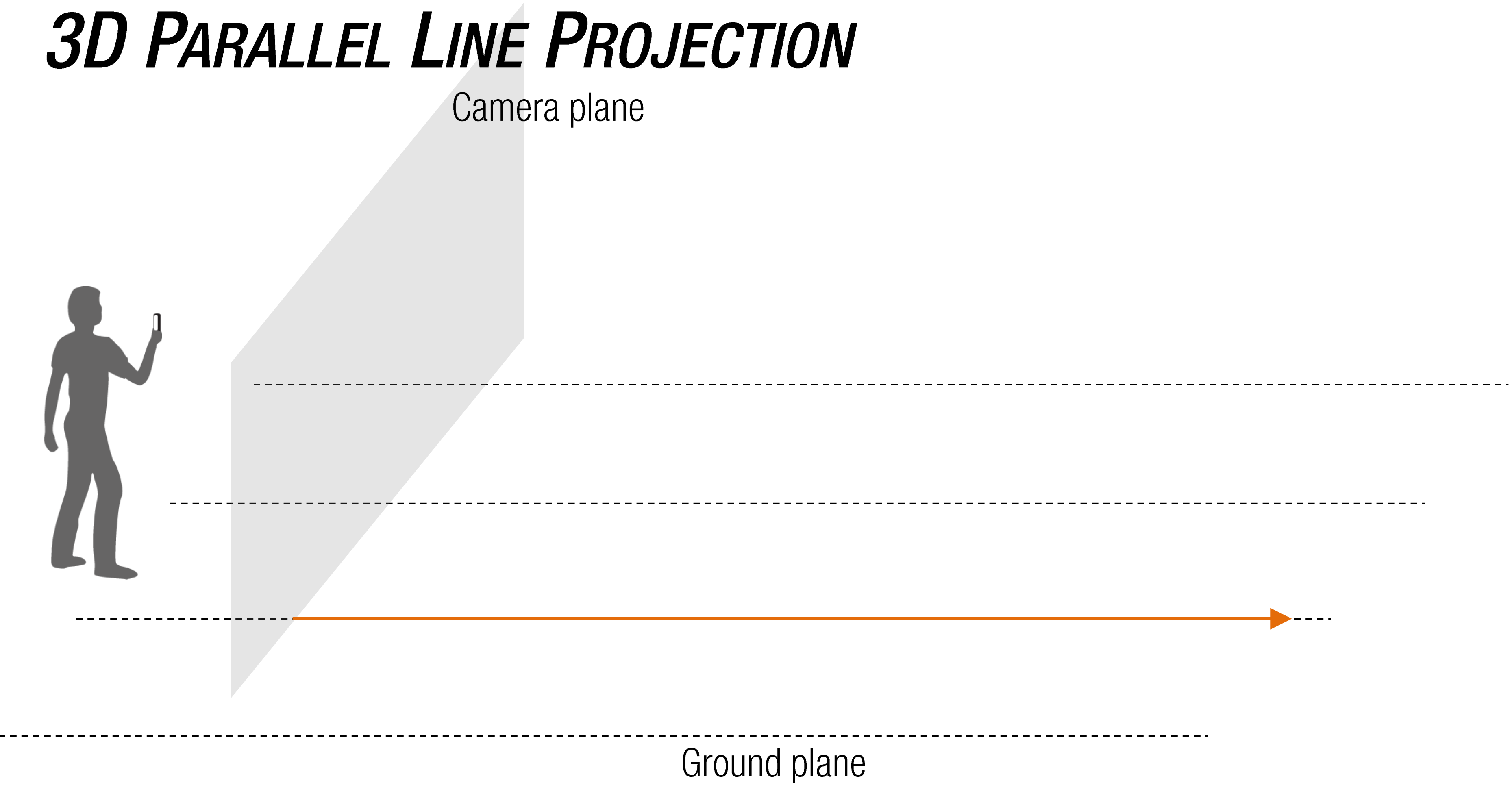
Hyun Soo Park


MLPS-St. Paul International Airport
BURGER KING
WOMEN
E5
Indoor point at infinity
Parallel lines in 3D converge to a point in the image.

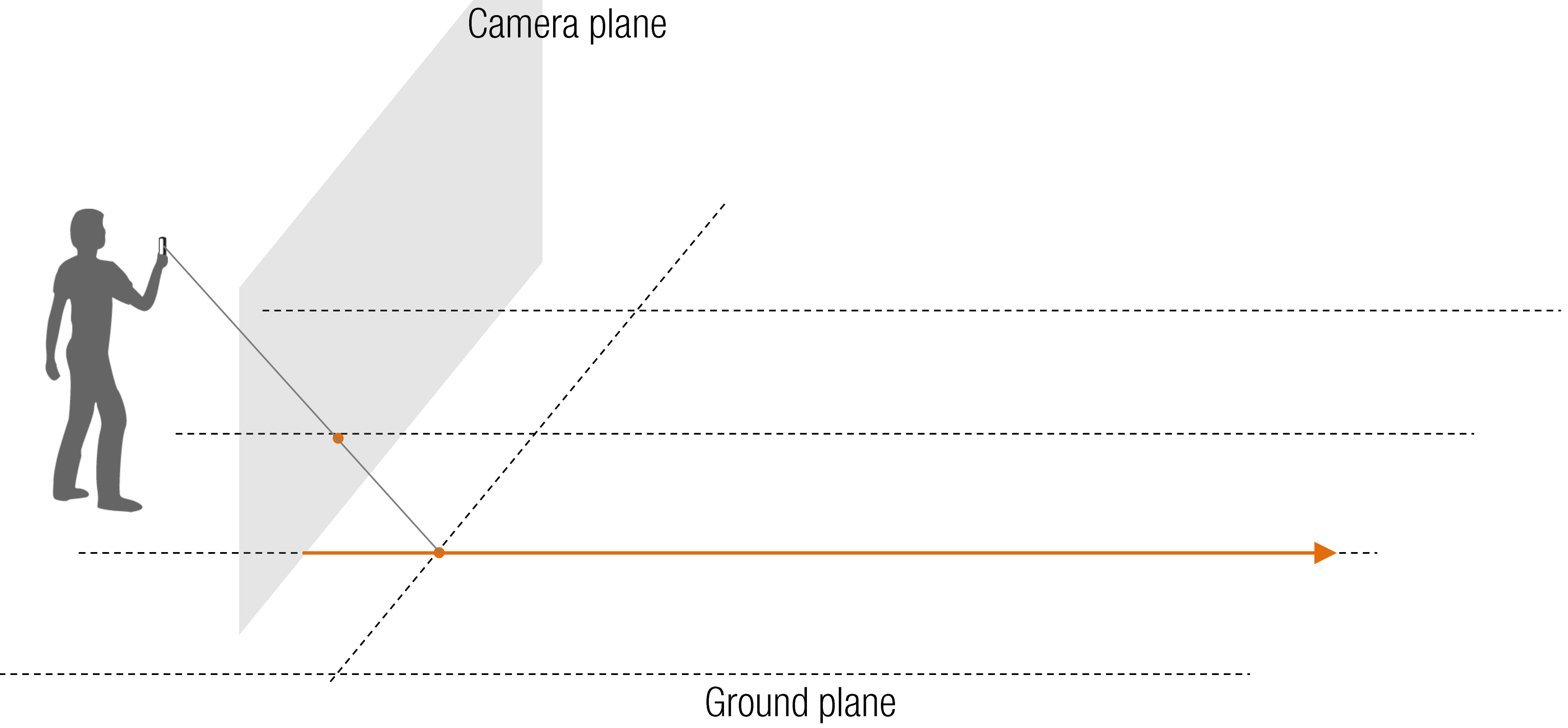
3D PARALLEL LINE PROJECTION



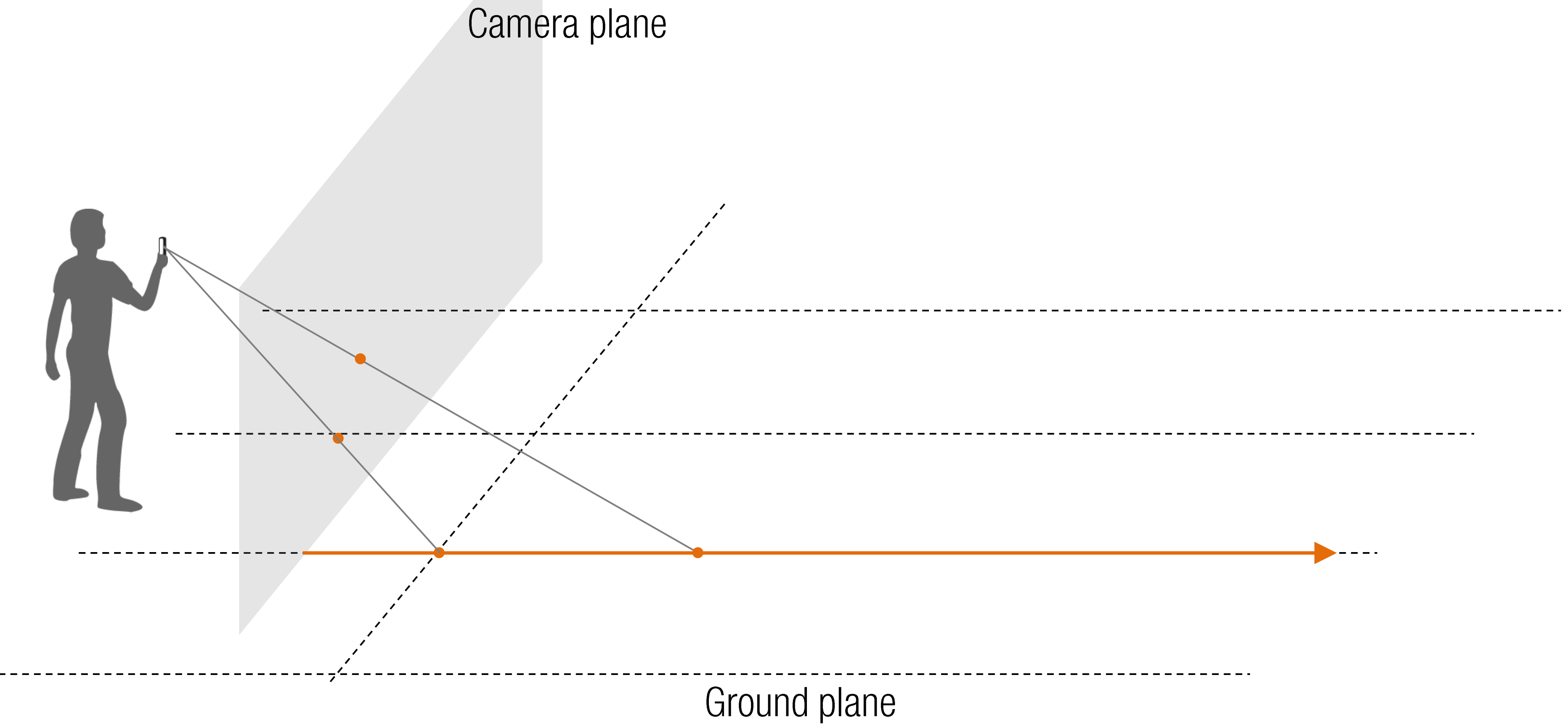
3D Parallel Line Projection



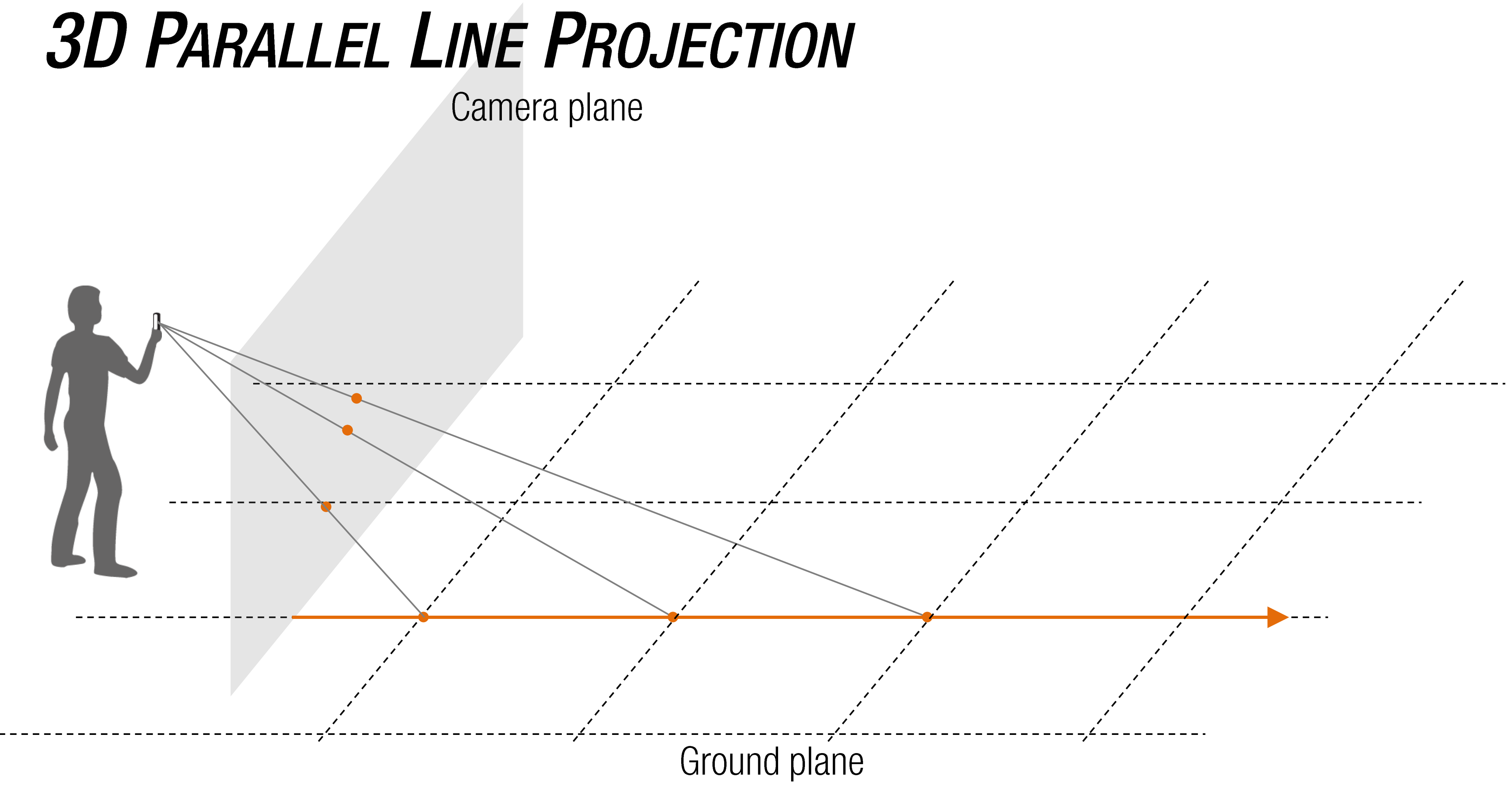
3D Parallel Line Projection



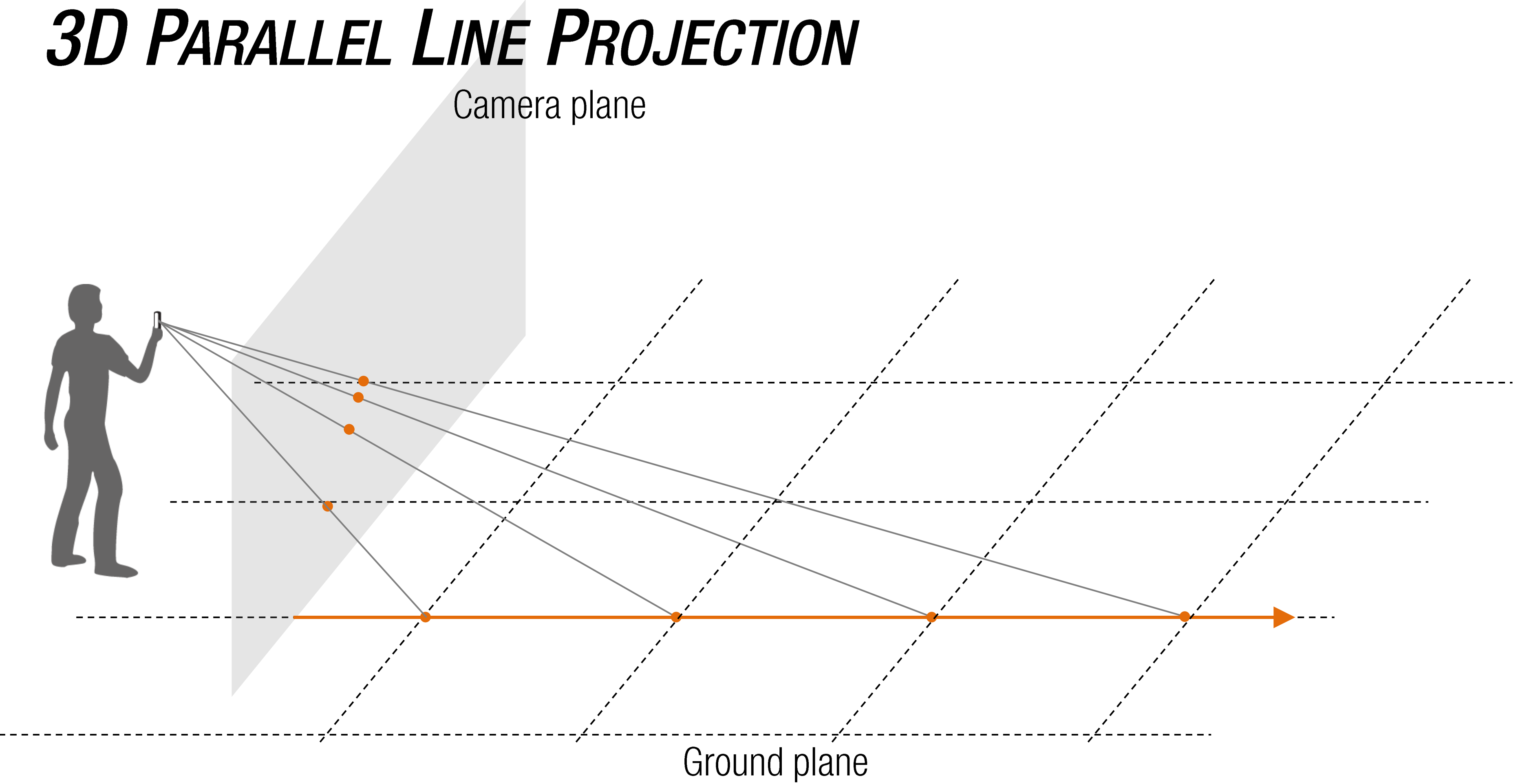
3D Parallel Line Projection



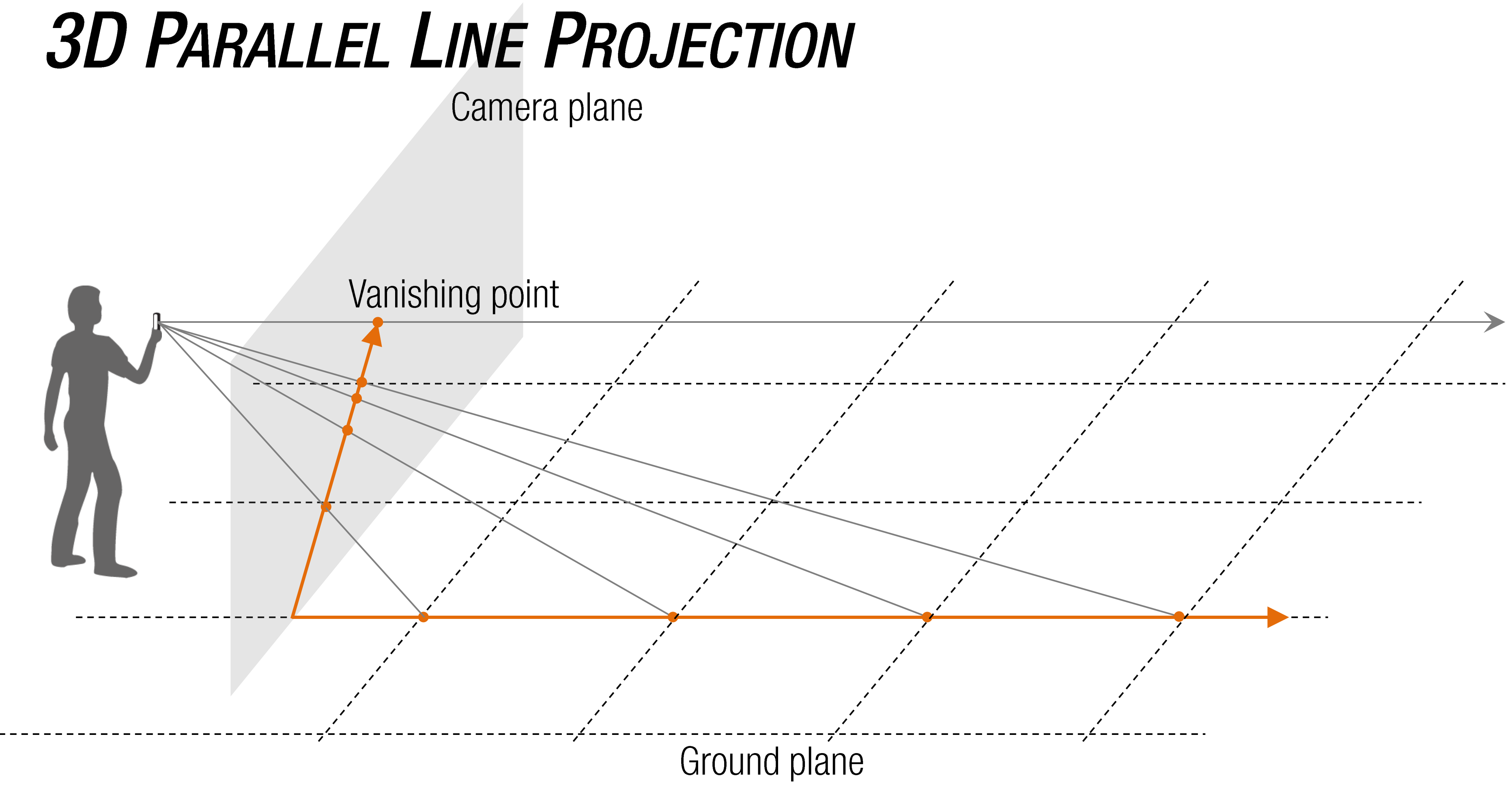
3D Parallel Line Projection



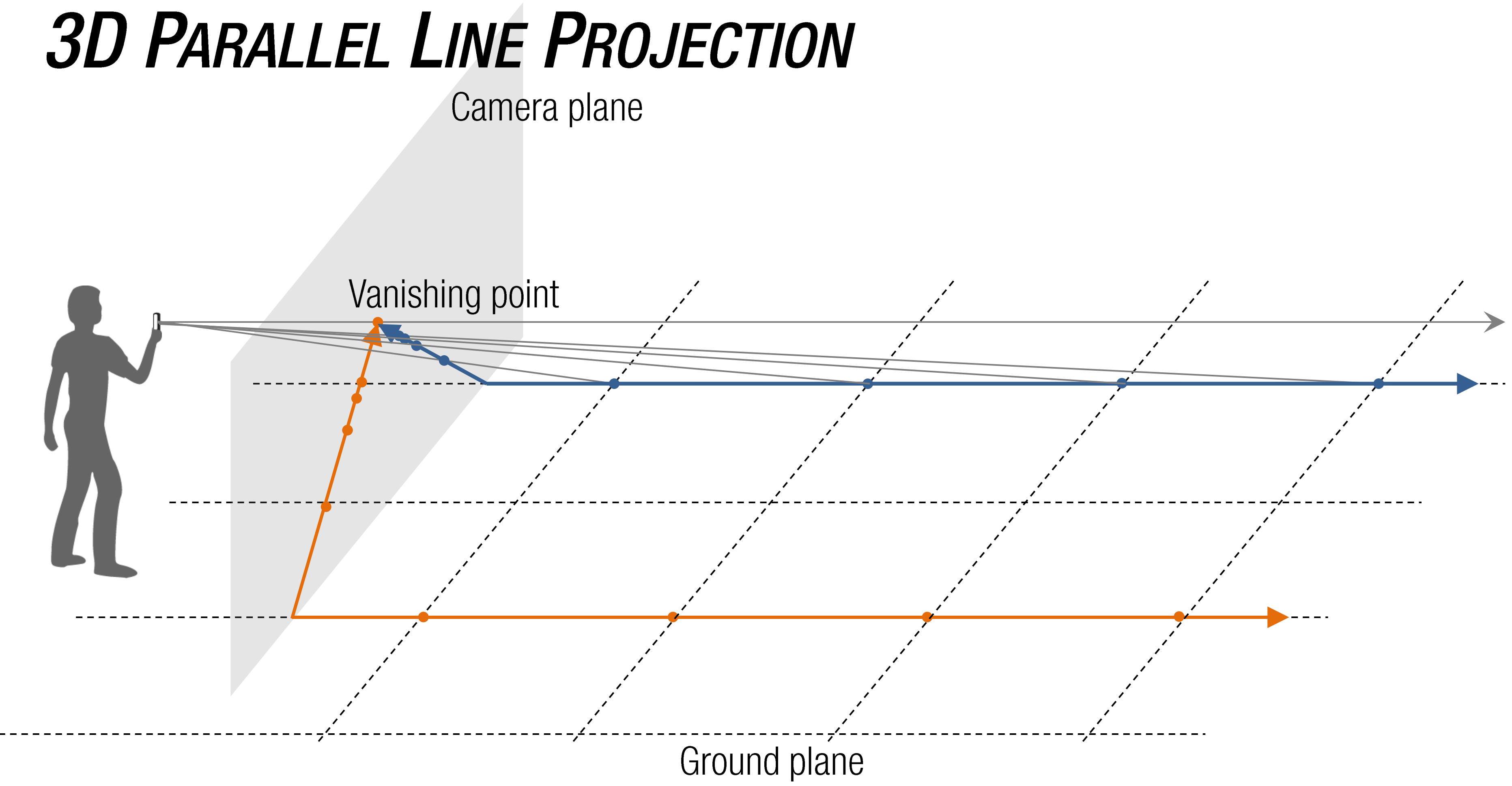
3D Parallel Line Projection



3D Parallel Line Projection

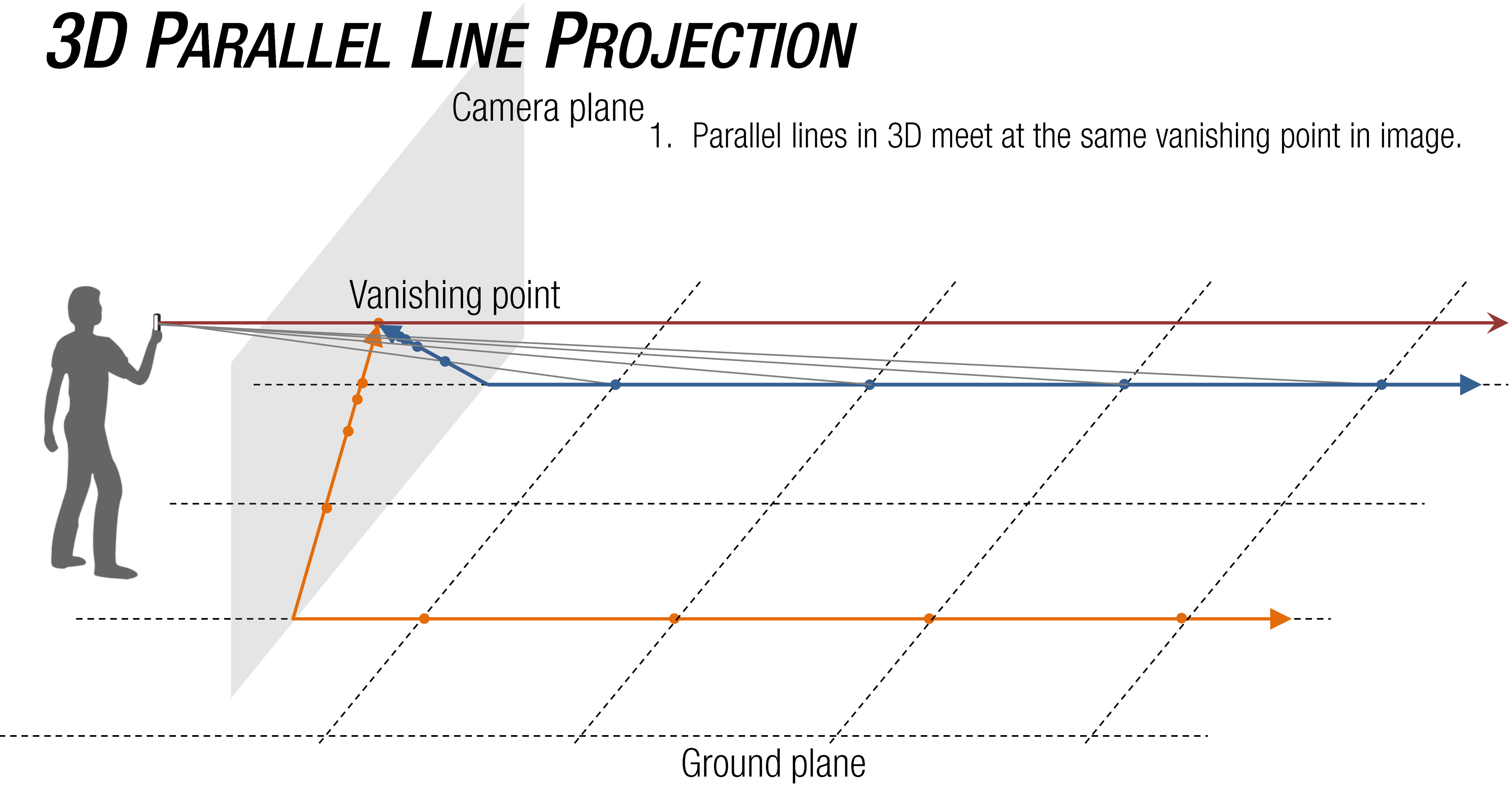


3D Parallel Line Projection



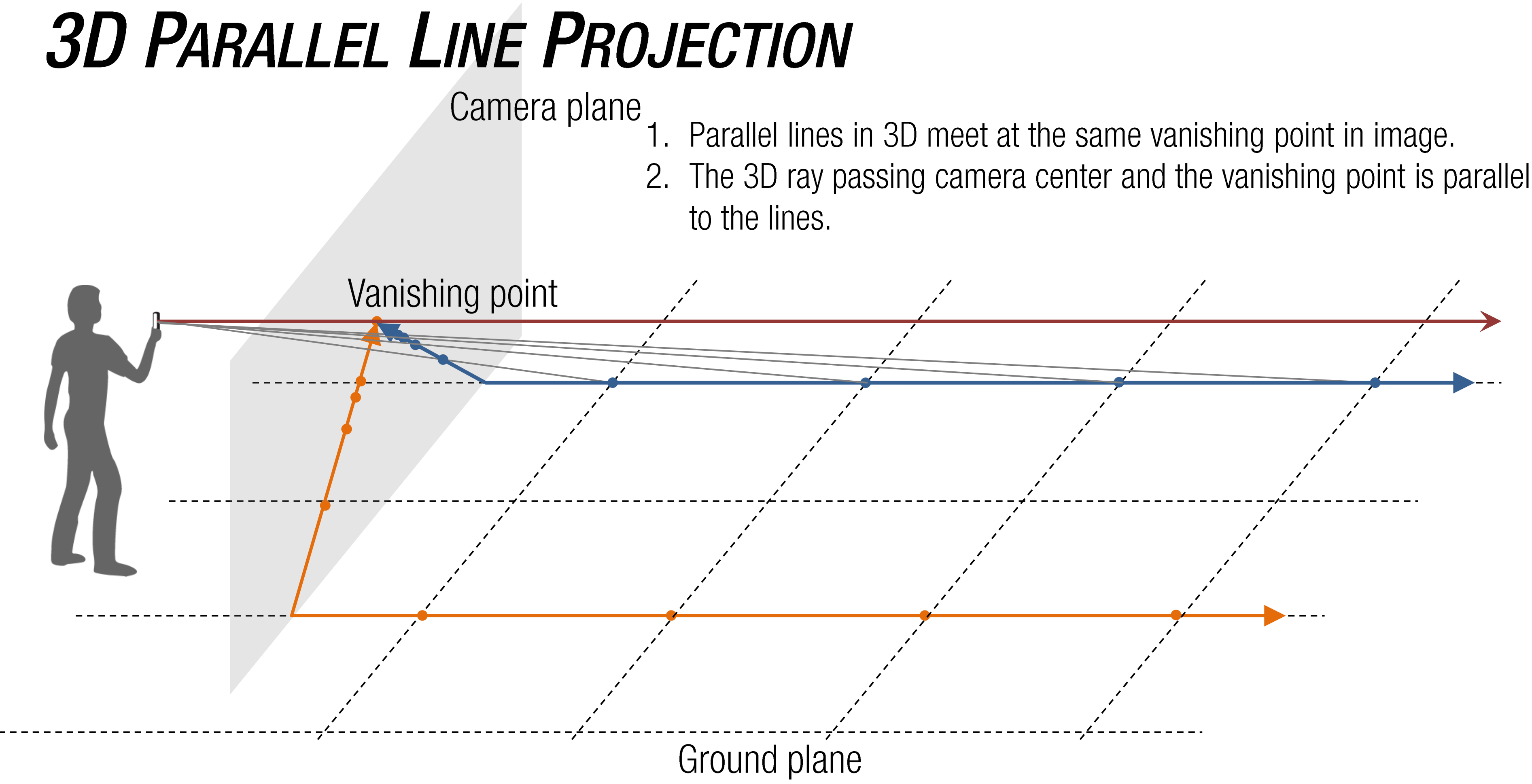
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.



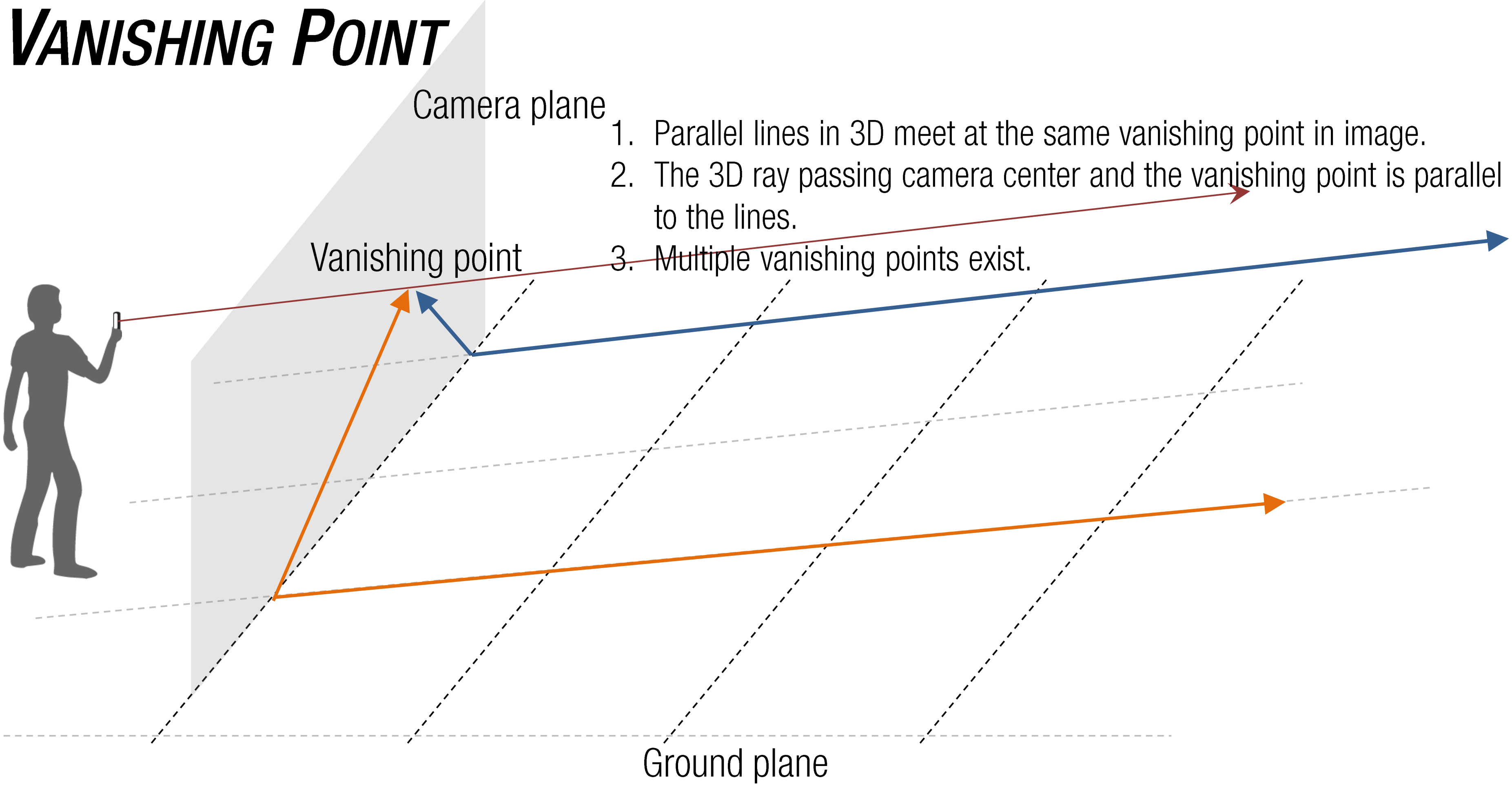
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.



VANISHING POINT

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
3. Multiple vanishing points exist.



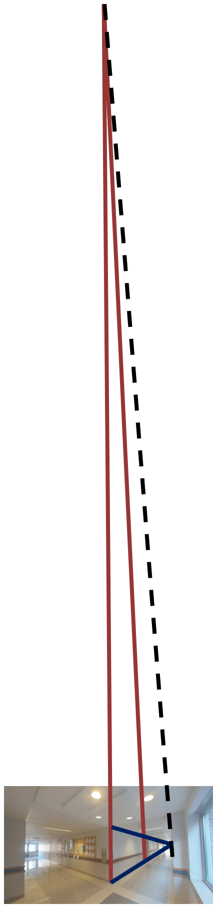

Vanishing point

Multiple vanishing point

Vanishing line: Horizon

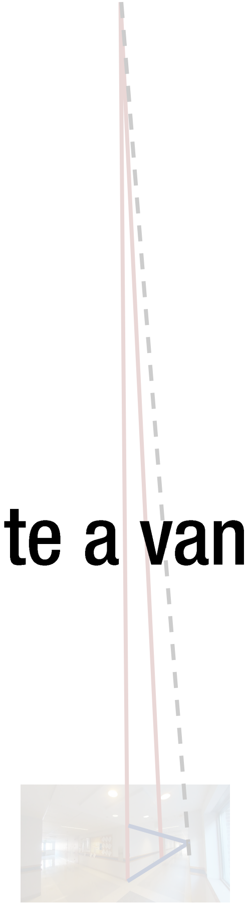
Parallel 3D planes share the vanishing line.

Different plane produces different vanishing line.



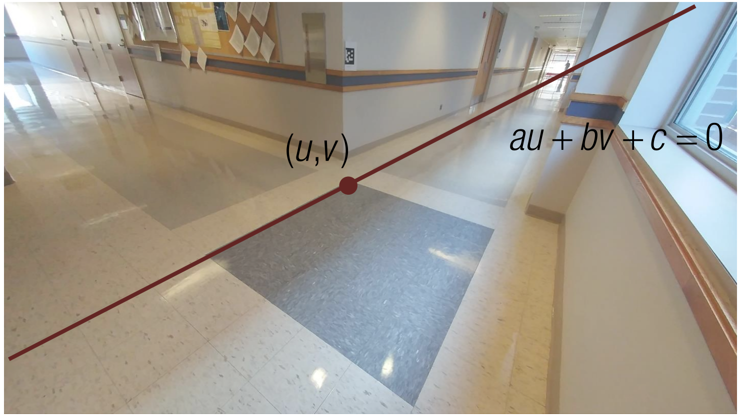
Different plane produces different vanishing line.

How to compute a vanishing point?



Different plane produces different vanishing line.

POINT-LINE

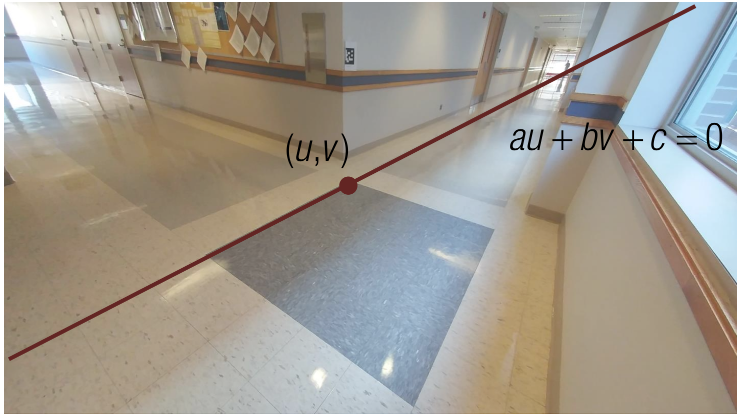


A 2D line passing through 2D point (u,v):

$au + bv + c = 0$

Line parameter: (a,b,c)

POINT-LINE



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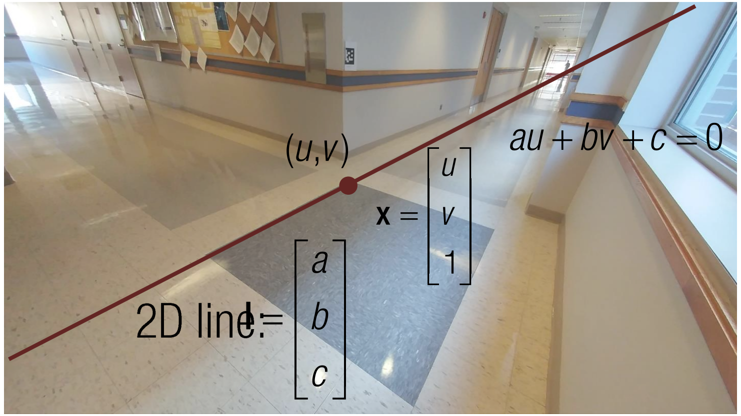
$au+bv+c=0$

Line parameter: (a,b,c)

$$au+bv+c=0 \rightarrow \begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{l}^{\mathsf{T}}\mathbf{x} = 0$$

Point-Line



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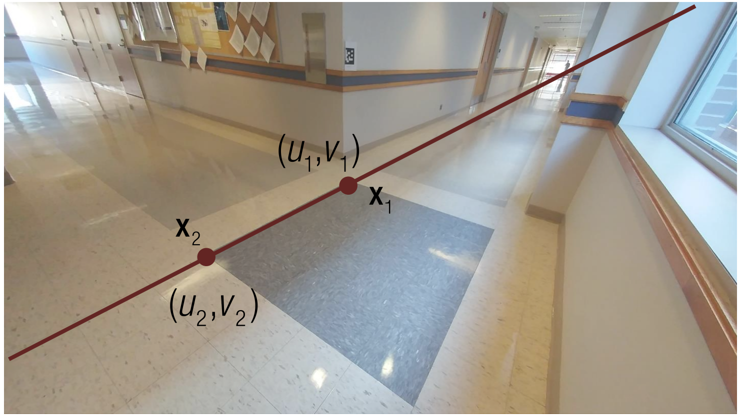
where $\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$ and $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

2D point Line parameter

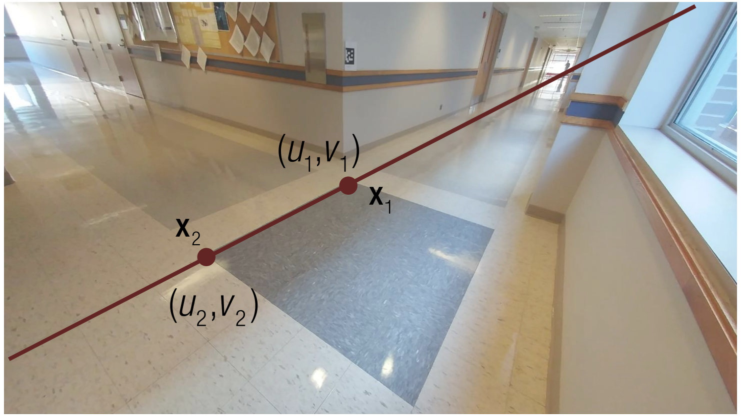
POINT-LINE



A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

Point-Line



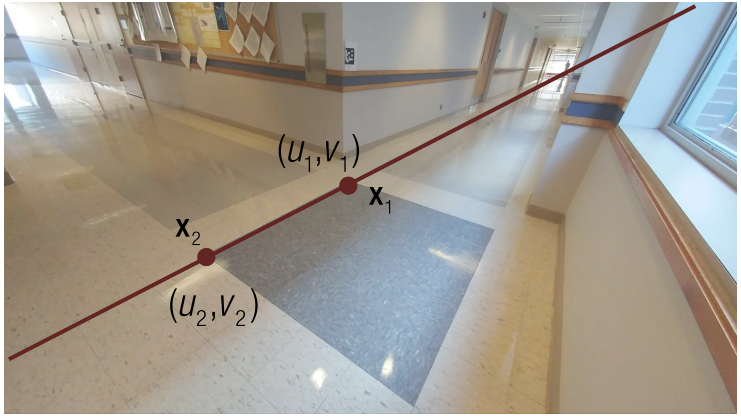
A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

$$\mathbf{x}_1^{\mathsf{T}}\mathbf{l} = 0 \qquad \mathbf{x}_2^{\mathsf{T}}\mathbf{l} = 0$$

where $\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$ $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

Point-Line



A 2D line passing through two 2D points:

$$au_1 + bv_1 + c = 0 \qquad au_2 + bv_2 + c = 0$$

$$\mathbf{x}_1^\top \mathbf{l} = 0 \qquad \mathbf{x}_2^\top \mathbf{l} = 0$$

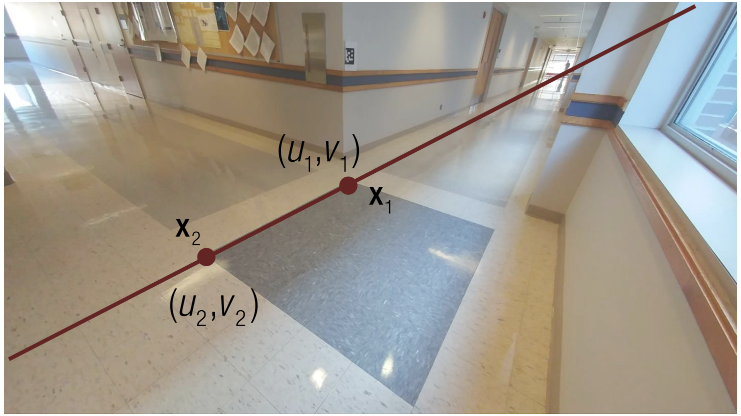
where $\mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}$ $\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$\rightarrow \begin{bmatrix} \mathbf{x}_1^\top \\ \mathbf{x}_2^\top \end{bmatrix} \mathbf{l} = \mathbf{0}$$

$$\mathbf{A} \; \blacksquare = \begin{matrix} 0 \\ 0 \end{matrix}$$

2x3

Point-Line



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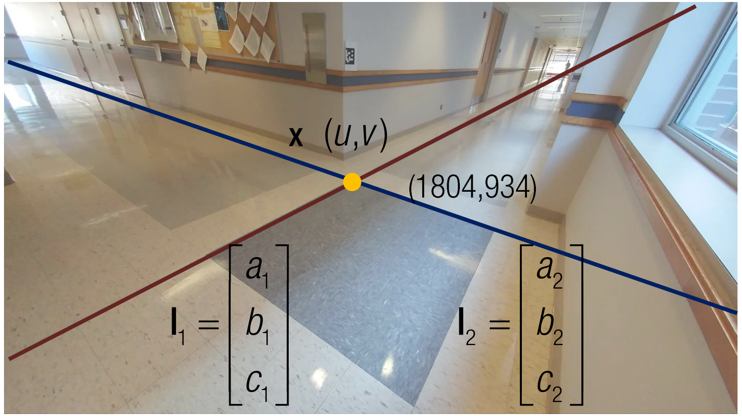
$$\rightarrow \begin{bmatrix} \mathbf{x}_1^{\mathsf{T}} \\ \mathbf{x}_2^{\mathsf{T}} \end{bmatrix} \mathbf{l} = \mathbf{0}$$

$$\mathbf{A}\,\blacksquare = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow \quad \blacksquare = \mathrm{null}(\mathbf{A}) \quad \text{or} \quad \mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2$$

2x3

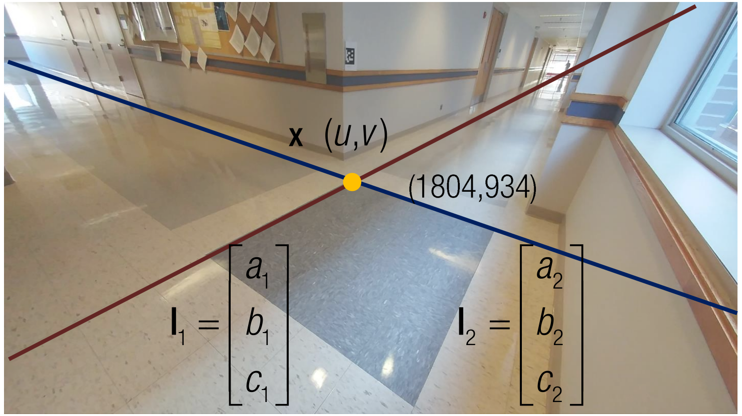
LINE-LINE



Two 2D lines in an image intersect at a 2D point:

$$a_1u + b_1v + c_1 = 0 \qquad a_2u + b_2v + c_2 = 0$$

LINE-LINE



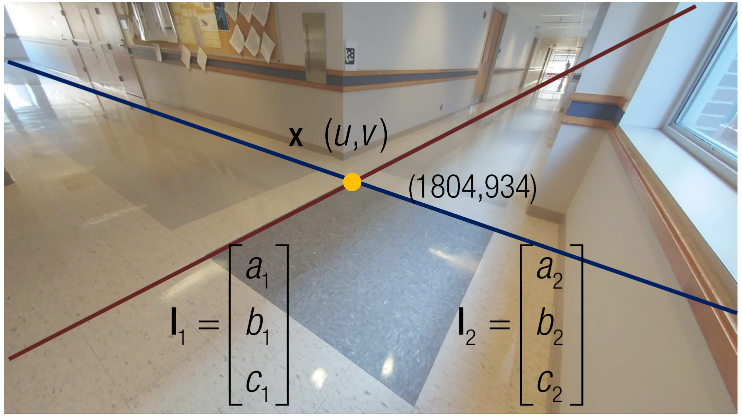
Two 2D lines in an image intersect at a 2D point:

$$a_1 u + b_1 v + c_1 = 0 \qquad a_2 u + b_2 v + c_2 = 0$$

$$\mathbf{l}_1^{\mathsf{T}} \mathbf{x} = 0 \qquad \mathbf{l}_2^{\mathsf{T}} \mathbf{x} = 0$$

where $\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

LINE-LINE



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where $\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

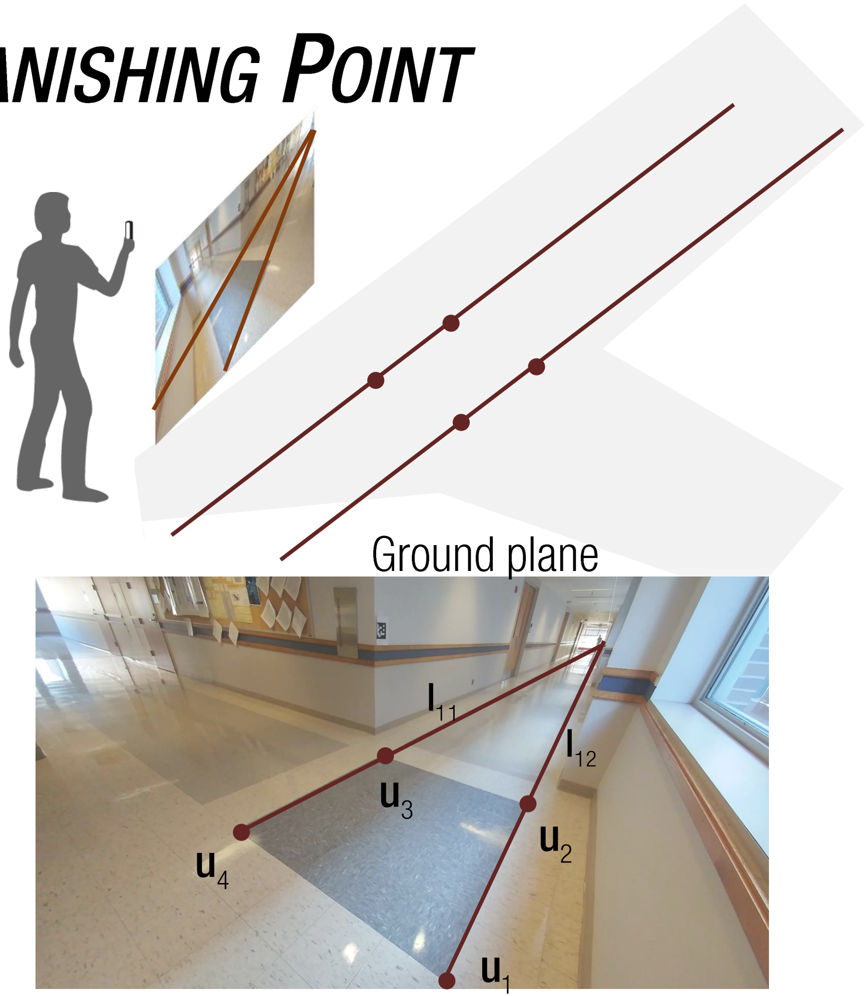
$$\rightarrow \begin{bmatrix} \mathbf{l}_1^{\mathsf{T}} \\ \mathbf{l}_2^{\mathsf{T}} \end{bmatrix} \mathbf{x} = \mathbf{0}$$

$$\mathbf{A} \, \blacksquare = \begin{matrix} 0 \\ 0 \end{matrix} \quad \rightarrow \quad \blacksquare = \text{null}\left(\mathbf{A} \right)$$

2x3

or $\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$

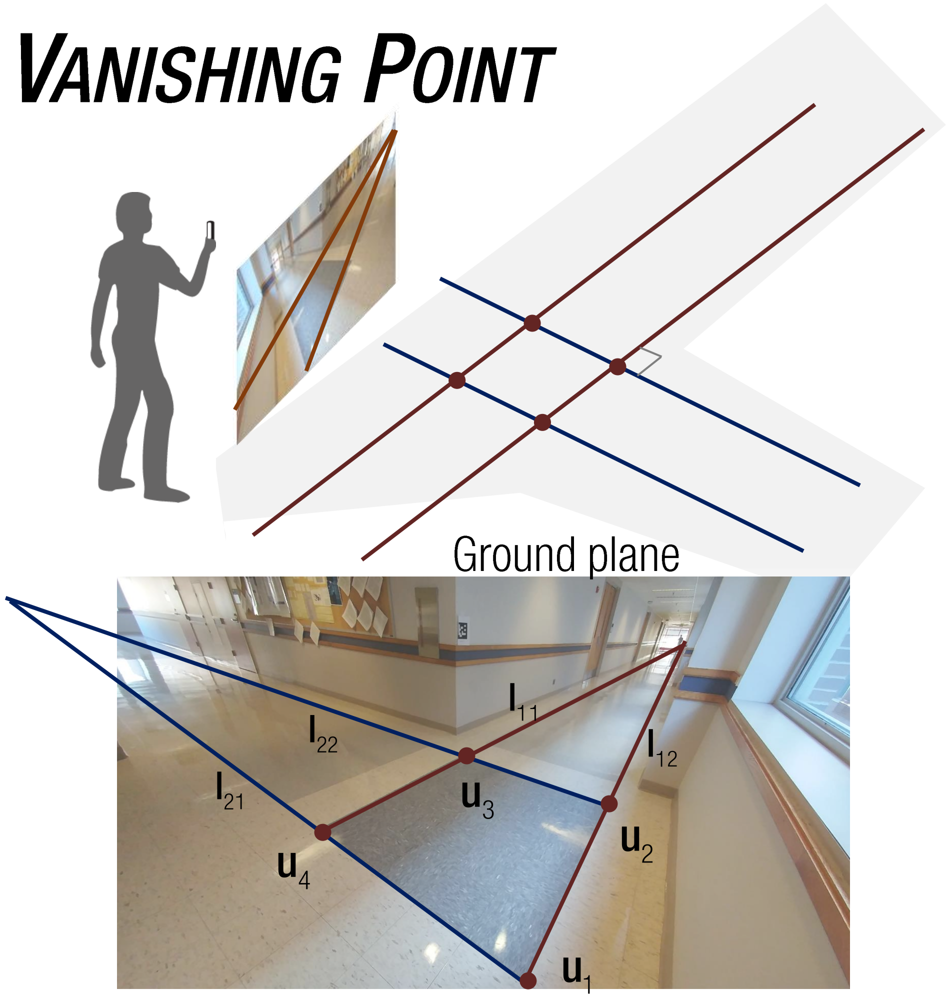
VANISHING POINT



Parallel lines:

$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3$ $\quad$ $\mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$

VANISHING POINT

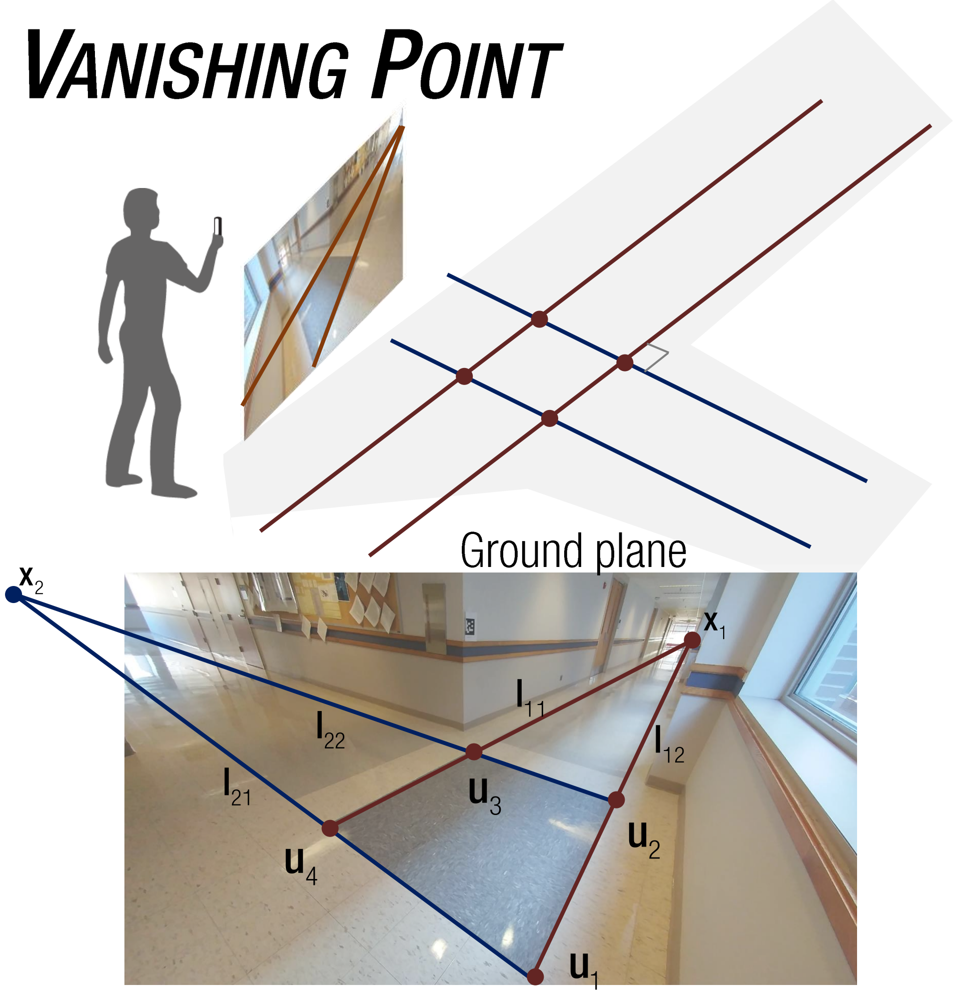


Parallel lines:

$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3$ $\quad$ $\mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$

$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1$ $\quad$ $\mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$

VANISHING POINT



Parallel lines:

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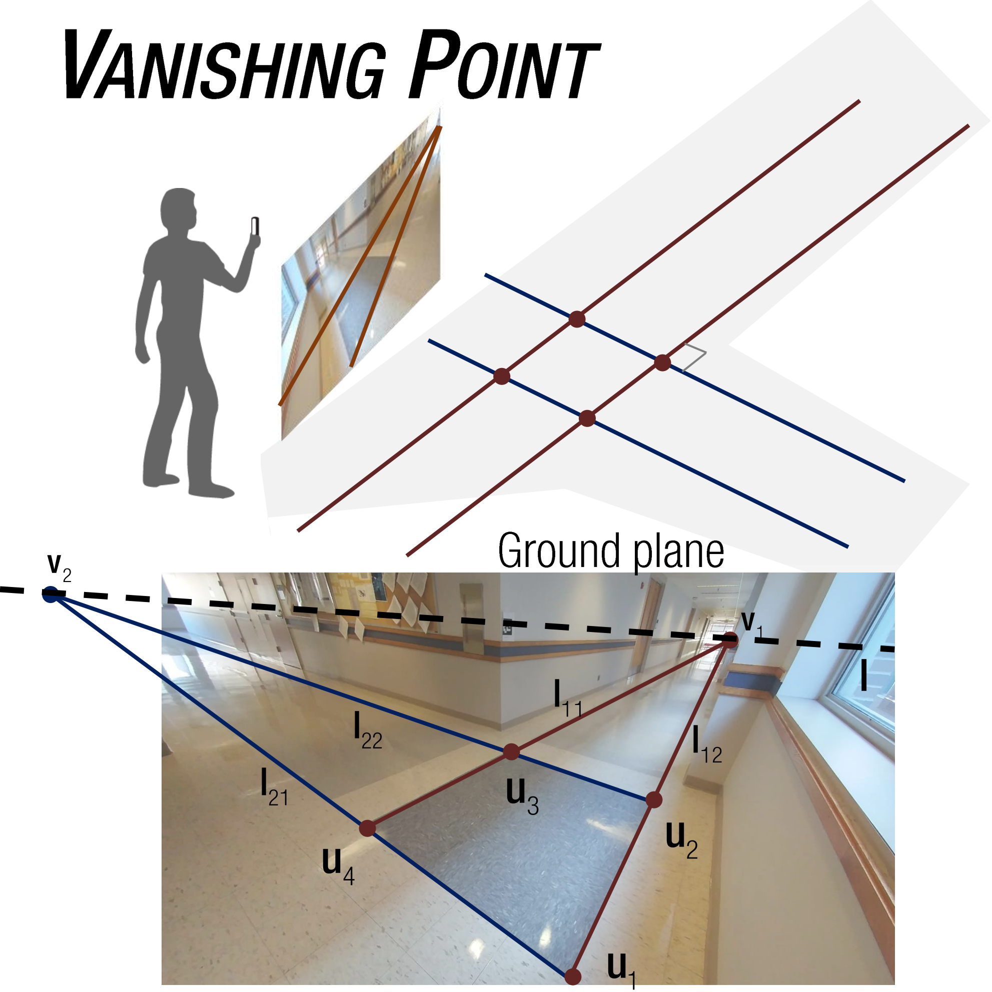
$$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \qquad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$$

Vanishing points:

$$\mathbf{x}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \qquad \mathbf{x}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22}$$

Vanishing Point



Parallel lines:

$$\mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \qquad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2$$

$$\mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \qquad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4$$

Vanishing points:

$$\mathbf{v}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \qquad \mathbf{v}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22}$$

Vanishing line:

$$\mathbf{l} = \mathbf{v}_1 \times \mathbf{v}_2$$

Geometric Interpretation of Vanishing Line

Side view

WHERE WAS I?


WHERE WAS I?

Taken from my hotel room (6th floor)

Taken from beach