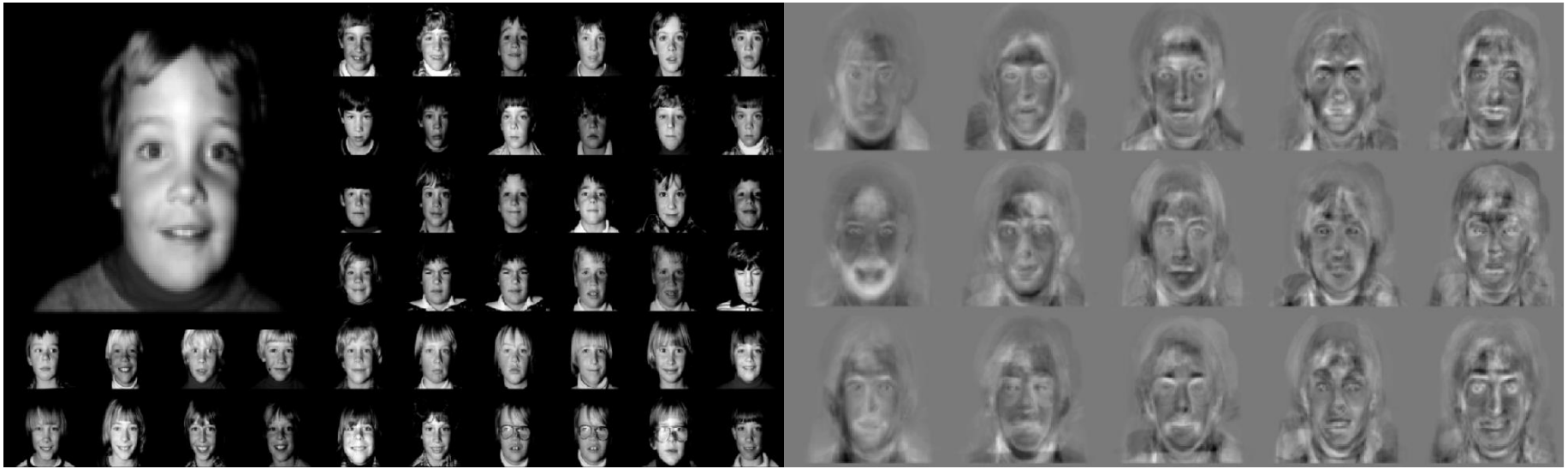

Fisher's Faces
Hyun Soo Park

RECALL: EIGENFACES



Reconstruction

$$\alpha^* = \underset{\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

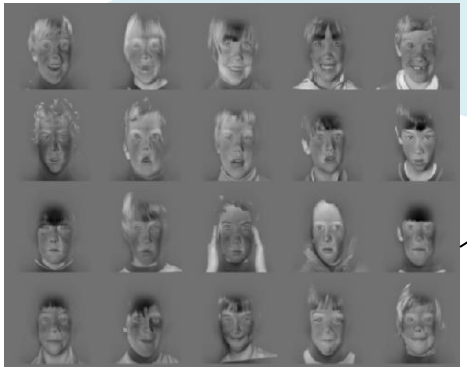
RECALL: FACE DETECTION

Residual

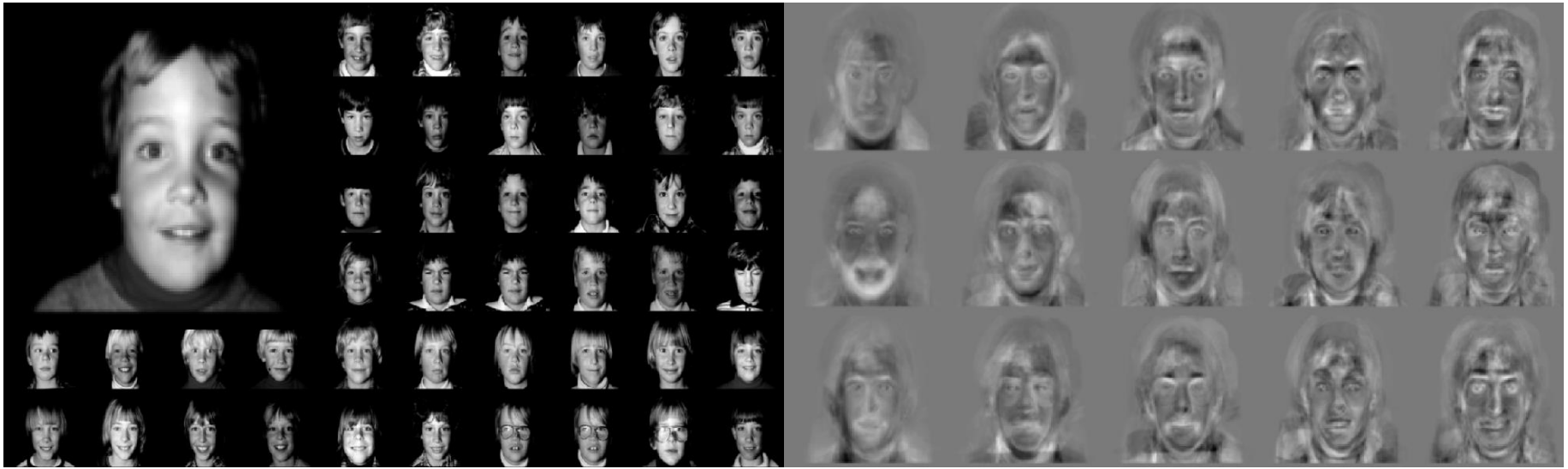
Residual

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\|b\|^2} b \right\|$$

Face subspace



RECALL: EIGENFACES



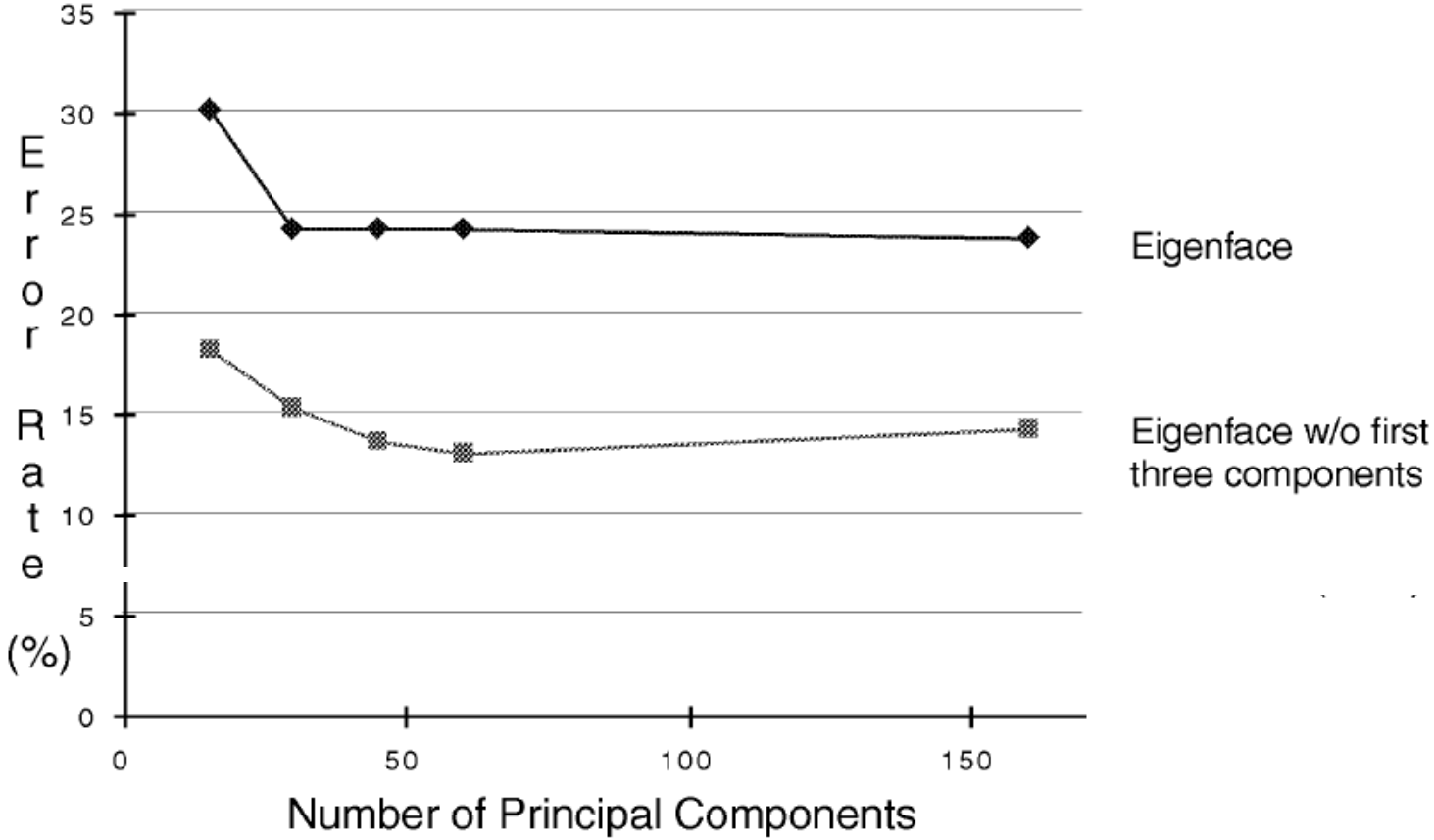
Reconstruction

$$\alpha^* = \underset{\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

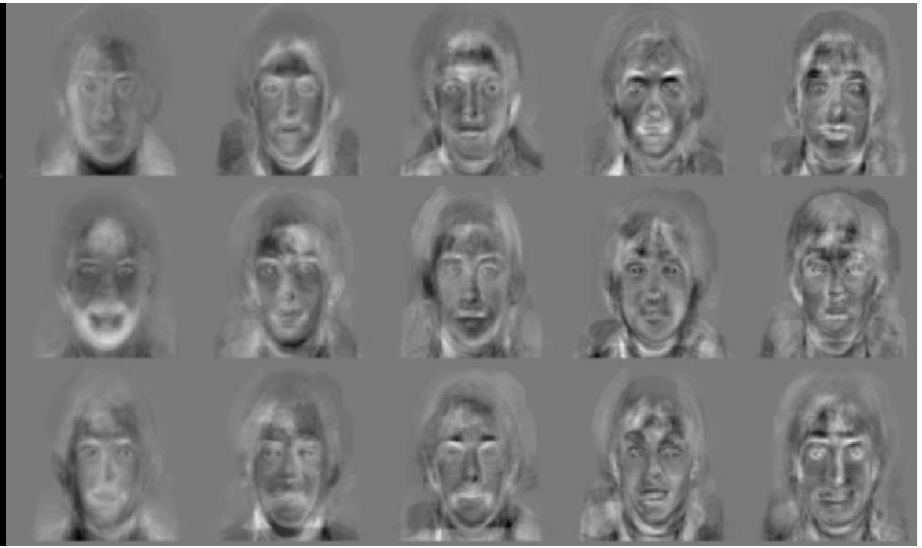
Detection

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\left\| b \right\|^2} b \right\|$$

FACE RECOGNITION WITH EIGENFACES



RECALL: EIGENFACES



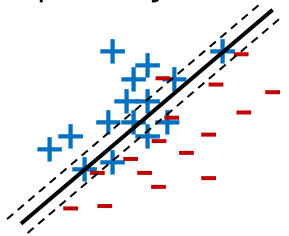
Reconstruction

$$\alpha^* = \underset{\alpha}{\text{minimize}} \left\| y - m - B\alpha \right\|^2$$

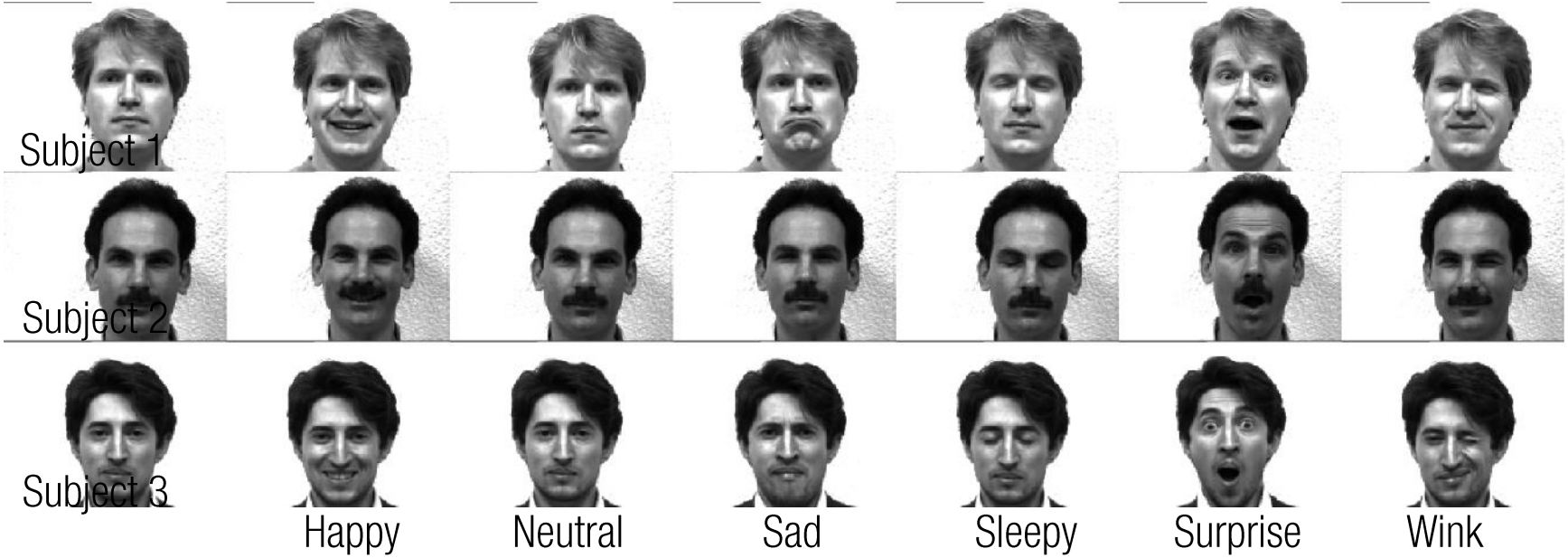
Detection

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\|b\|^2} b \right\|$$

Not optimally discriminative.



LINEAR DISCRIMINANT ANALSYS

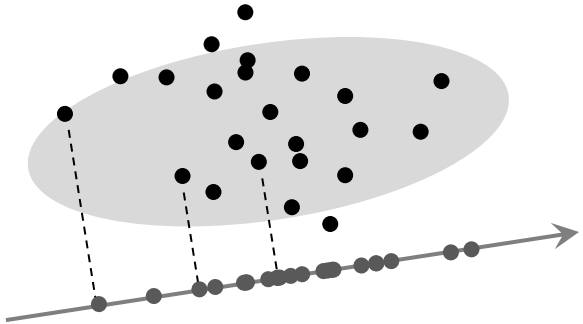


LDA ~ PCA + Class label

Find projection that maximizes scatter between classes and minimizes scatter within classes

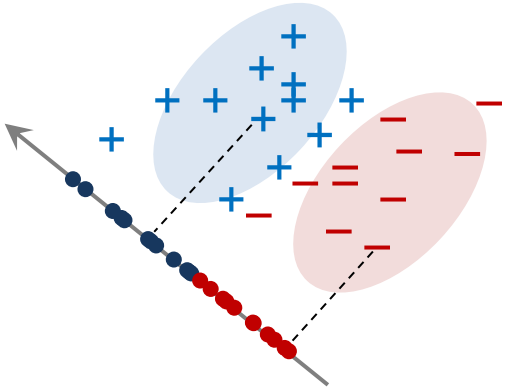
PCA vs. LDA

PCA

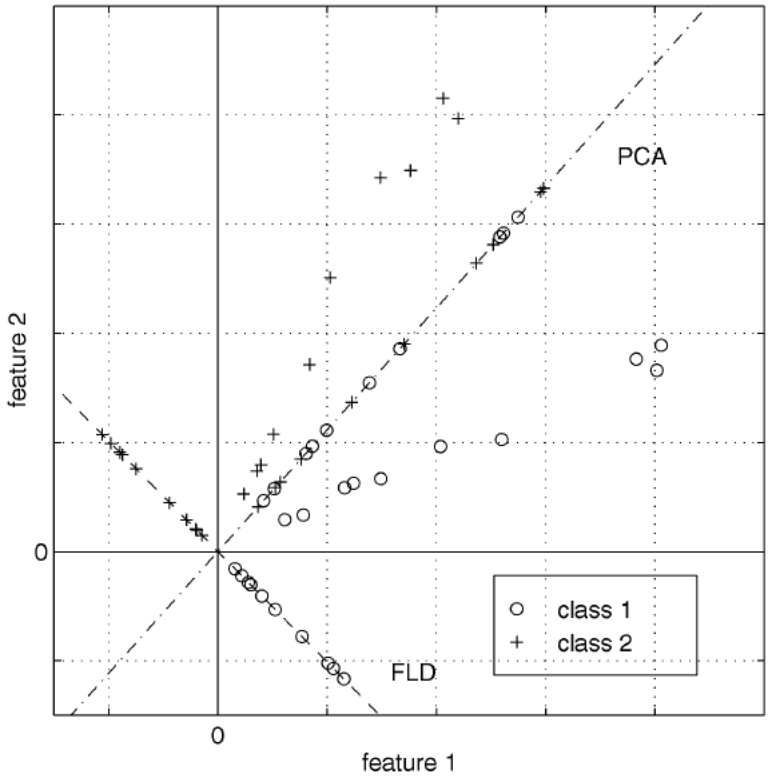


Maximize covariance

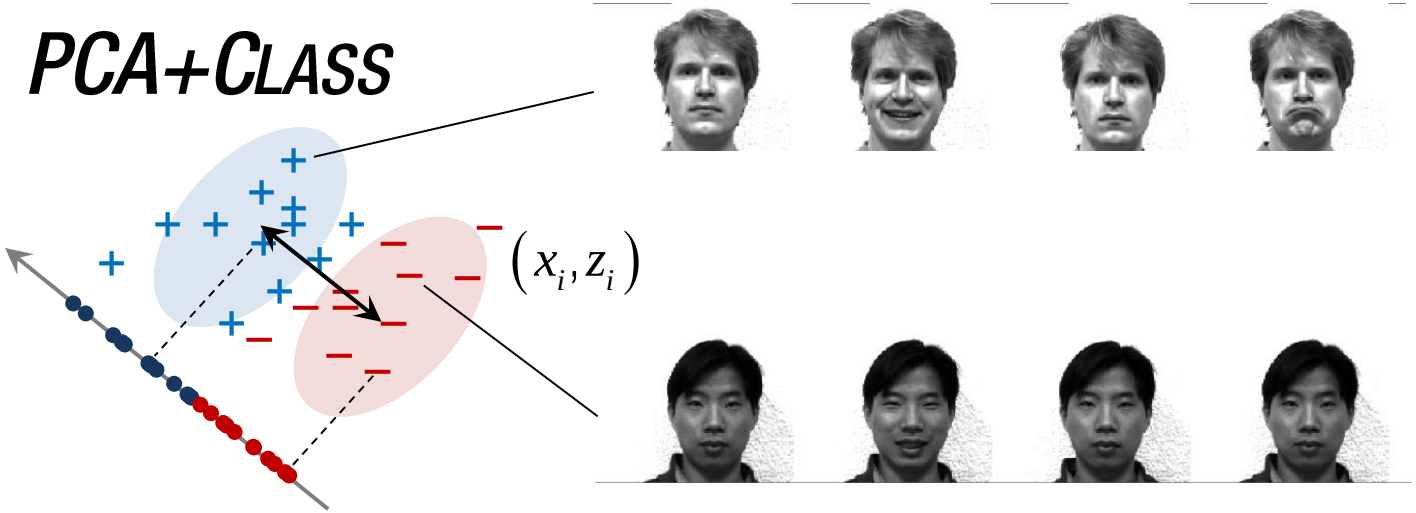
LDA



Maximize discriminance

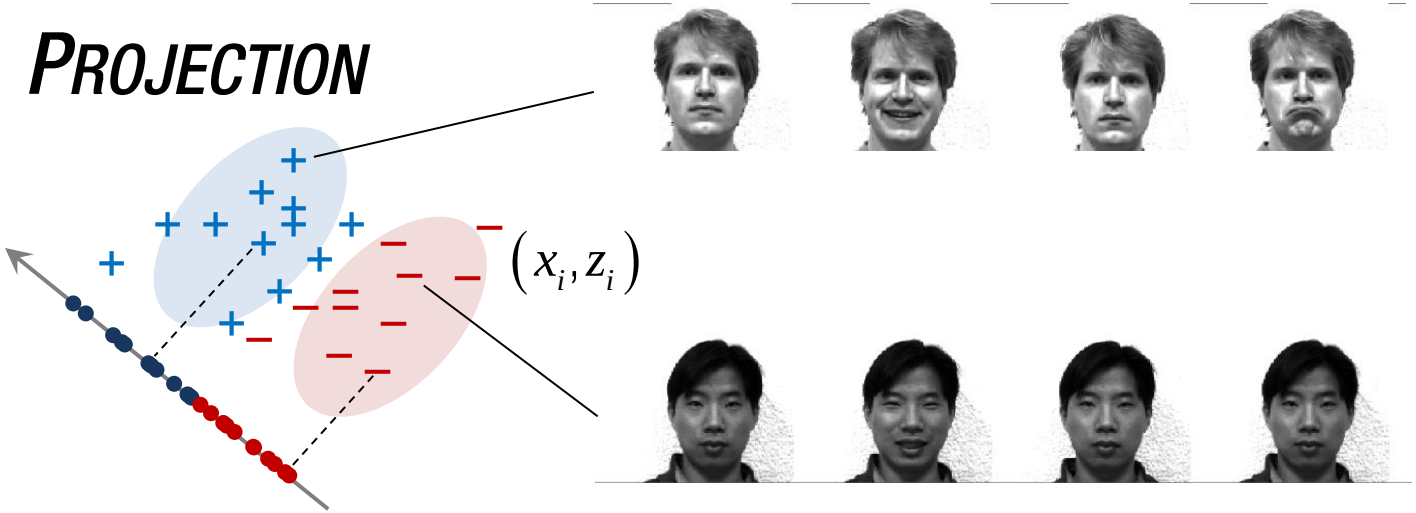

PCA
FLD
class 1
class 2
feature 2
feature 1
0
0

PCA+CLASS



(x_i, z_i): image x with label z

PROJECTION

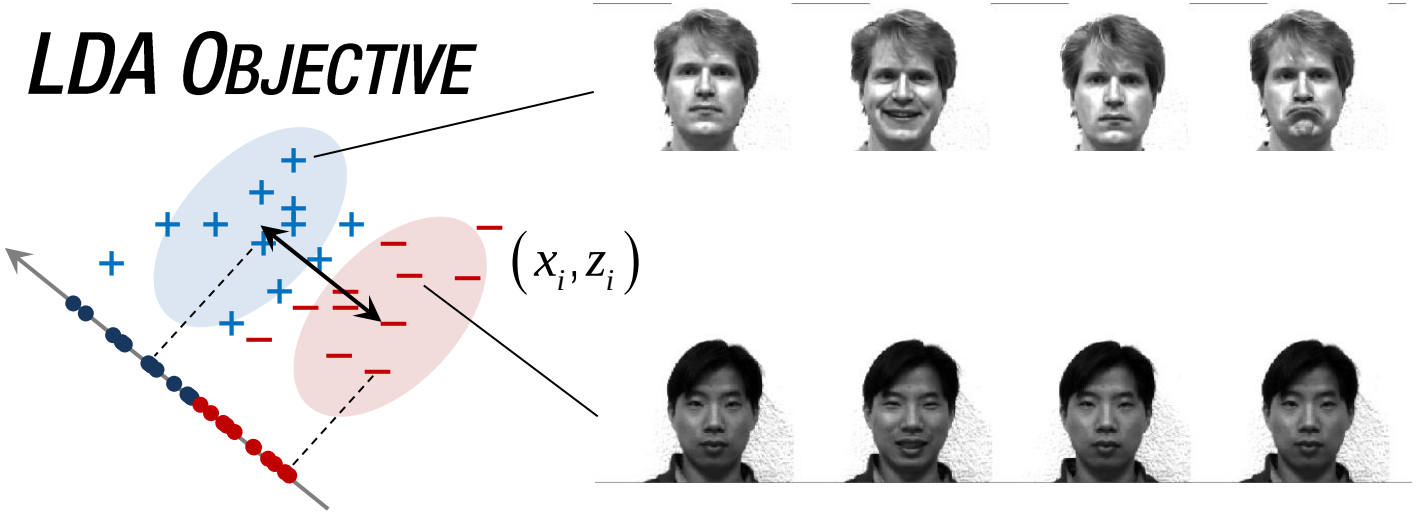


(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

LDA Objective



(x_i, z_i): image x with label z

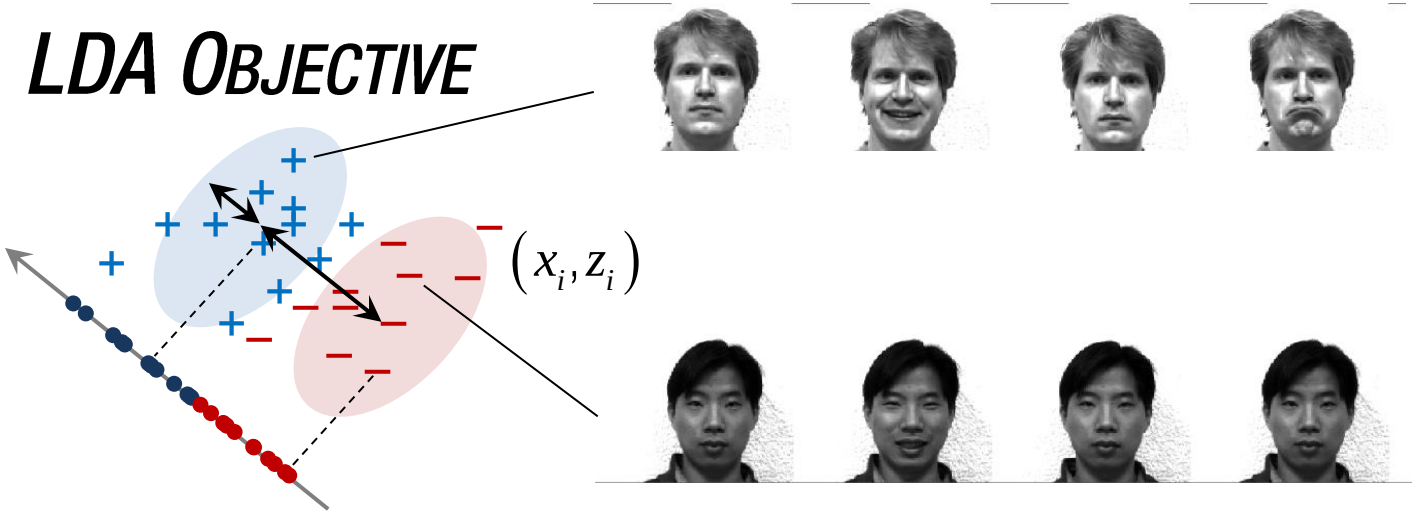
Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

Objective:

- Maximize variance between classes

LDA Objective



(x_i, z_i): image x with label z

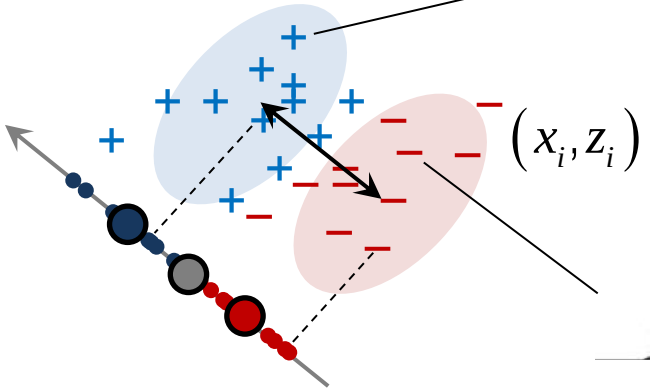
Recall: Eigen basis

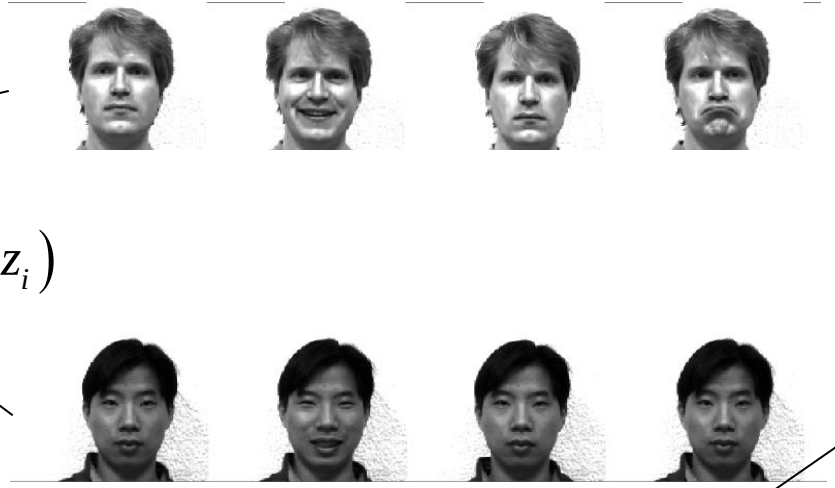
Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

Objective:

- Maximize variance between classes
- Minimize variance within class

LDA Objective





Number of data for each class

(x_i, z_i): image x with label z

$$\tilde{S}_{\text{inter}} = \sum_{j \in C} n_j \left(\bar{y} - \bar{y}_j\right)\left(\bar{y} - \bar{y}_j\right)^T$$

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

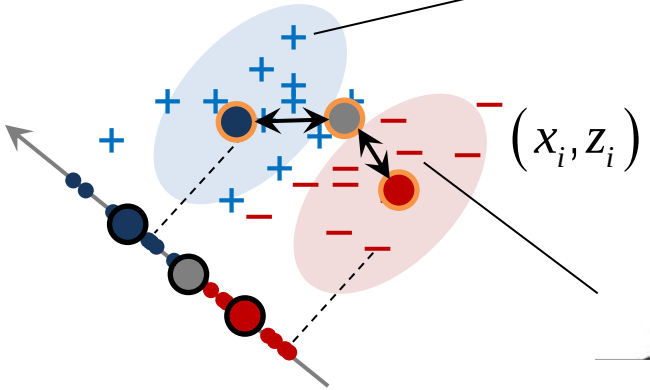
Objective:

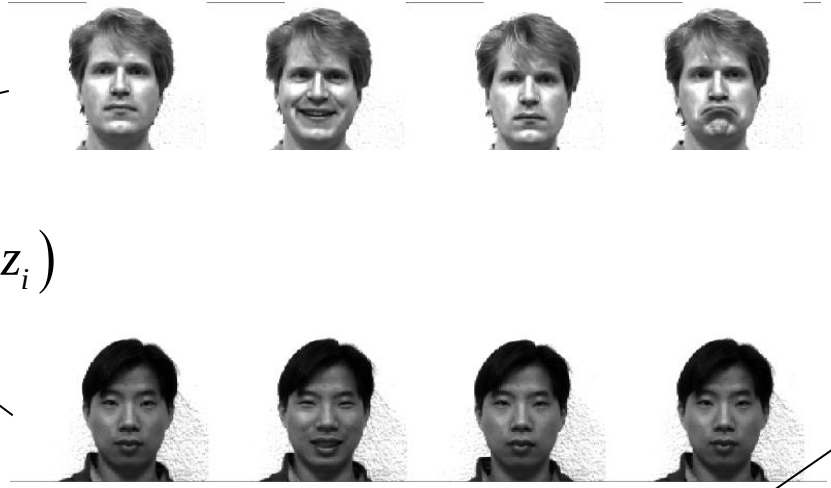
- **Maximize variance between classes**
- Minimize variance within class

$\bar{y}$: mean of projected features

$\bar{y}_j$: class mean of projected features (C is class label set)

LDA Objective





Number of data for each class

(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

Objective:

- **Maximize variance between classes**
- Minimize variance within class

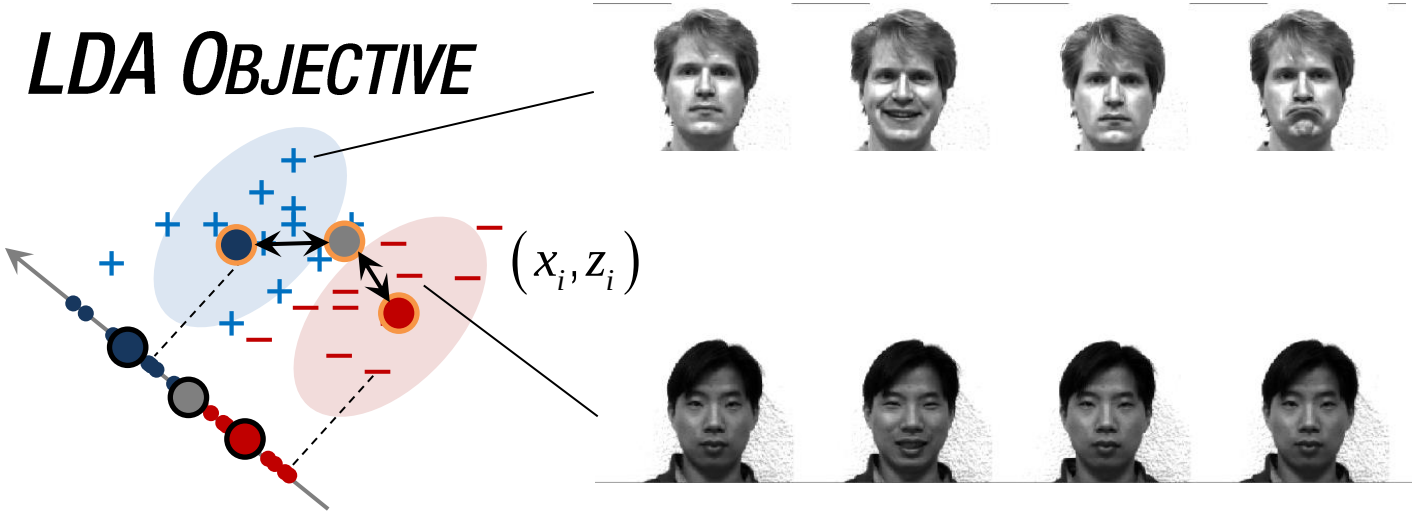
$$\tilde{S}_{\text{inter}} = \sum_{j \in C} n_j \left(\bar{y} - \bar{y}_j\right)\left(\bar{y} - \bar{y}_j\right)^T$$

$$= \sum_{j \in C} n_j b^T \left(\bar{x} - \bar{x}_j\right)\left(\bar{x} - \bar{x}_j\right)^T b$$

$\bar{x}$: mean of features

$\bar{x}_j$: class mean of features (C is class label set)

LDA Objective



(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

Objective:

- **Maximize variance between classes**
- Minimize variance within class

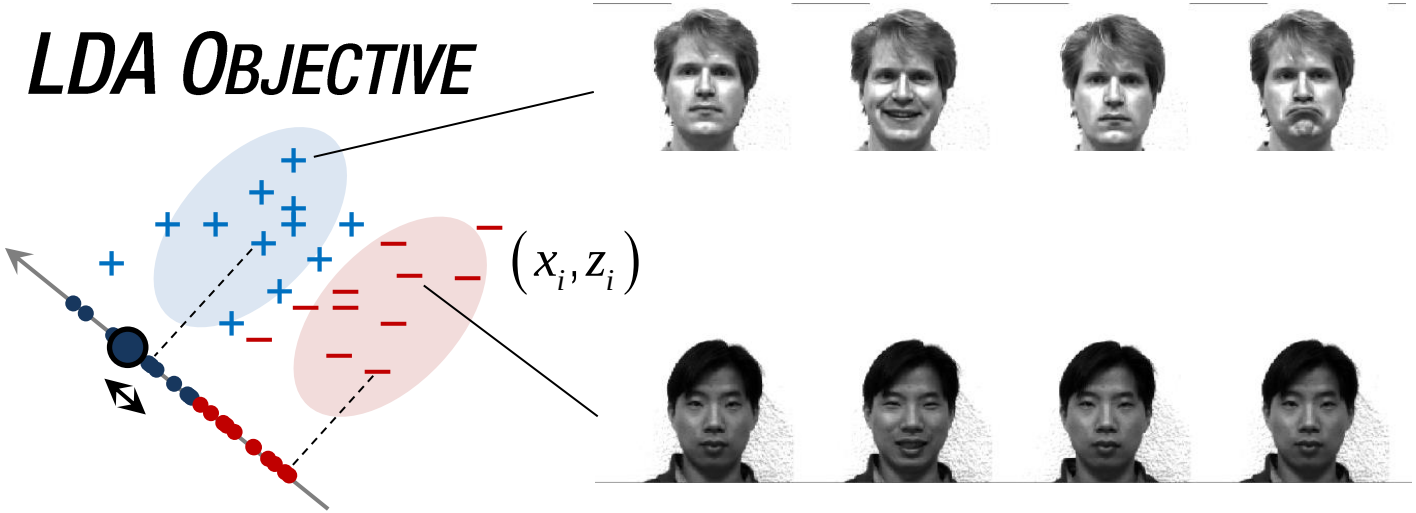
$$\tilde{S}_{\text{inter}} = b^T S_{\text{inter}} b$$

where $S_{\text{inter}} = \sum_{j \in C} n_j \left(\bar{x} - \bar{x}_j\right)\left(\bar{x} - \bar{x}_j\right)^T$

$\bar{x}$: mean of features

$\bar{x}_j$: class mean of features (C is class label set)

LDA Objective



(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

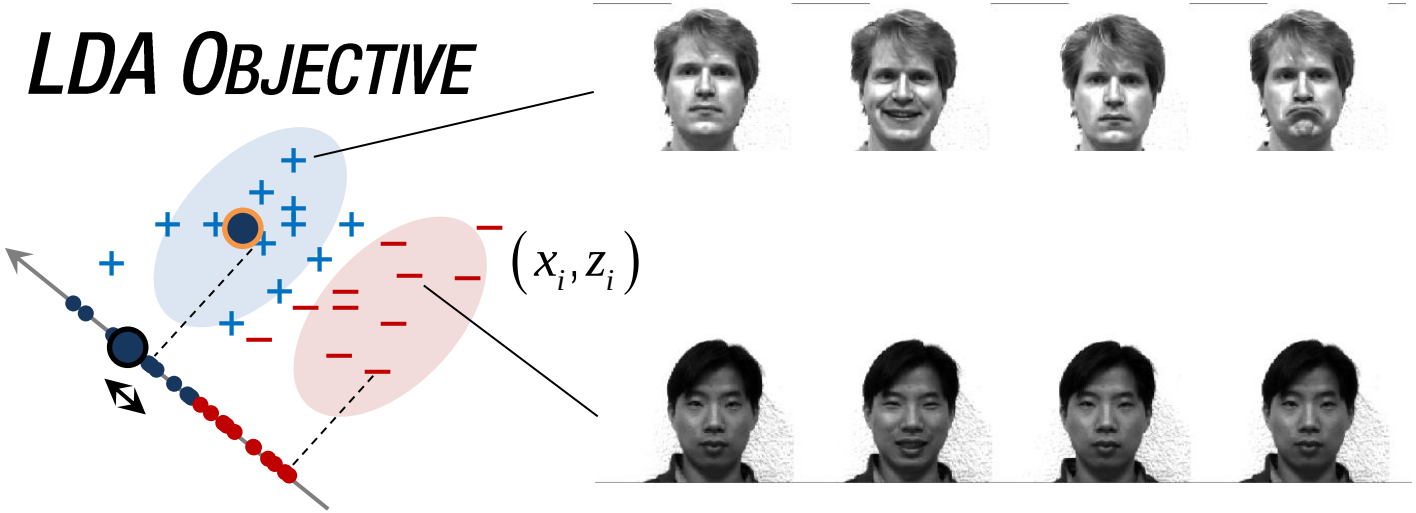
Objective:

- Maximize variance between classes
- **Minimize variance within class**

$$\tilde{S}_{\text{intra}} = \sum_{j \in C} \sum_{i \in C_j} \left(\bar{y}_j - y_j^i\right)\left(\bar{y}_j - y_j^i\right)^T$$

- $\bar{y}_j$: class mean of projected features
- y_j^i : projected features
- C_j : class index set

LDA Objective



(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

Objective:

- Maximize variance between classes
- **Minimize variance within class**

$$\tilde{S}_{\text{intra}} = \sum_{j \in C} \sum_{i \in C_j} \left(\bar{y}_j - y_j^i\right)\left(\bar{y}_j - y_j^i\right)^T$$

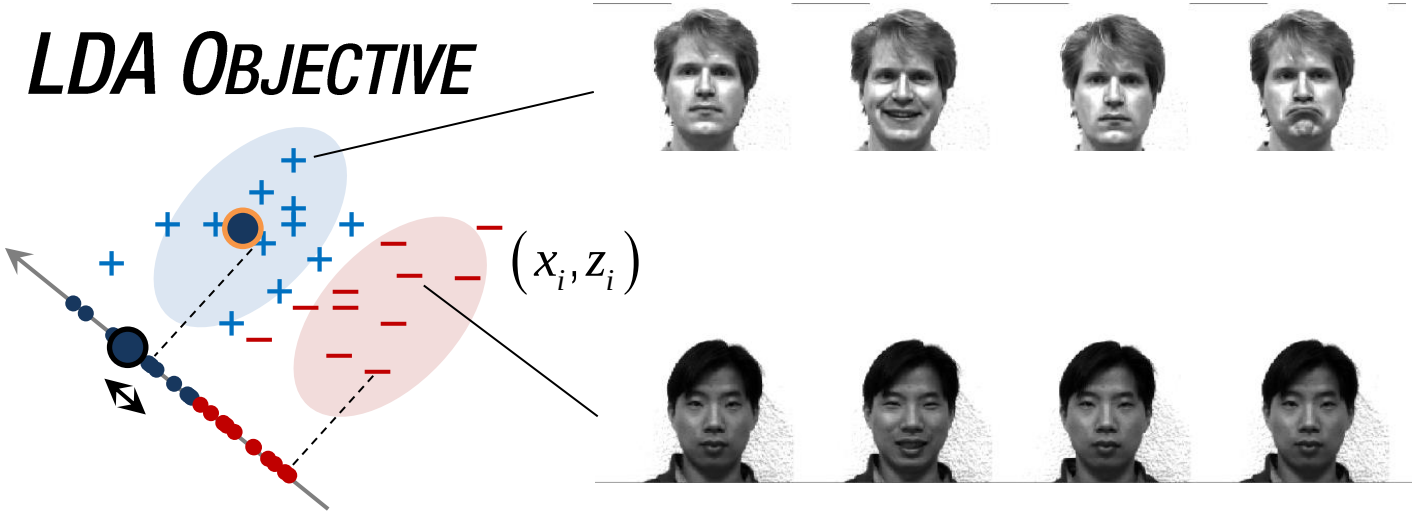
$$= \sum_{j \in C} \sum_{i \in C_j} b^T \left(\bar{x}_j - x_j^i\right)\left(\bar{x}_j - x_j^i\right)^T b$$

$\bar{x}_j$: class mean of features

x_j^i : features

C_j : class index set

LDA Objective



(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx \qquad y \in \mathbb{R}^d$

Objective:

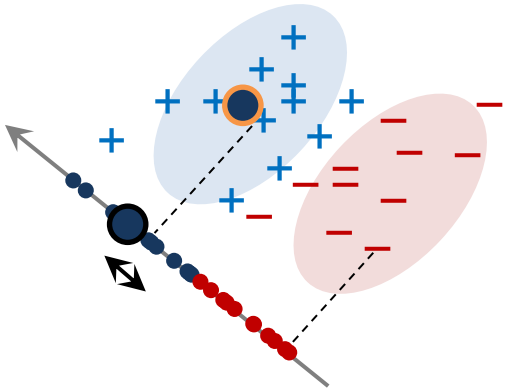
- Maximize variance between classes
- **Minimize variance within class**

$$\tilde{S}_{\text{intra}} = b^T S_{\text{intra}} b$$

where $S_{\text{intra}} = \sum_{j \in C} \sum_{i \in C_j} \left(\bar{x}_j - x_j^i\right)\left(\bar{x}_j - x_j^i\right)^T$

- $\bar{x}_j$: class mean of features
- x_j^i : features
- C_j : class index set

LDA Objective



(x_i, z_i): image x with label z

Recall: Eigen basis

Projection onto basis set: $y = bx$

Objective:

- Maximize variance between classes $\underset{b}{\text{maximize}} \left|\tilde{S}_{\text{inter}}\right|$
- Minimize variance within class $\underset{b}{\text{minimize}} \left|\tilde{S}_{\text{intra}}\right|$

$$\longrightarrow b^* = \underset{b}{\text{argmax}} \frac{\left|\tilde{S}_{\text{inter}}\right|}{\left|\tilde{S}_{\text{intra}}\right|}$$

$$= \underset{b}{\text{argmax}} \frac{\left|b^T S_{\text{inter}} b\right|}{\left|b^T S_{\text{intra}} b\right|}$$

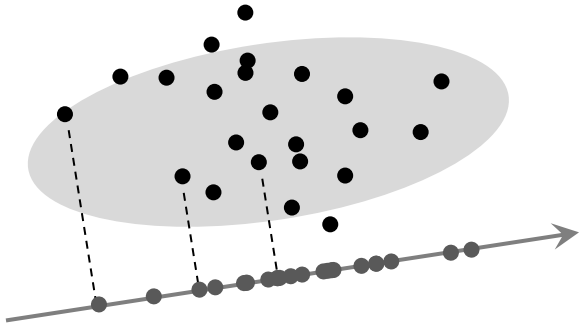
~ Solution of a generalized Eigenvalue problem:

$$S_{\text{inter}} b^* = \lambda S_{\text{intra}} b^*$$

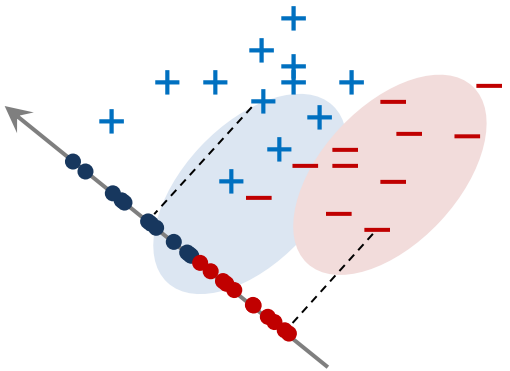
LEARNED BASIS

EIGENFACES VS. FISHER FACES

PCA



LDA



$$b^* = \underset{b}{\operatorname{argmax}}\, b^T X^T X b$$

$$X^T X b^* = \lambda b^*$$

$$b^* = \underset{b}{\operatorname{argmax}} \frac{\left| b^T S_{\text{inter}} b \right|}{\left| b^T S_{\text{intra}} b \right|}$$

$$S_{\text{inter}} b^* = \lambda S_{\text{intra}} b^*$$

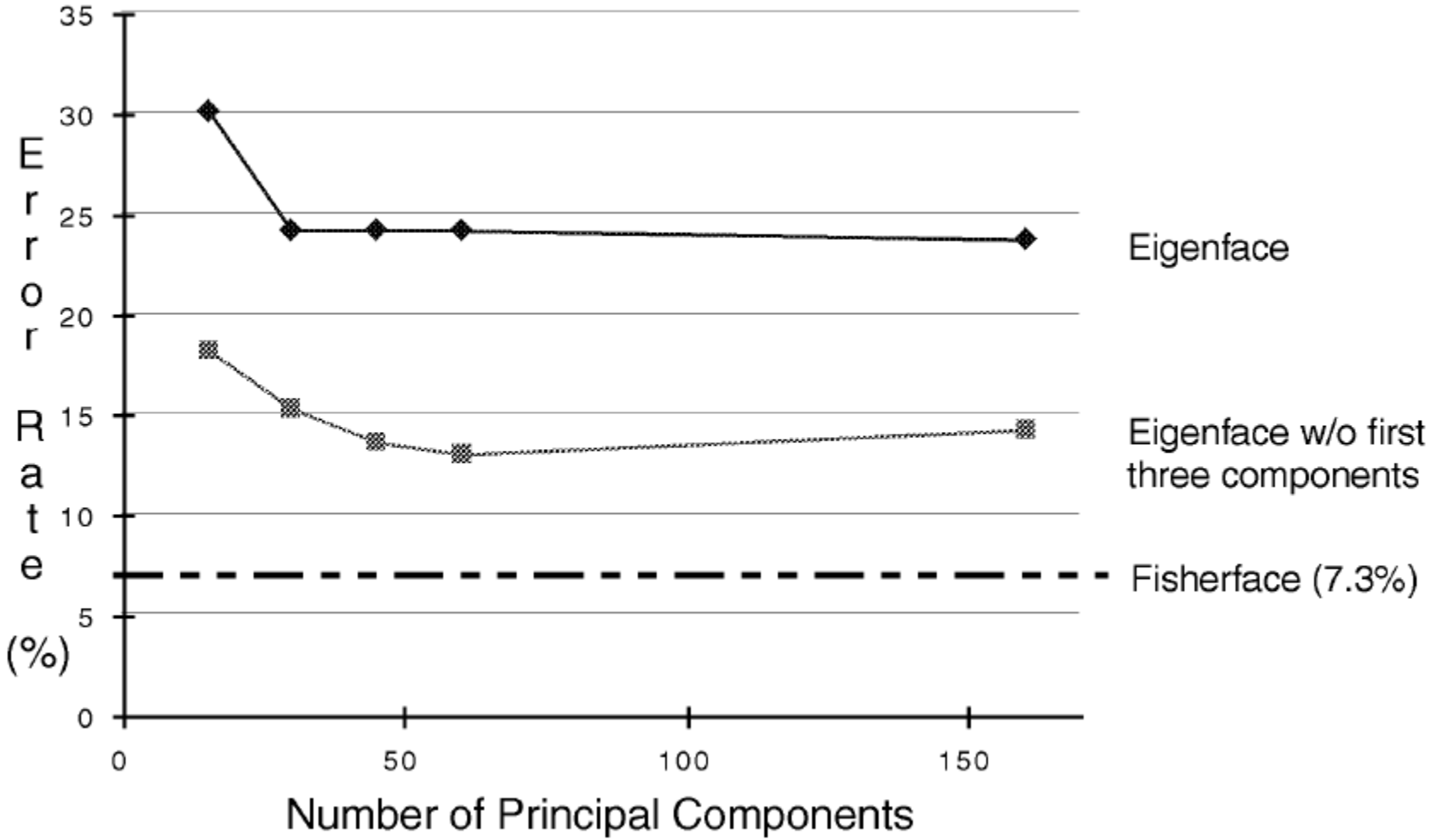
FISHER FACES RECOGNITION

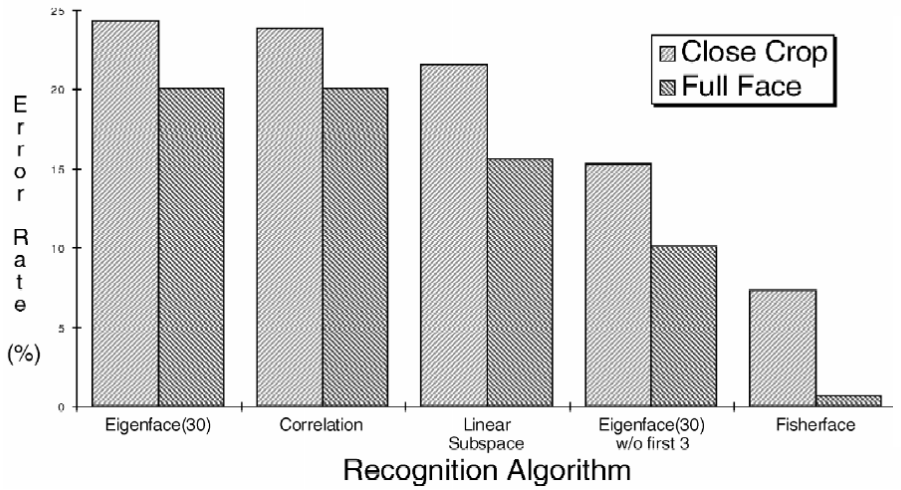
Residual

Residual

$$\left\| \overline{x} - \frac{\overline{x} \cdot b}{\|b\|^2} b \right\|$$

Face subspace


35
30
25
20
15
10
5
0
Error Rate (%)
0
50
100
150
Number of Principal Components
Eigenface
Eigenface w/o first three components
Fisherface (7.3%)



"Leaving-One-Out" of Yale Database			
Method	Reduced Space	Error Rate (%)	
		Close Crop	Full Face
Eigenface	30	24.4	19.4
Eigenface w/o 1st 3	30	15.3	10.8
Correlation	160	23.9	20.0
Linear Subspace	48	21.6	15.6
Fisherface	15	7.3	0.6

GLASS RECOGNITION