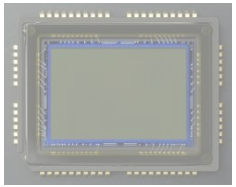
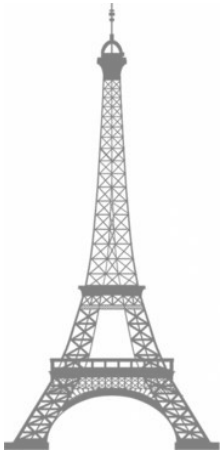
Camera Model

Hyun Soo Park

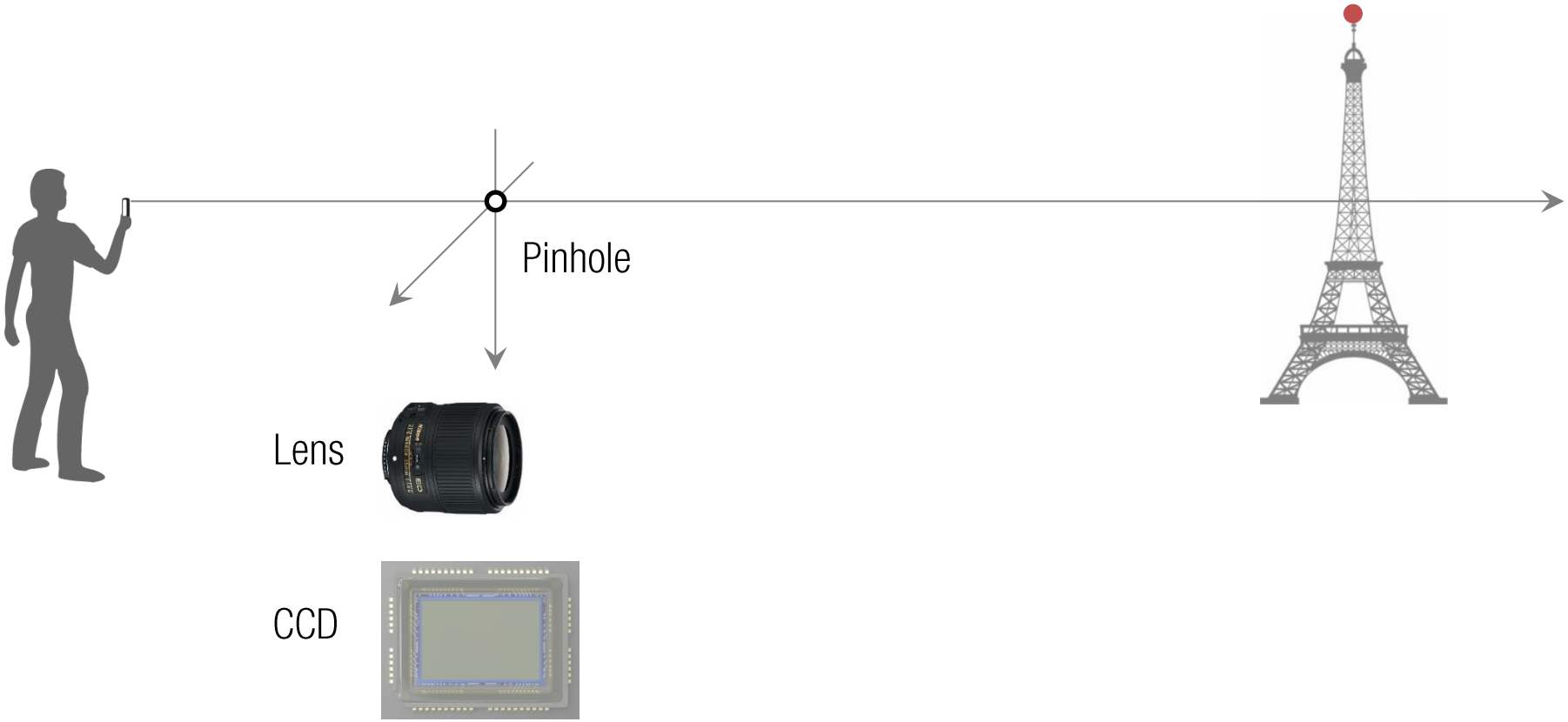
CAMERA ANATOMY

Lens 

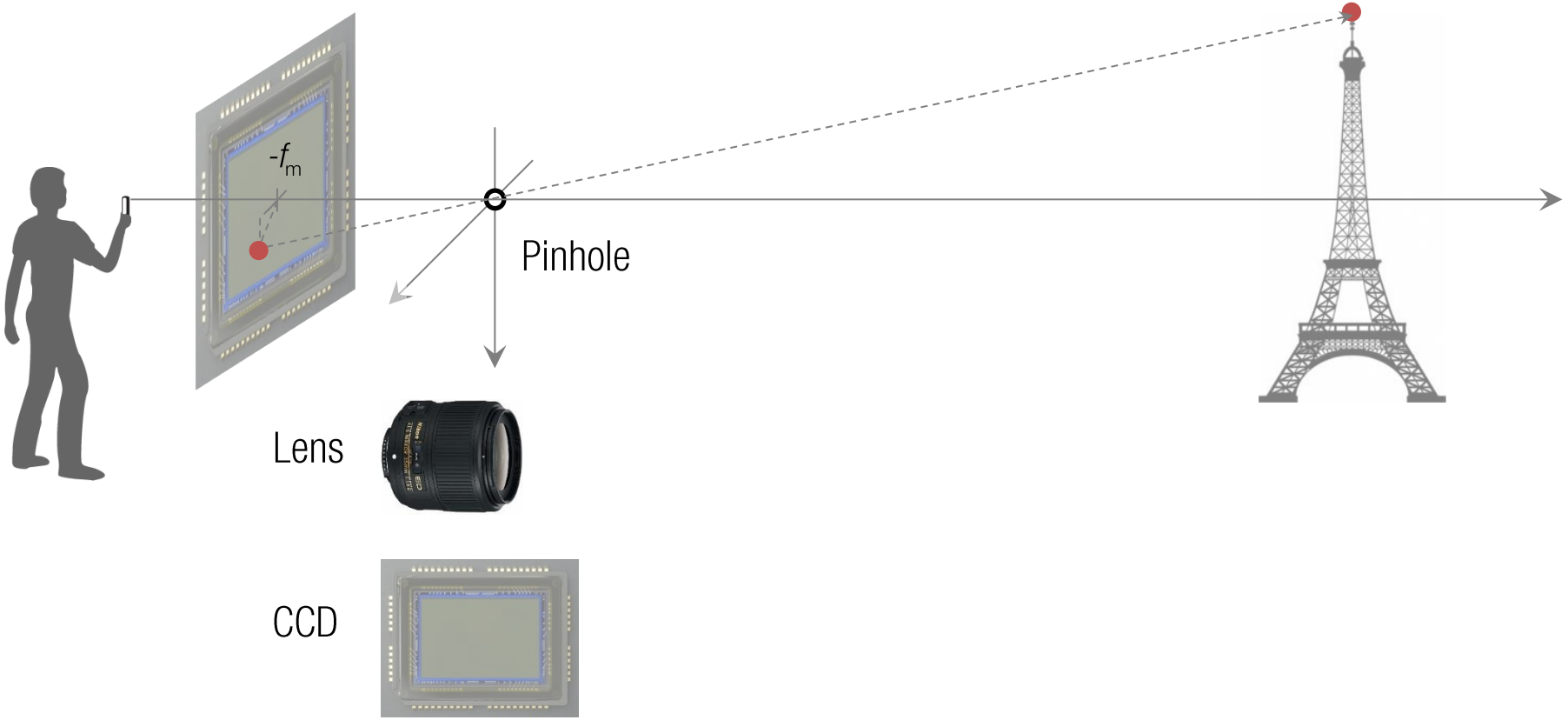
CCD 



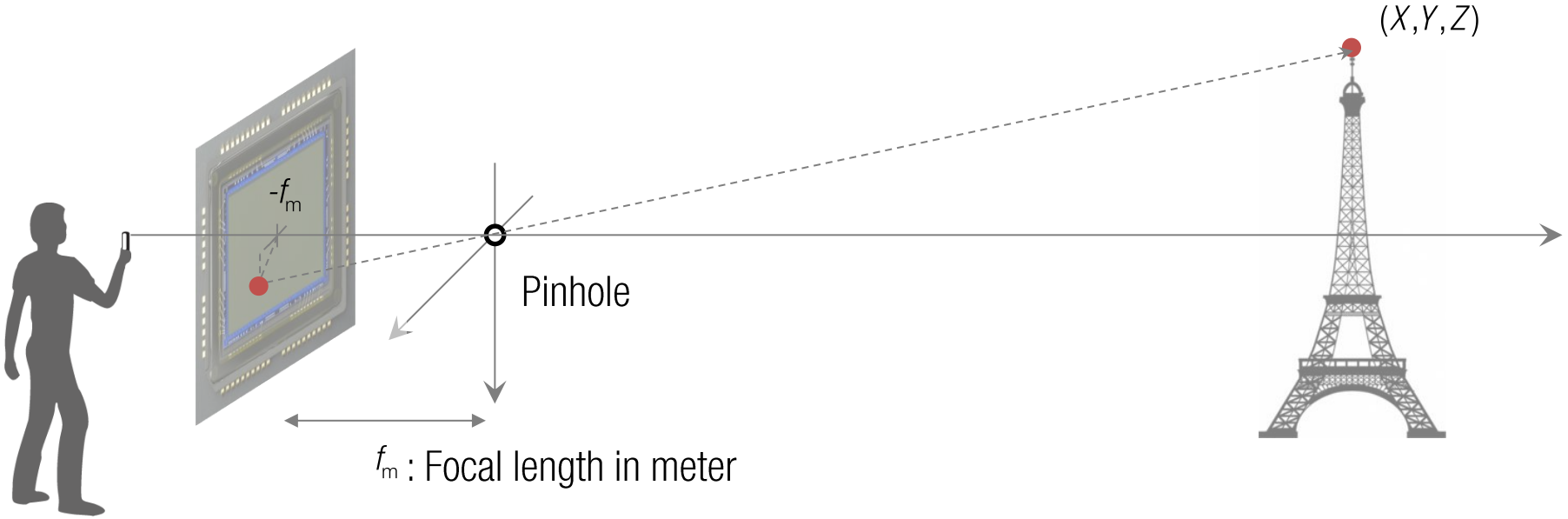
CAMERA ANATOMY



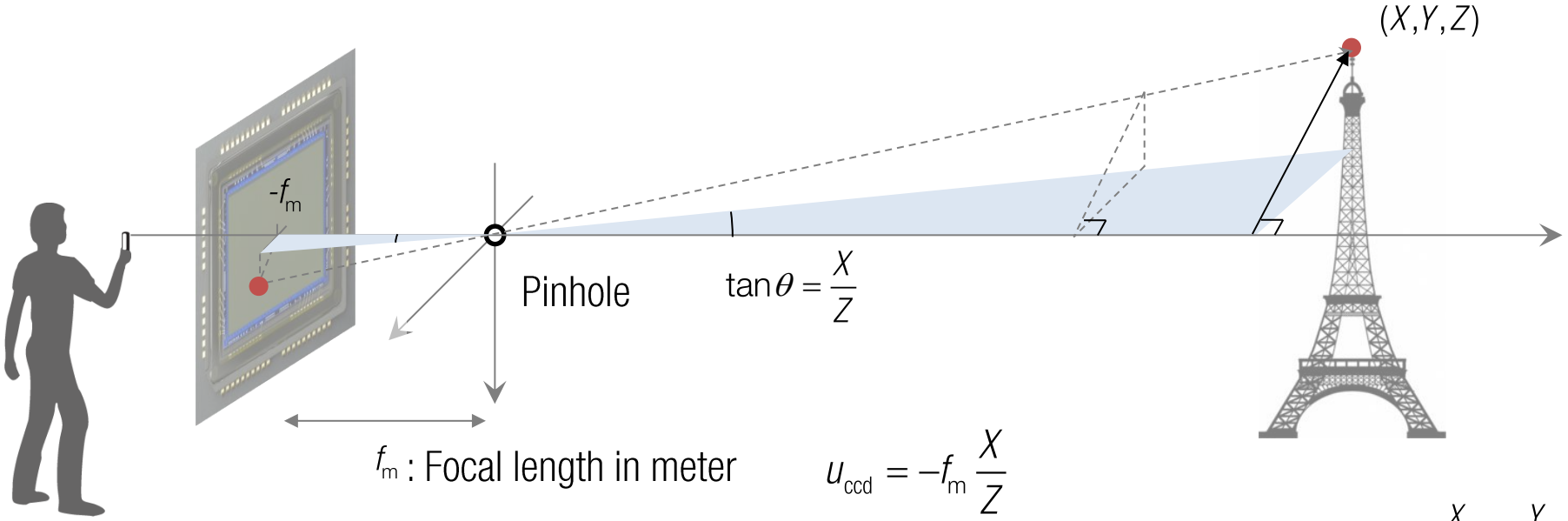
CAMERA ANATOMY



CAMERA ANATOMY



CAMERA ANATOMY



f_m : Focal length in meter

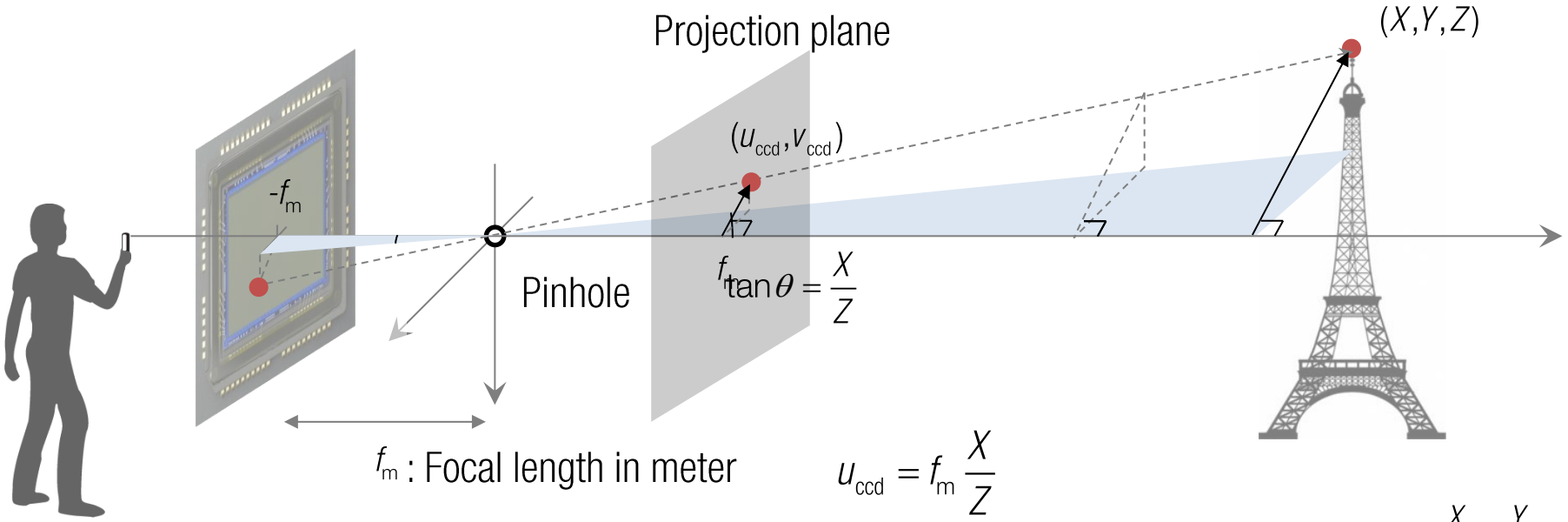
$$u_{ccd} = -f_m \frac{X}{Z}$$

$$v_{ccd} = -f_m \frac{Y}{Z}$$

$$(X,Y,Z) \rightarrow (u_{ccd}, v_{ccd}) = (-f_m \frac{X}{Z}, -f_m \frac{Y}{Z})$$

2D projection onto CCD plane

CAMERA ANATOMY



f_m : Focal length in meter

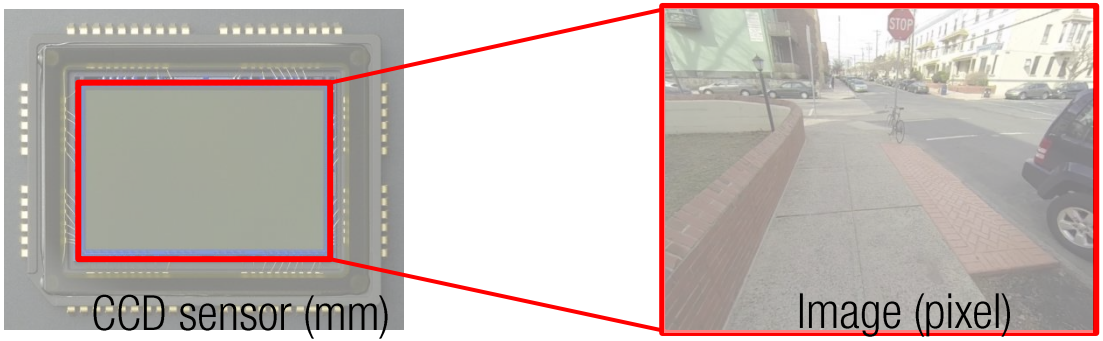
$$u_{ccd} = f_m \frac{X}{Z}$$

$$v_{ccd} = f_m \frac{Y}{Z}$$

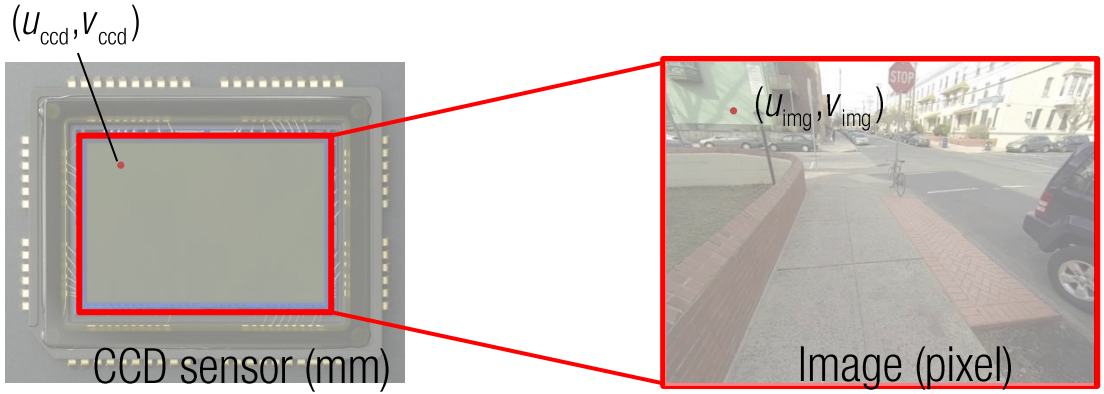
$$(X,Y,Z) \rightarrow \quad (u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$

2D projection onto CCD plane

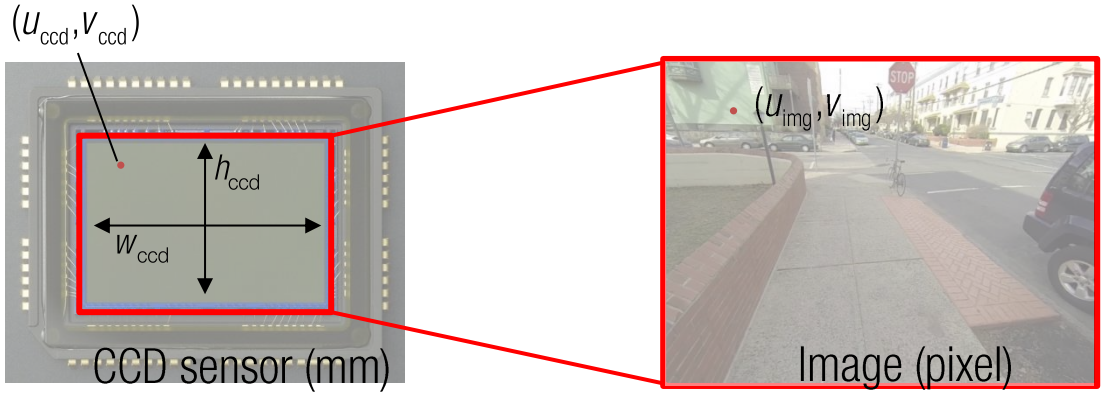
CCD to Image

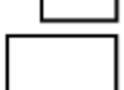
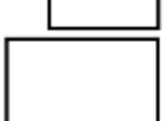
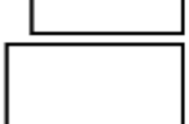


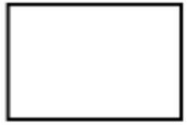
CCD to Image



CCD to Image



Imager Sizes	Formats (Type)	~Diag.	Uses
	1/7" - 1.85 x 1.39mm	2.3	Cell phones, web cams, etc....
	1/6" - 2.15 x 1.61mm	2.7	Cell phones, web cams, etc....
	1/5" - 2.55 x 1.91mm	3.2	Cell phones, web cams, etc....
	1/4" - 3.2 x 2.4mm	4.0	Cell phones, web cams, etc....
	1/3.6" - 4.0 x 3.0mm	5.0	P&S DSC
	1/3.2" - 4.536 x 3.416mm	5.678	P&S DSC
	1/3" - 4.8 x 3.6mm	6.0	Casio QV-8000SX (1.2MP), Epson PhotoPC 700 (1.2MP)
	1/2.7" - 5.27 x 3.96mm	6.592	Canon PowerShot A20 (1.92MP), HP PhotoSmart C618 (1.92)
	1/2" - 6.4 x 4.8mm	8.0	Olympus C-2100Z (1.92MP), Epson PhotoPC 850Z (1.92)
	1/1.8" - 7.176 x 5.319mm	8.932	Nikon Coolpix 995 (3.14MP), Olympus C-4040Z (3.9MP), Canon PowerShot G2 (3.8MP), Sony DSC-S85 (3.8MP)
	2/3" - 8.8 x 6.6mm	11.0	Nikon Coolpix 5000 (4.92MP), Sony DSC-F707 (4.92MP), Olympus E-10 (3.7MP), Minolta DiMAGE 7 (4.92MP)
	1" - 12.8 x 9.6mm	16.0	Not used in DSCs. Used in some high-end video cameras
	Kodak KAF-5100CE CCD 17.8 x 13.4mm (4/3")	22.28	Olympus announced deveolpment of a new camera and new lenses for this 4/3" size. 2614 x 1966 - 5.1MP - 6.8µm pixel
	Foveon X3 F7-35X3-A25B 20.7 x 13.8mm	24.9	Sigma SD9 (X3) 2268 x 1512 = 3.43MP - 9.12µm pixel 1.74x Focal Length Multiplier (35mm film)



Canon D30 CMOS
21.8 x 14.5mm

26.2

Canon D30
2160 x 1440 = 3.11MP - 10.1µm pixel
1.65x Focal Length Multiplier (35mm film)

Canon D60 CMOS
22.7 x 15.1mm

27.3

Canon D60
3072 x 2048 = 6.3MP - 7.4µm pixel
1.59x Focal Length Multiplier (35mm film)

Nikon D100 CCD
Nikon D1x CCD
23.7 x 15.6mm

28.2

Nikon D100 - 3008 x 2000 = 6.1MP - 7.8µm pixel
Nikon D1x - 4024 x 1324 = 5.24MP - 5.9 x 11.7µm pixel
1.52x Focal Length Multiplier (35mm film)

APS Film
25.1 x 16.7mm

30.148

APS cameras
1.44x Focal Length Multiplier (35mm film)

Canon EOS-1D CCD
27.0 x 17.8mm

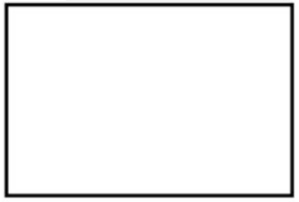
32.3

Canon EOS-1D
2464 x 1648 = 4.06MP - 10.8µm pixel
1.34x Folcal Length Multiplier (35mm Film)

Kodak KAF-6303CE CCD
27.8 x 18.5mm

33.4

Kodak 760
3088 x 2056 = 6.35MP - 9.0µm pixel
1.30x Focal Length Multiplier (35mm film)

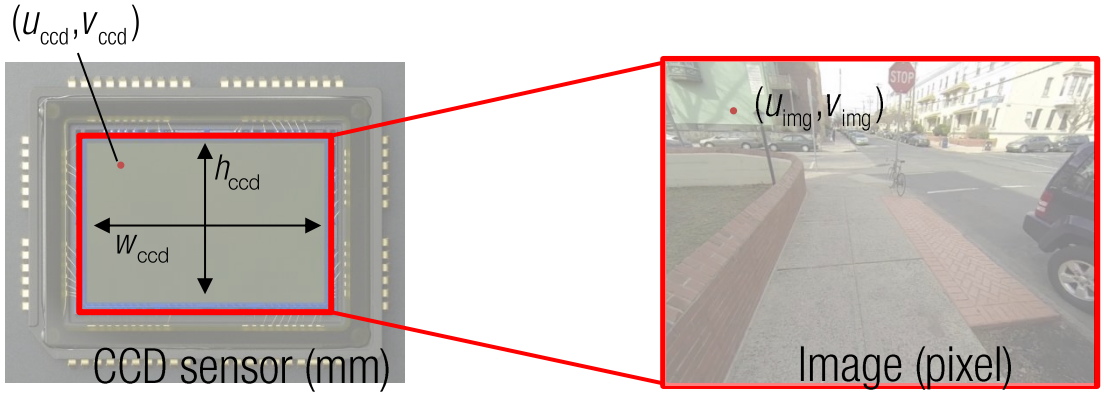


35mm Film
Canon 1Ds
Kodak 14n
36.0 x 24.0mm

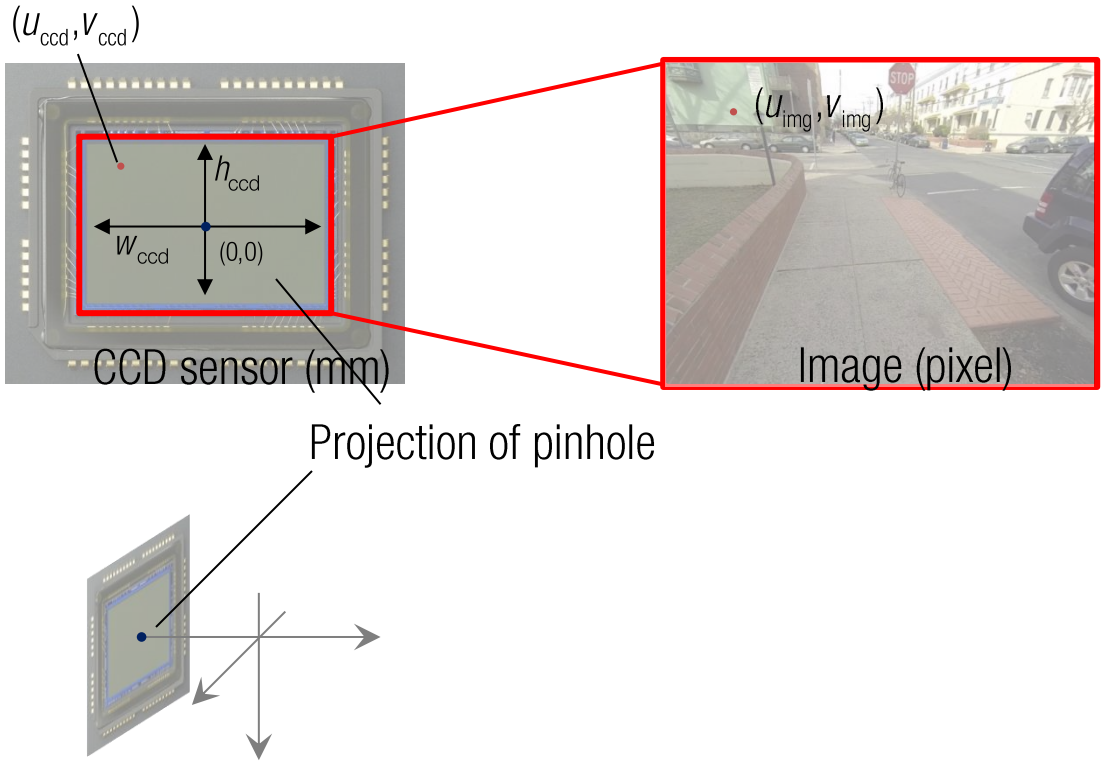
43.27

35mm film cameras
Canon 1Ds - 4064 x 2704 = 10.99MP - 8.85µm pixel
Kodak DCS Pro 14n - 4536 x 3024 = 13.7MP - 7.94µm pixel

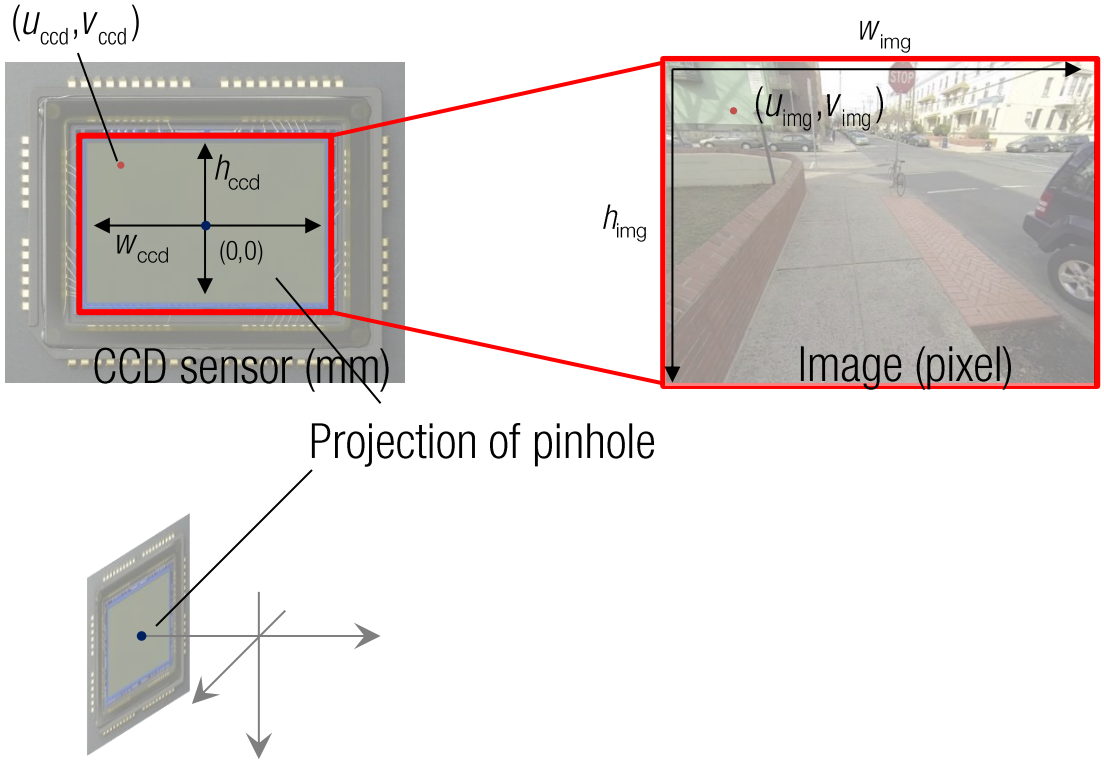
CCD to Image



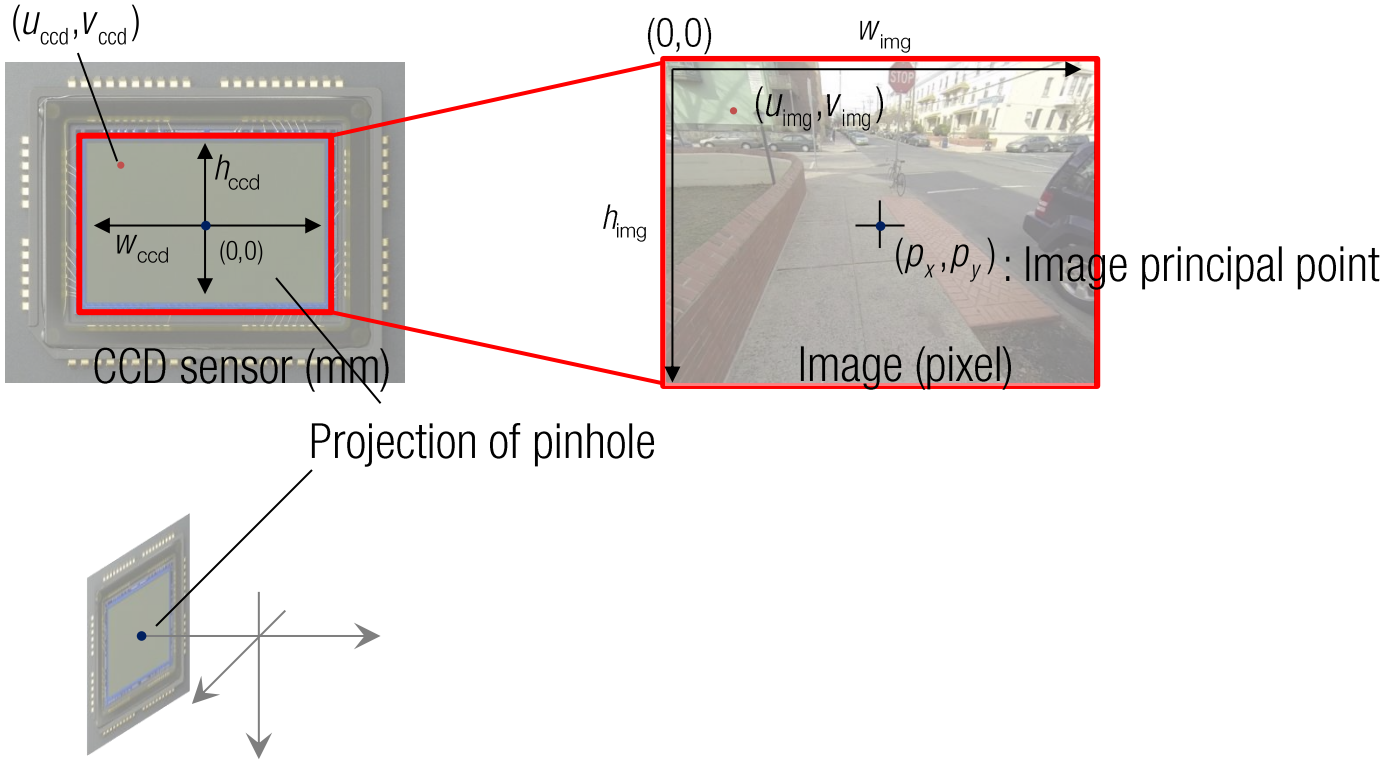
CCD to Image


(u_{ccd}, v_{ccd})
h_{ccd}
w_{ccd}
(0,0)
CCD sensor (mm)
(u_{img}, v_{img})
Image (pixel)
Projection of pinhole

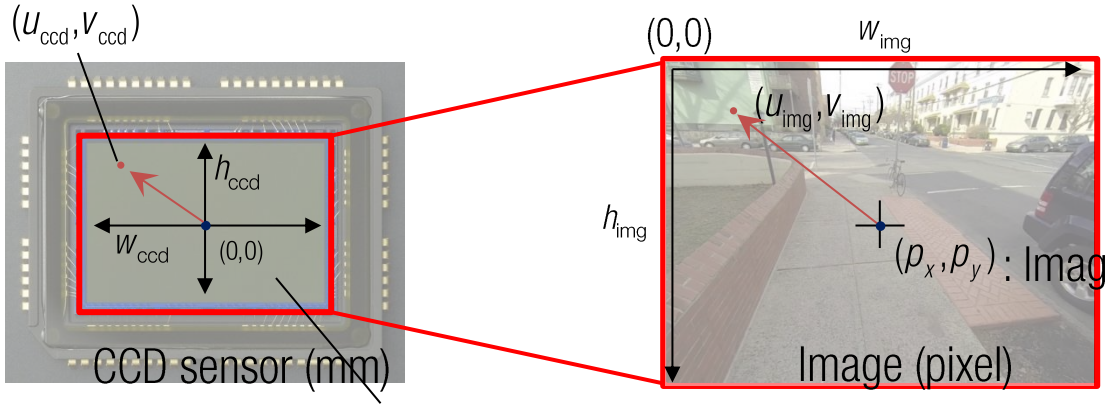
CCD to Image


(u_{ccd}, v_{ccd})
h_{ccd}
w_{ccd}
(0,0)
CCD sensor (mm)
w_{img}
(u_{img}, v_{img})
h_{img}
Image (pixel)
Projection of pinhole

CCD to Image

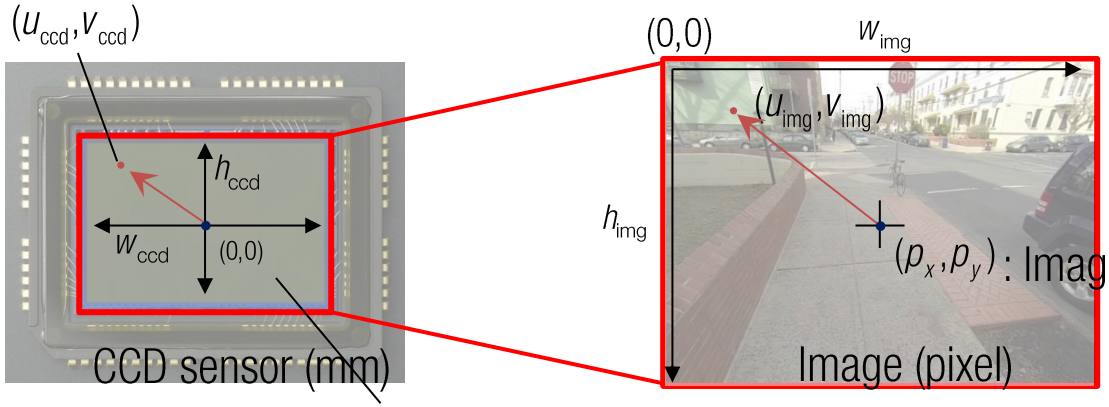

(u_{ccd}, v_{ccd})
h_{ccd}
w_{ccd}
(0,0)
CCD sensor (mm)
(0,0)
w_{img}
(u_{img}, v_{img})
h_{img}
(p_x, p_y) : Image principal point
Image (pixel)
Projection of pinhole

CCD to Image



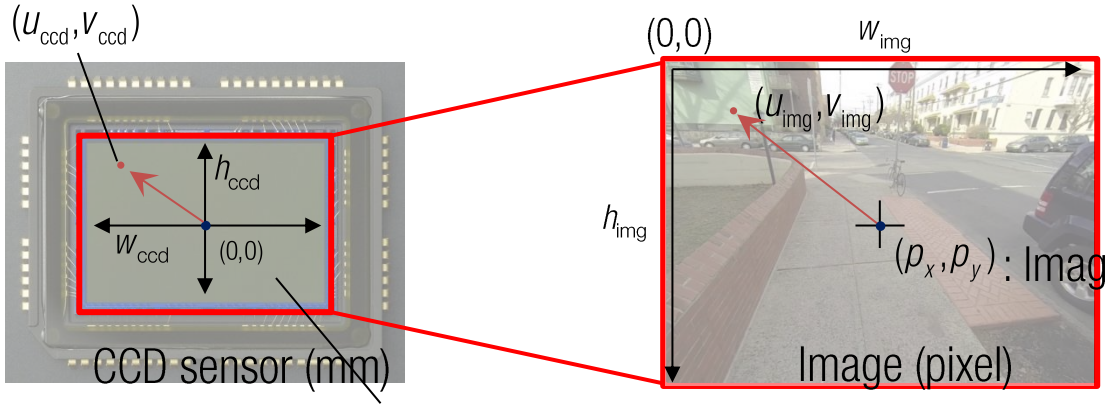
$$\frac{u_{ccd}}{w_{ccd}} = \frac{u_{img} - p_x}{w_{img}}$$

CCD to Image



$$\frac{u_{ccd}}{w_{ccd}} = \frac{u_{img} - p_x}{w_{img}} \qquad \frac{v_{ccd}}{h_{ccd}} = \frac{v_{img} - p_y}{h_{img}}$$

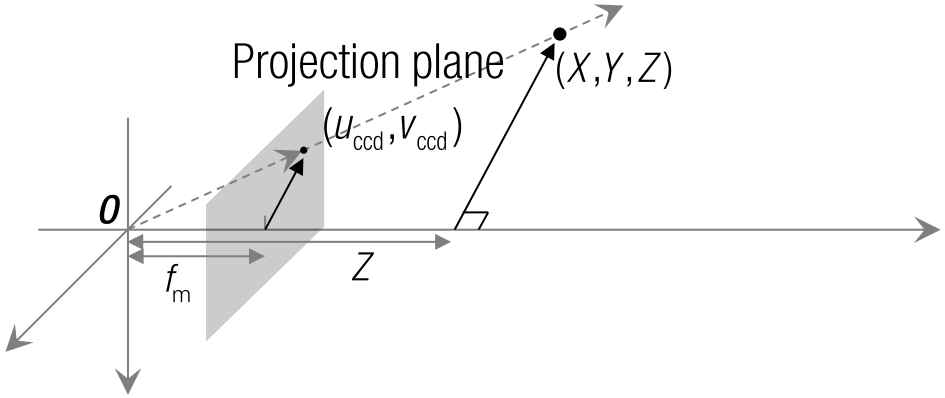
CCD to Image



$$\frac{u_{ccd}}{w_{ccd}} = \frac{u_{img} - p_x}{w_{img}} \qquad \frac{v_{ccd}}{h_{ccd}} = \frac{v_{img} - p_y}{h_{img}}$$

$$\rightarrow \quad u_{img} = u_{ccd}\frac{w_{img}}{w_{ccd}} + p_x \qquad v_{img} = v_{ccd}\frac{h_{img}}{h_{ccd}} + p_y$$

IMAGE PROJECTION



$$(u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$: Metric projection

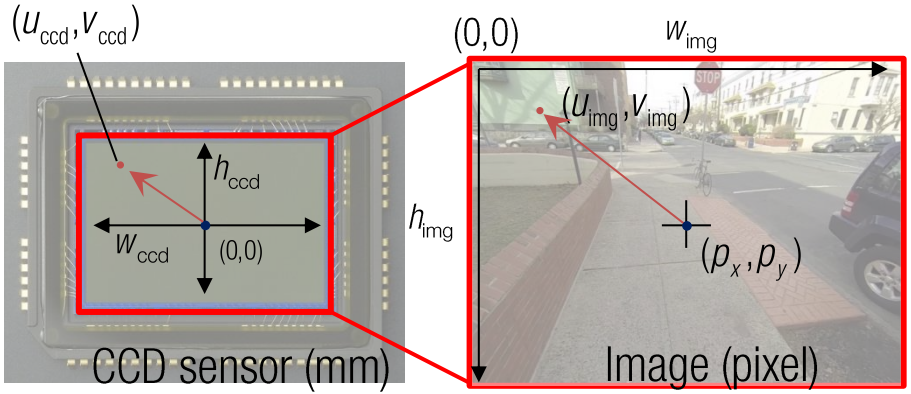
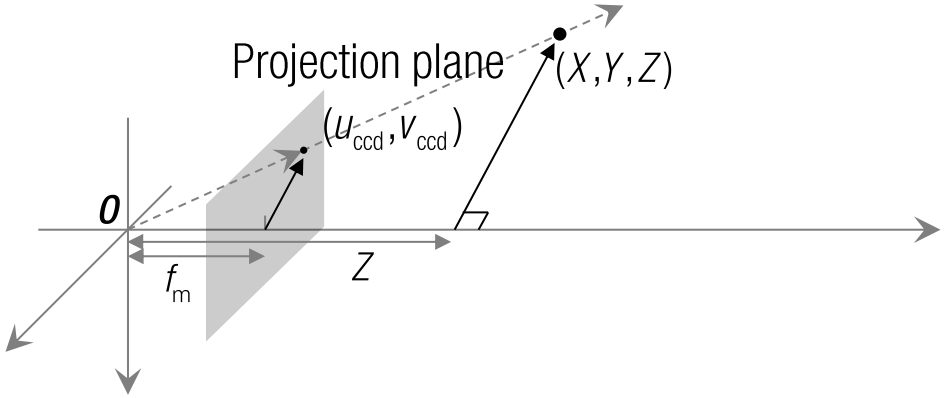


IMAGE PROJECTION



$$(u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$: Metric projection

Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y$$

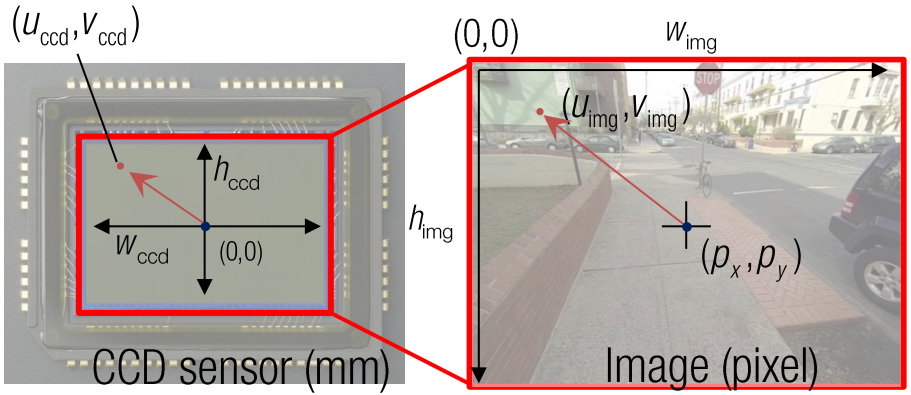
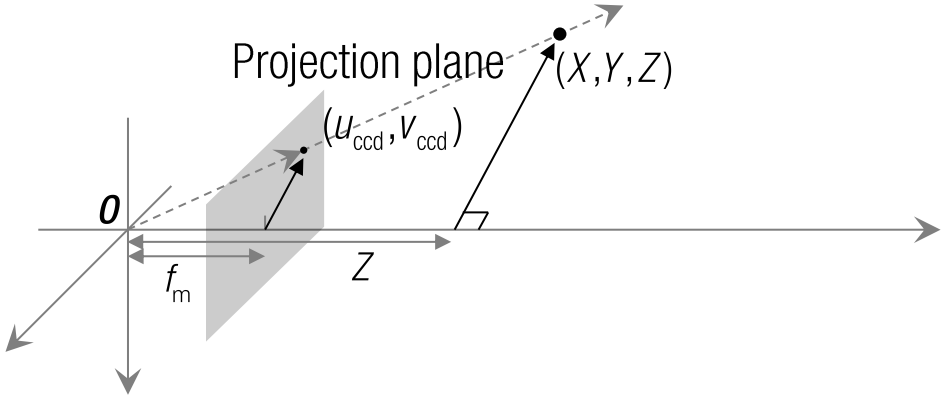
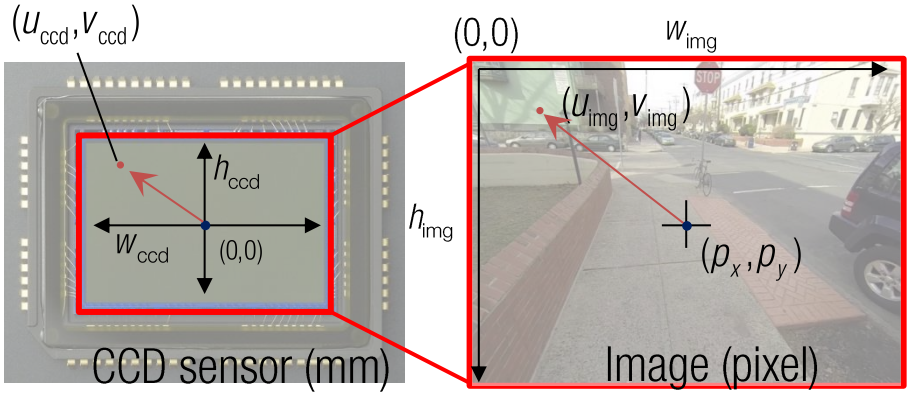


IMAGE PROJECTION



$$(u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$

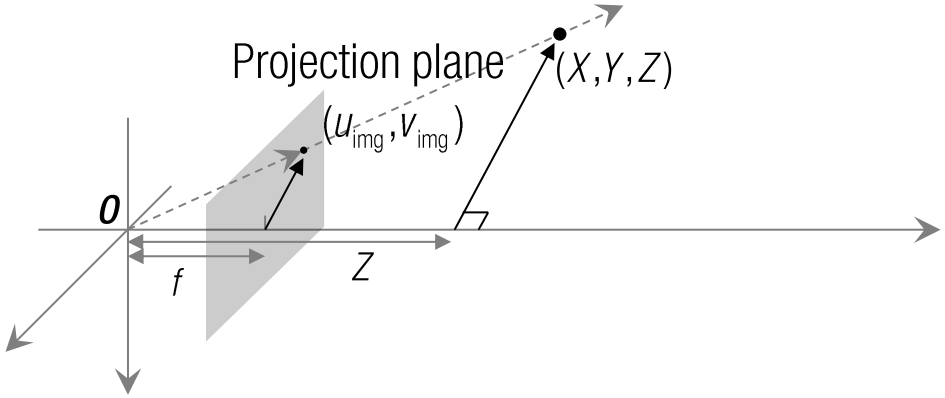


Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img}}{w_{ccd}} \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} + p_y$$

IMAGE PROJECTION



$$(u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z})$$: Metric projection

Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_m \frac{w_{img}}{w_{ccd}} \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_m \frac{h_{img}}{h_{ccd}} \frac{Y}{Z} + p_y$$

Focal length in pixel

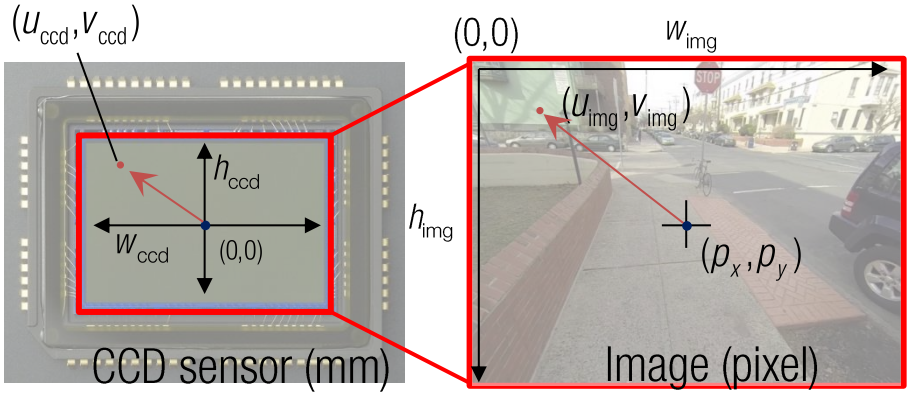
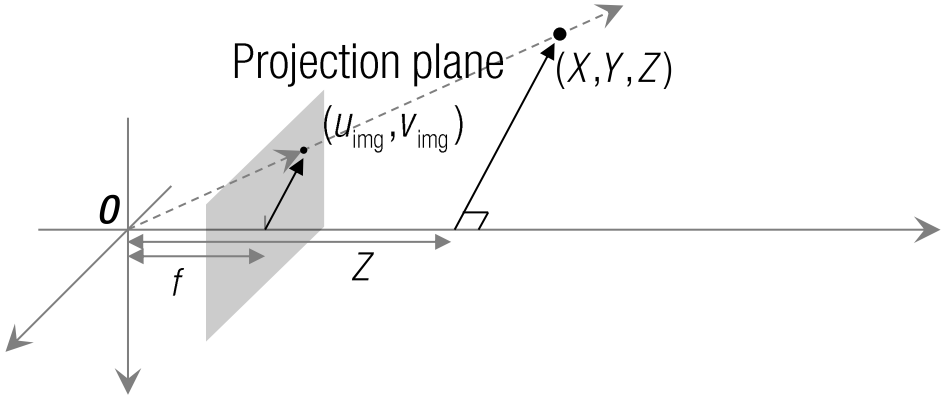


IMAGE PROJECTION



$$(u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$

Pixel projection

$$\rightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f_x \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f_y \frac{Y}{Z} + p_y$$

Focal length in pixel

$$\text{where } f_x = f_m \frac{w_{img}}{w_{ccd}} \qquad f_y = f_m \frac{h_{img}}{h_{ccd}}$$

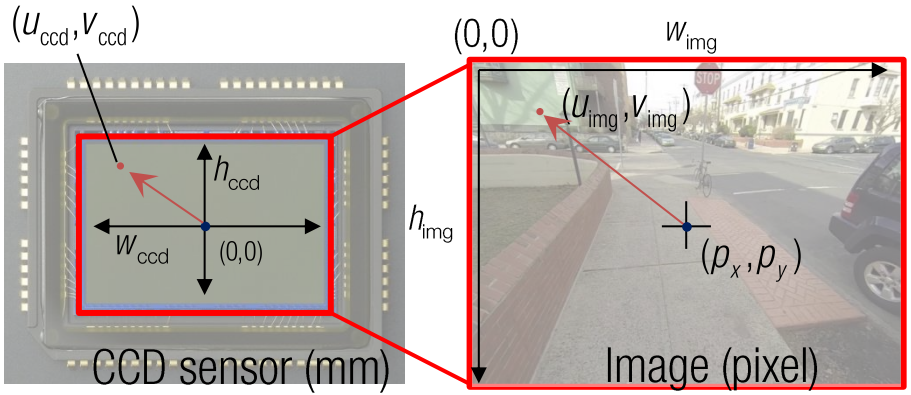
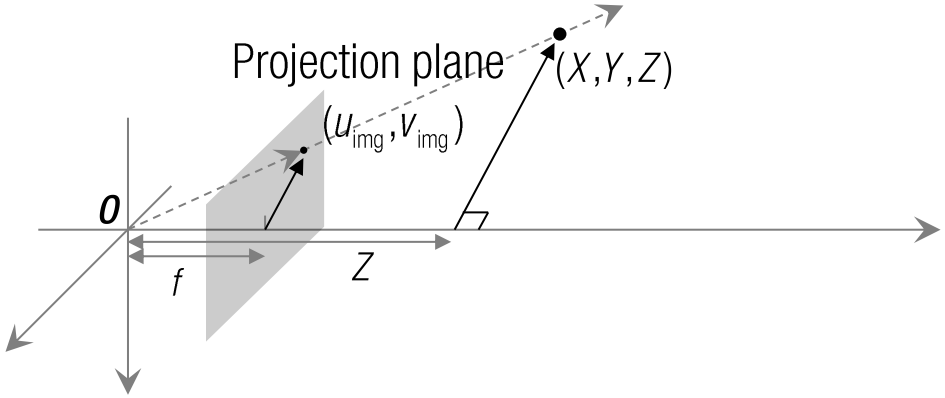
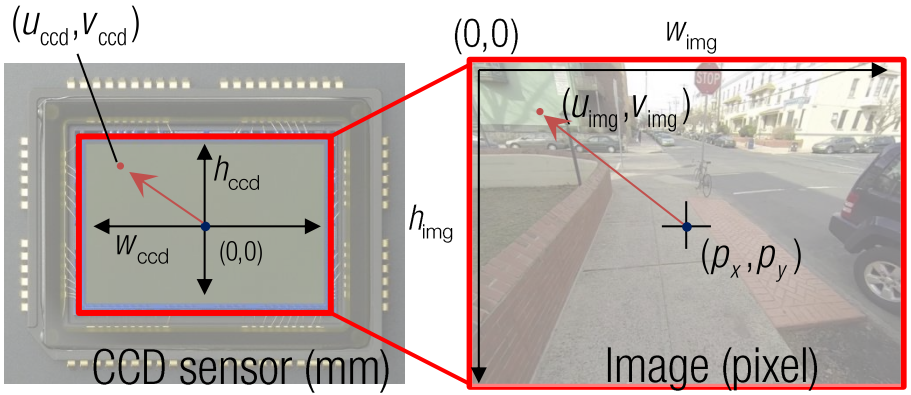


IMAGE PROJECTION



$$(u_{ccd},v_{ccd}) = (f_m \frac{X}{Z}, f_m \frac{Y}{Z}) \quad : \text{Metric projection}$$



Pixel projection

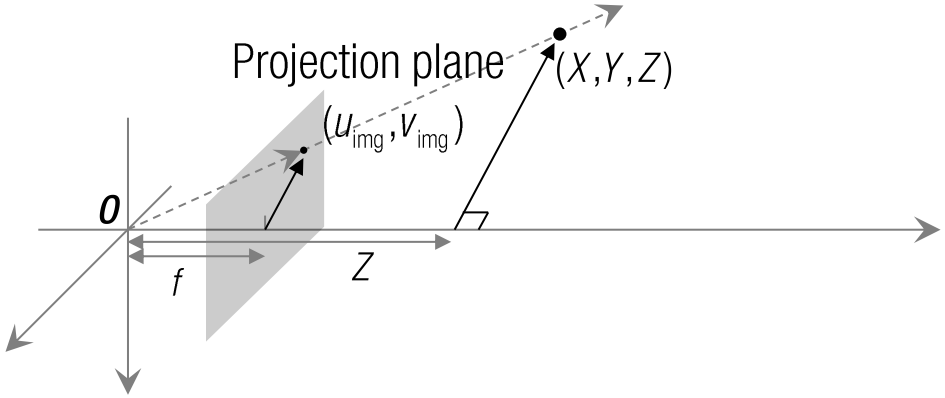
$$\longrightarrow u_{img} = u_{ccd} \frac{w_{img}}{w_{ccd}} + p_x = f \frac{X}{Z} + p_x$$

$$v_{img} = v_{ccd} \frac{h_{img}}{h_{ccd}} + p_y = f \frac{Y}{Z} + p_y$$

Focal length in pixel

where $f = f_m \frac{w_{img}}{w_{ccd}} = f_m \frac{h_{img}}{h_{ccd}}$ if $\frac{w_{img}}{w_{ccd}} = \frac{h_{img}}{h_{ccd}}$

IMAGE PROJECTION



$$u_{img} = f\frac{X}{Z} + p_x$$

$$v_{img} = f\frac{Y}{Z} + p_y$$

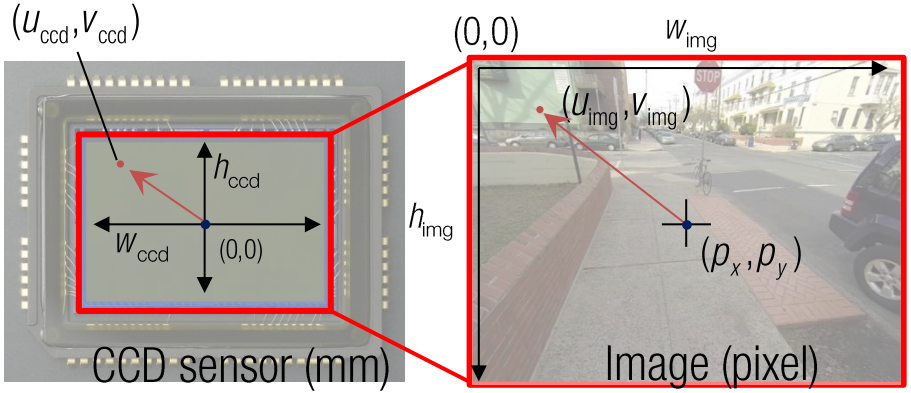
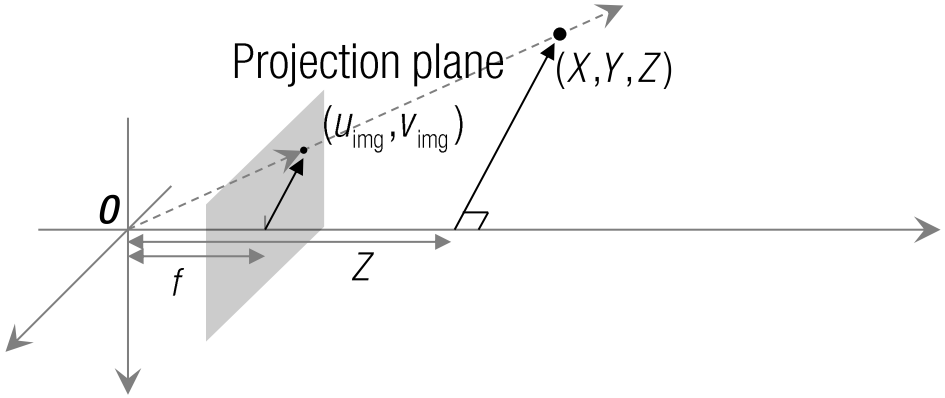


IMAGE PROJECTION



$$u_{img} = f\frac{X}{Z} + p_x \longrightarrow Zu_{img} = fX + p_xZ$$

$$v_{img} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{img} = fY + p_yZ$$

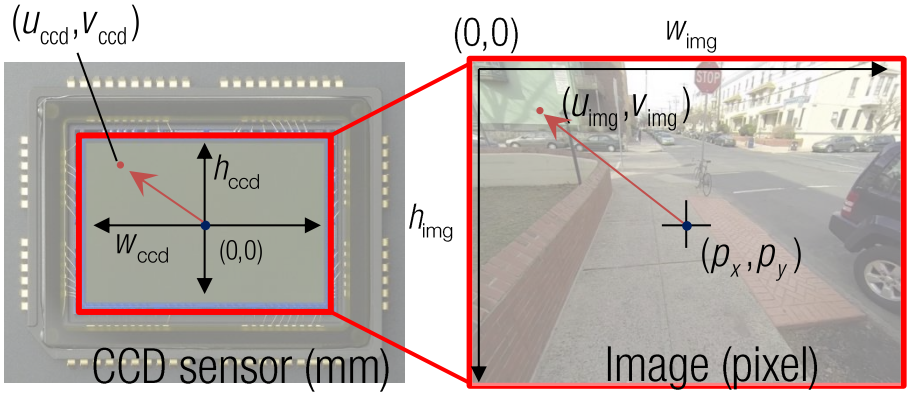
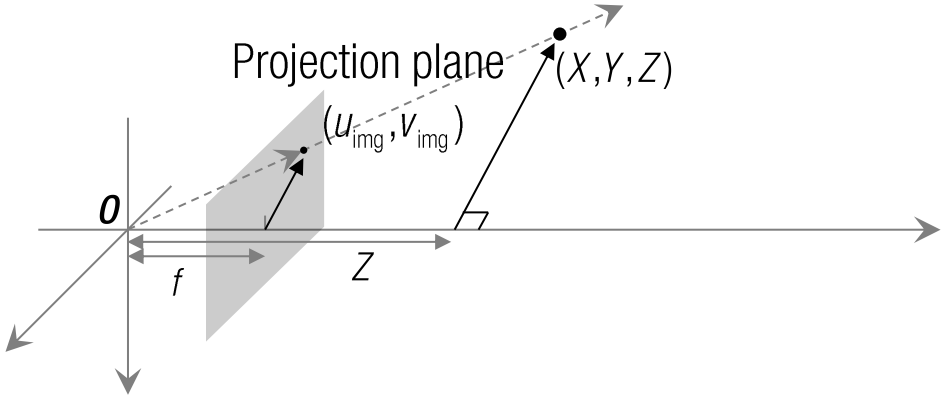


IMAGE PROJECTION



$$u_{img} = f\frac{X}{Z} + p_x \longrightarrow Zu_{img} = fX + p_x Z$$

$$v_{img} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{img} = fY + p_y Z$$

$$Z\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

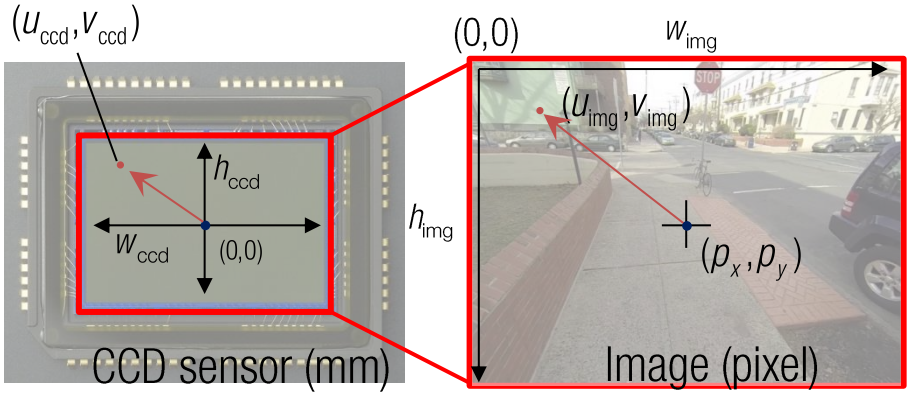
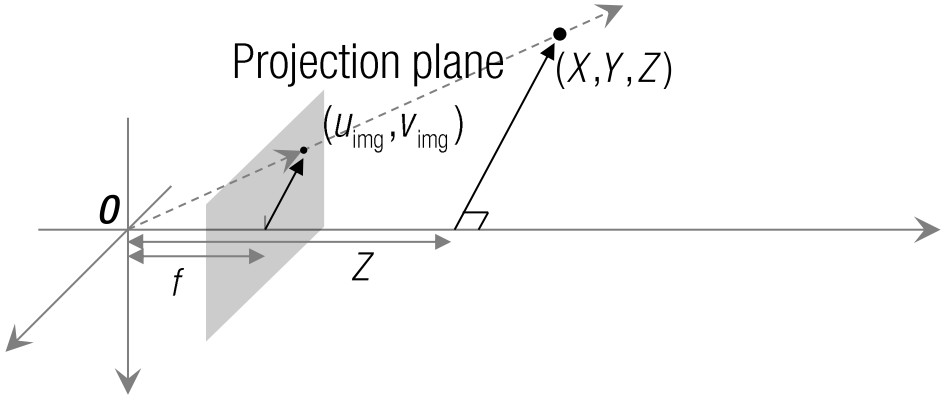


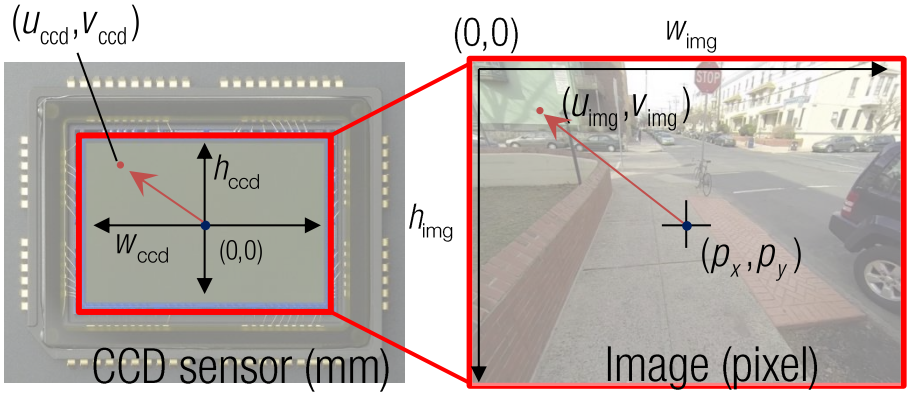
IMAGE PROJECTION



$$u_{img} = f\frac{X}{Z} + p_x \longrightarrow Zu_{img} = fX + p_x Z$$

$$v_{img} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{img} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



RENDERING

$$u_{\text{img}} = f\frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

$$v_{\text{img}} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

← Graphics

3D Reconstruction~Inverse Rendering

$$u_{\text{img}} = f\frac{X}{Z} + p_x \longrightarrow Zu_{\text{img}} = fX + p_x Z$$

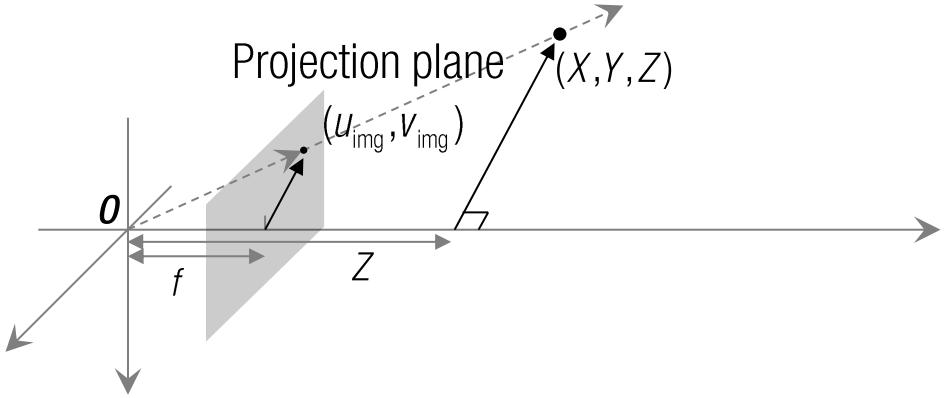
$$v_{\text{img}} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{\text{img}} = fY + p_y Z$$

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Graphics

Vision

IMAGE PROJECTION



$$u_{img} = f\frac{X}{Z} + p_x \longrightarrow Zu_{img} = fX + p_x Z$$

$$v_{img} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{img} = fY + p_y Z$$

Pixel space Metric space

$$\lambda\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

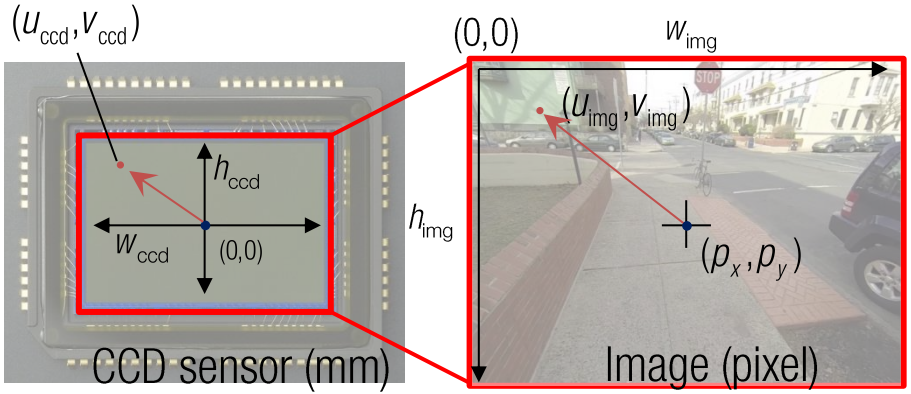
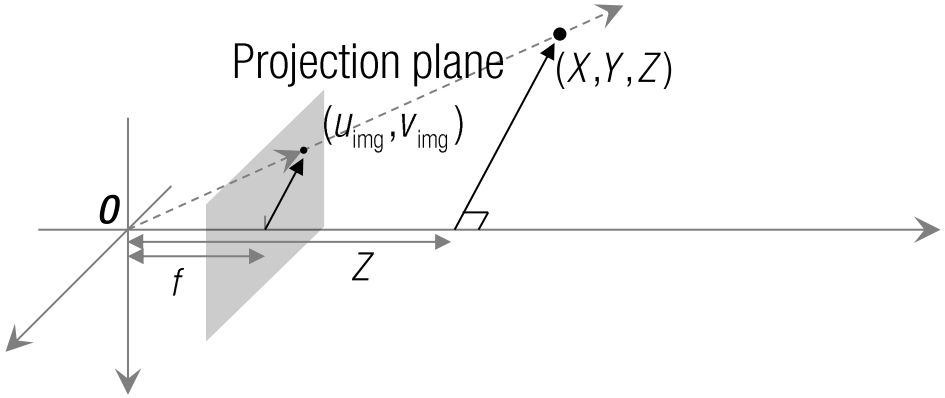


IMAGE PROJECTION

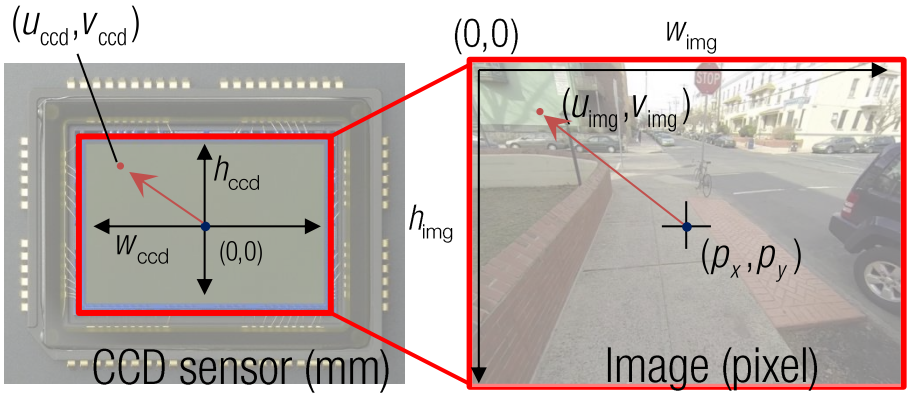


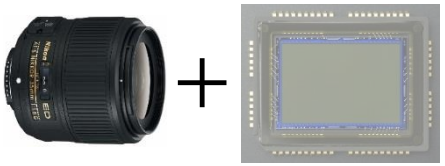
$$u_{img} = f\frac{X}{Z} + p_x \longrightarrow Zu_{img} = fX + p_x Z$$

$$v_{img} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{img} = fY + p_y Z$$

Pixel space Metric space

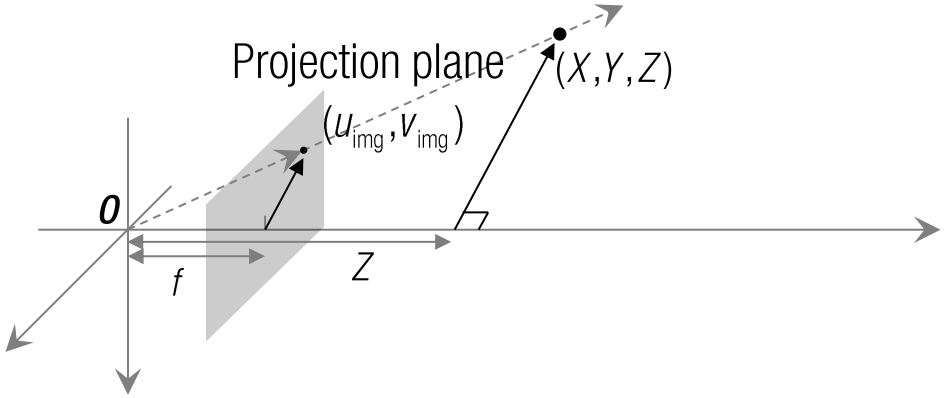
$$\lambda\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$





Camera intrinsic parameter
: metric space to pixel space

IMAGE PROJECTION

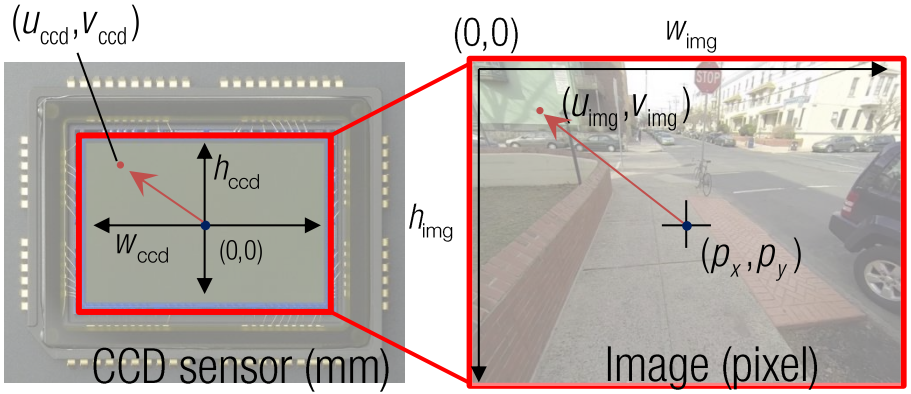


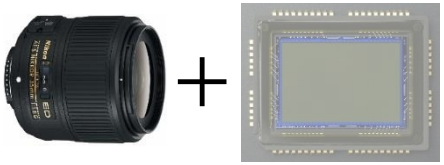
$$u_{img} = f\frac{X}{Z} + p_x \longrightarrow Zu_{img} = fX + p_x Z$$

$$v_{img} = f\frac{Y}{Z} + p_y \longrightarrow Zv_{img} = fY + p_y Z$$

Pixel space Metric space

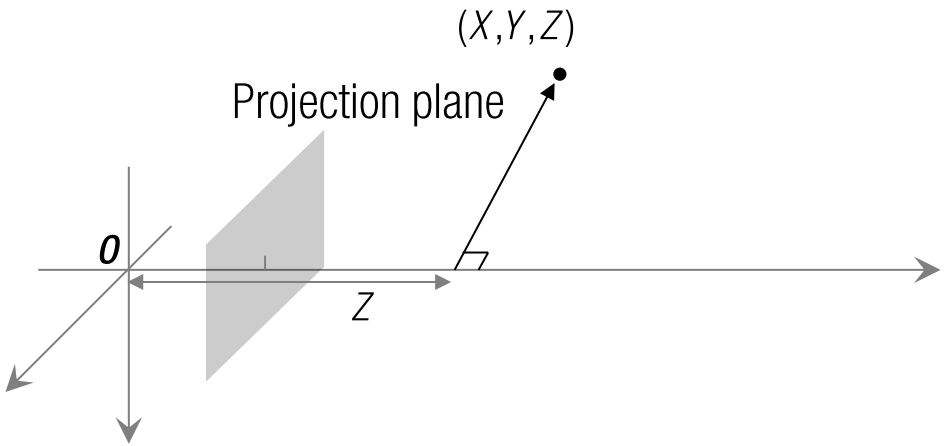
$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & \mathbf{K} & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$





Camera intrinsic parameter
: metric space to pixel space

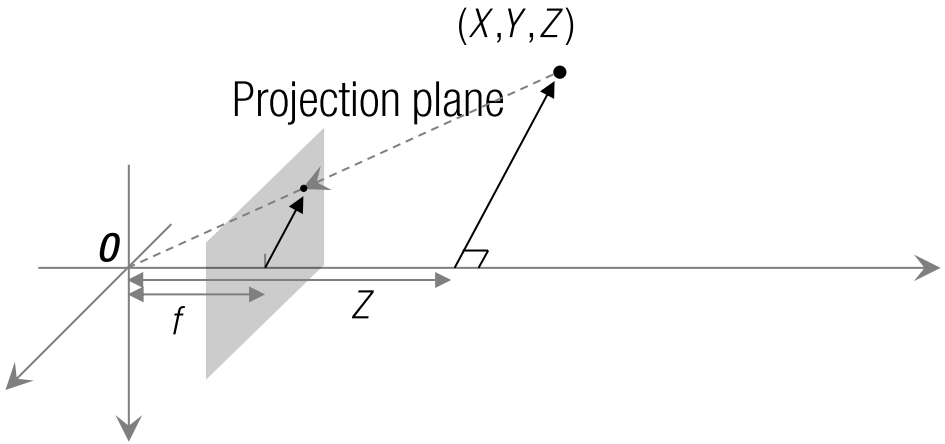
INTRINSIC PARAMETER



Metric space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

INTRINSIC PARAMETER

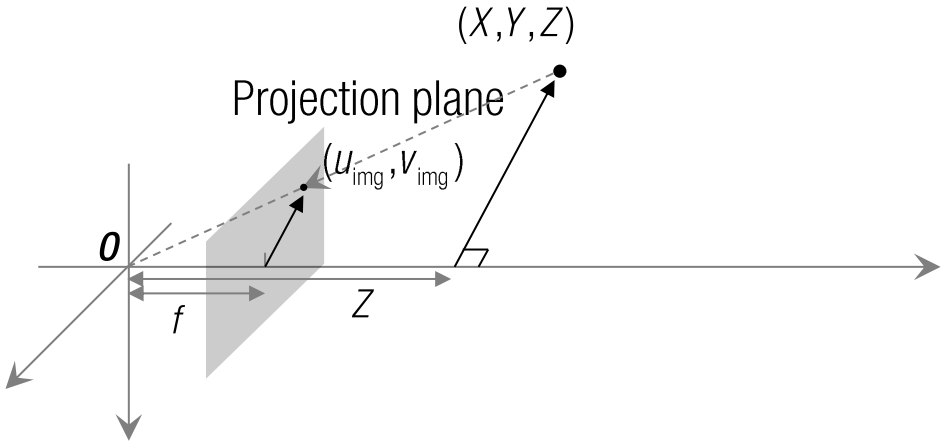


Metric space

$$\begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\mathbf{K}$$

INTRINSIC PARAMETER

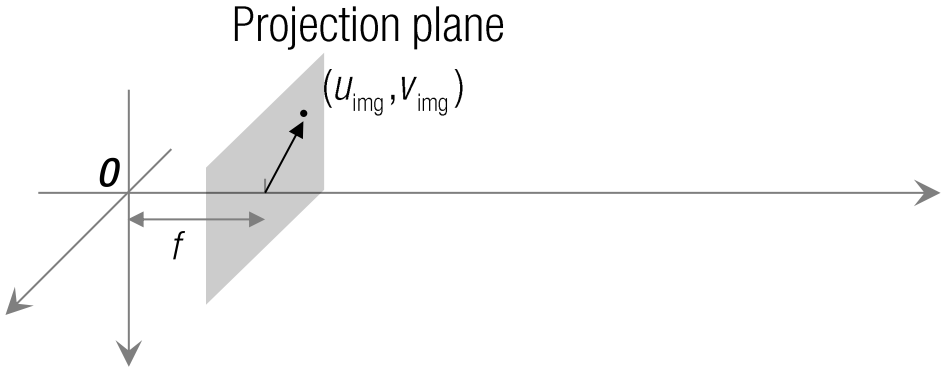


Pixel space

Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Inverse Projection



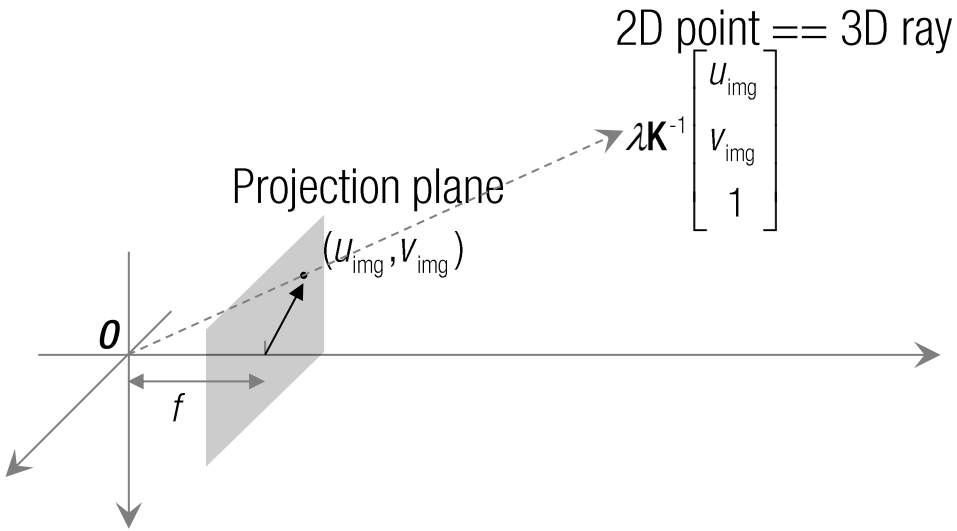
Pixel space

Metric space

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix}$$

INVERSE PROJECTION



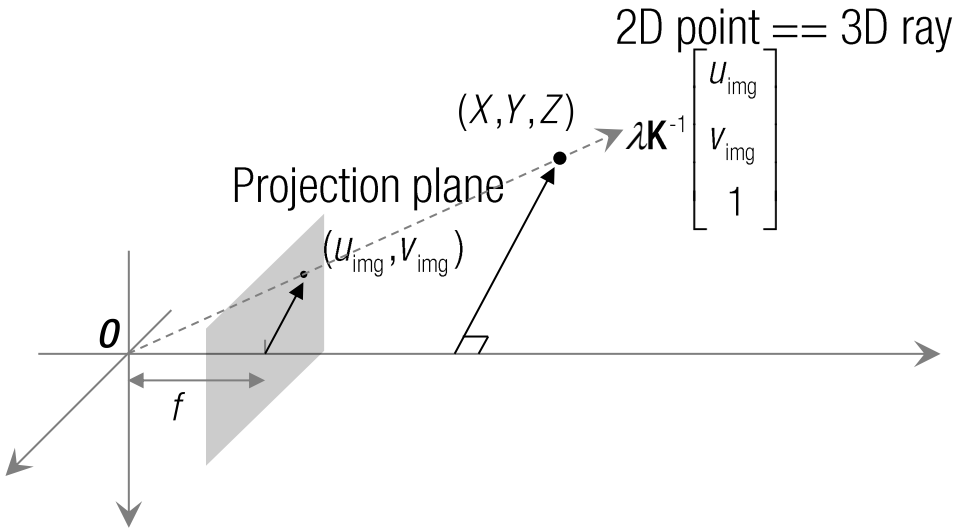
Pixel space → Metric space

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \mathbf{K} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix}$$

3D ray

INVERSE PROJECTION



Pixel space

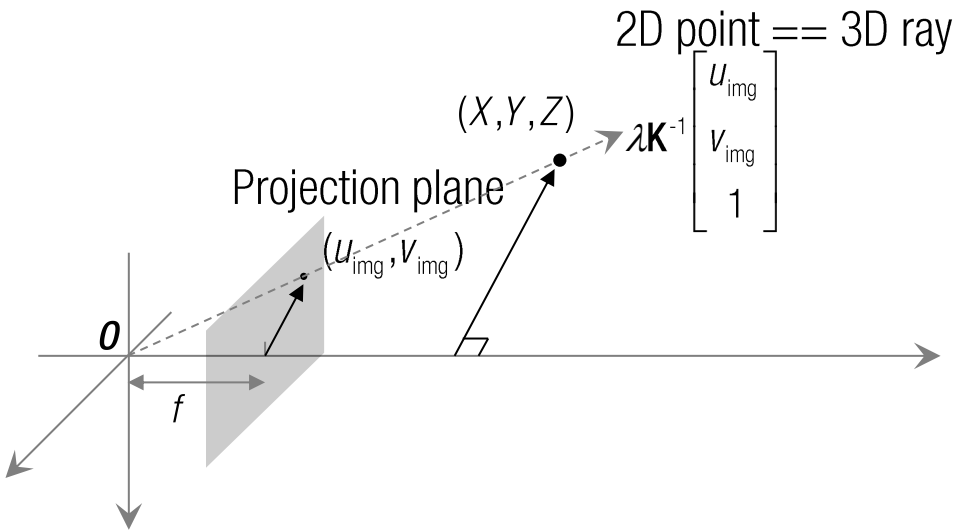
Metric space

$$\lambda \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3D ray

INVERSE PROJECTION



Pixel space

Metric space

$$\lambda \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \mathbf{K} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\lambda \mathbf{K}^{-1} \begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

3D ray

The 3D point must lie in
the *3D ray* passing through the origin and 2D image point.