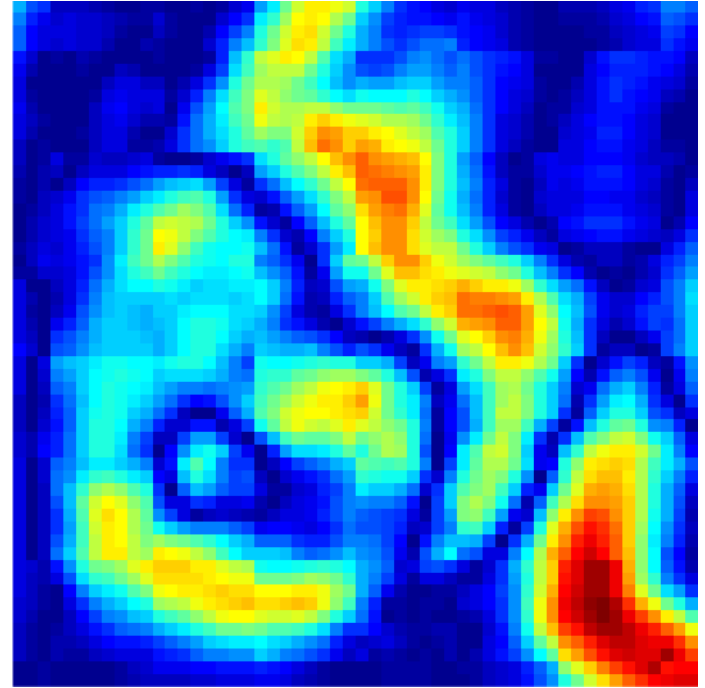
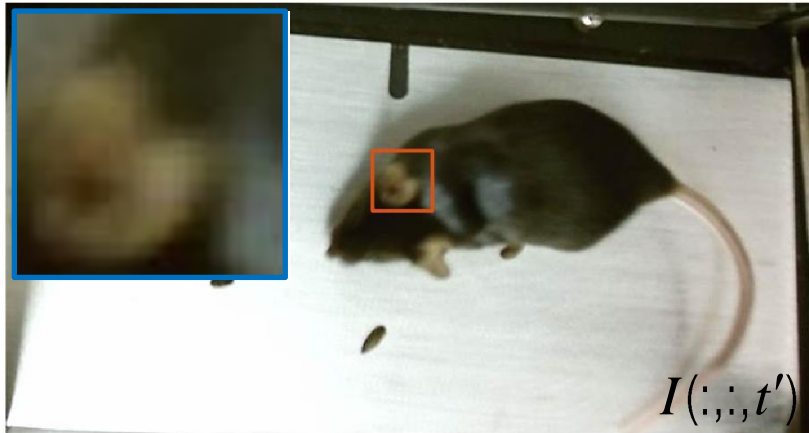
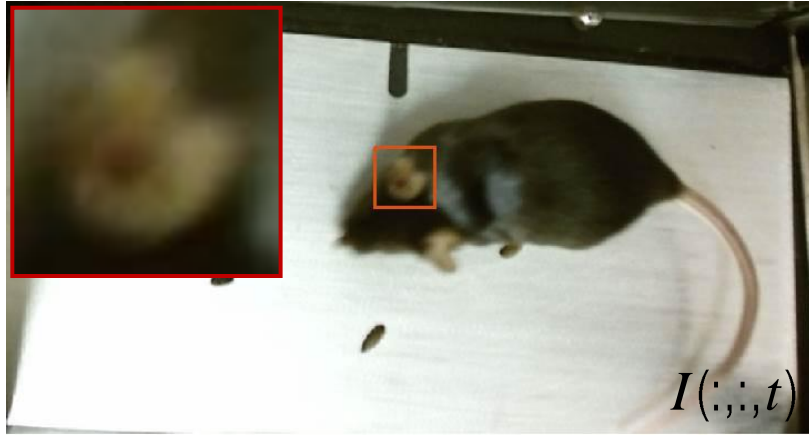


IMAGE ALIGNMENT

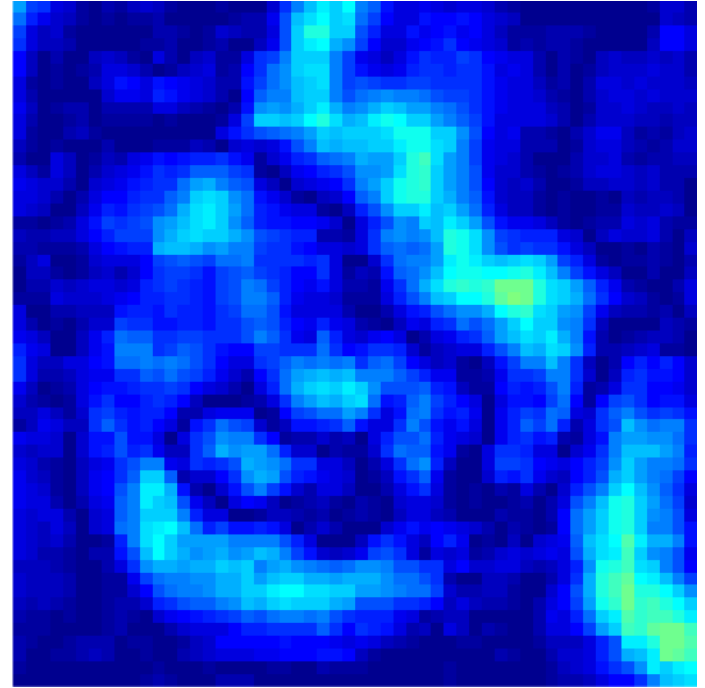
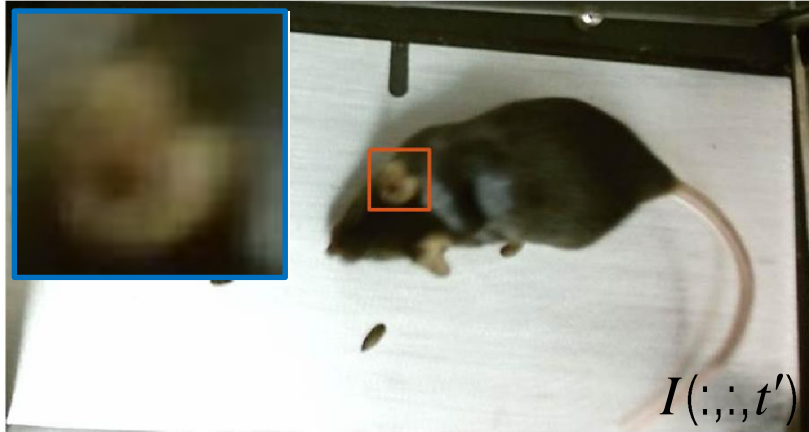
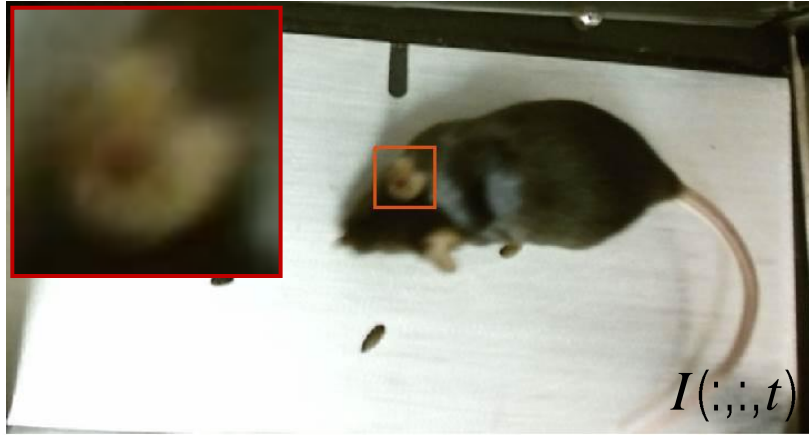
HYUN SOO PARK



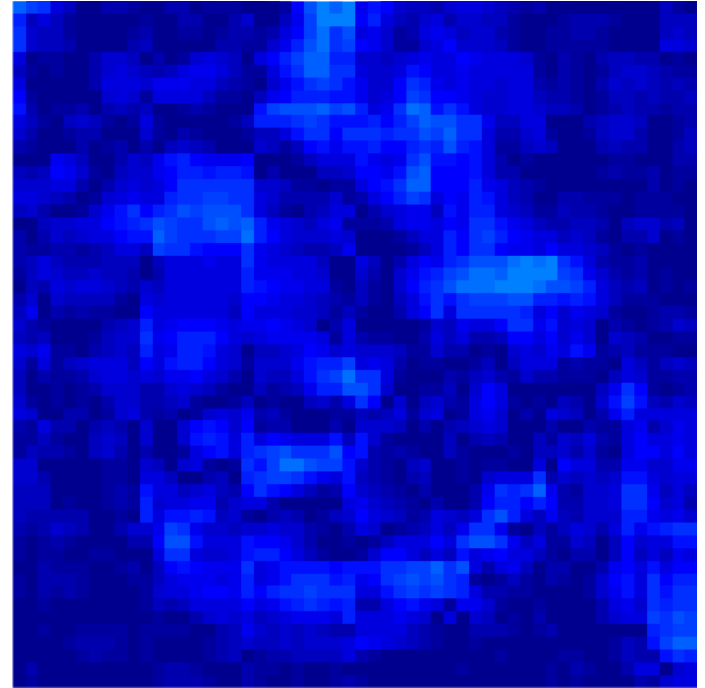
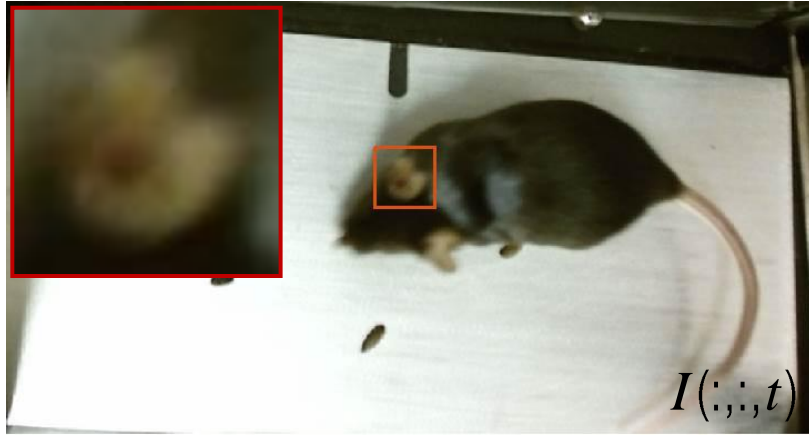
1ST ITERATION



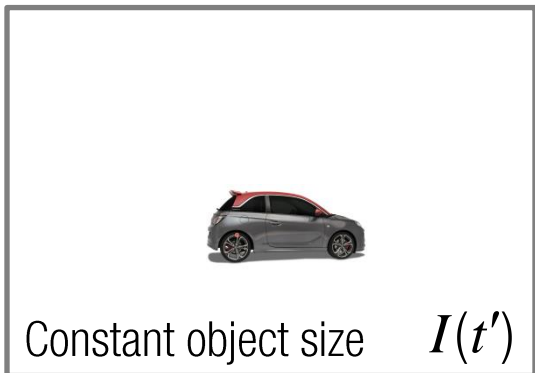
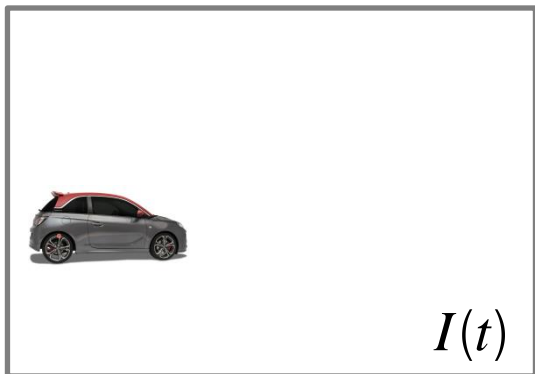
3RD ITERATION



6TH ITERATION



WHEN (u,v) FLOW MAKES SENSE?

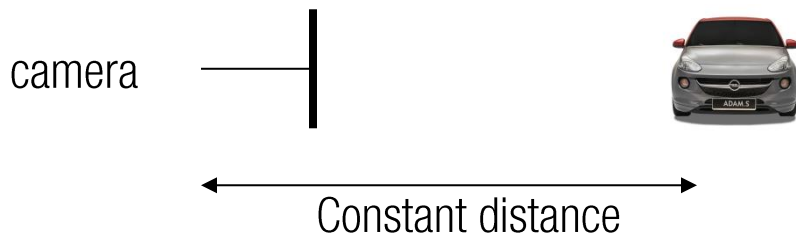


$$\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t} = 0$$

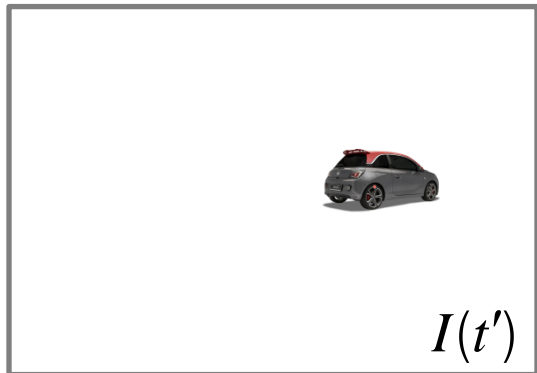
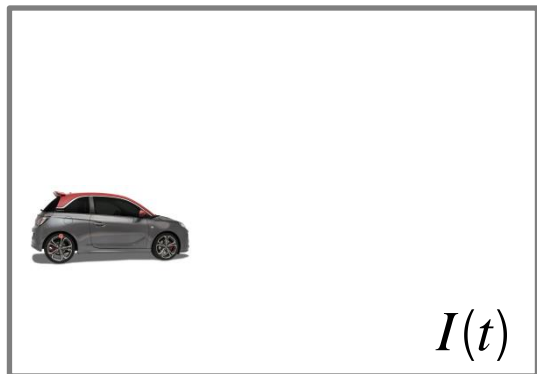
$$x' = x + u$$

$$y' = y + v$$

Side view



GENERAL OBJECT MOTION



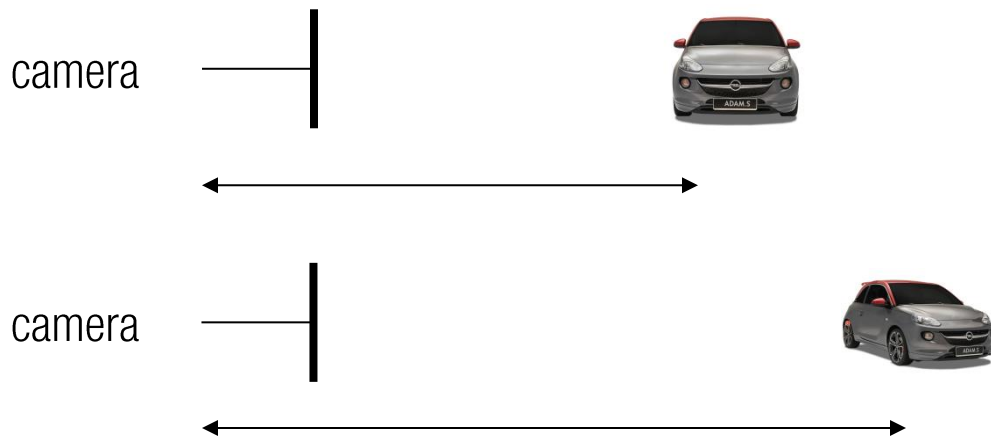
Different depth/orientation

$$\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t} = 0$$

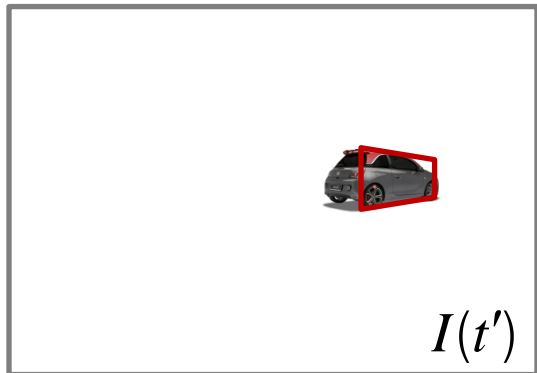
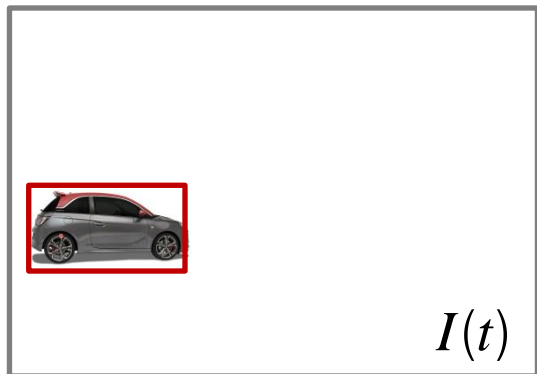
$$x' \neq x + u$$

$$y' \neq y + v$$

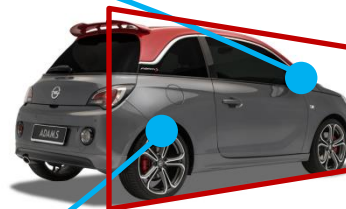
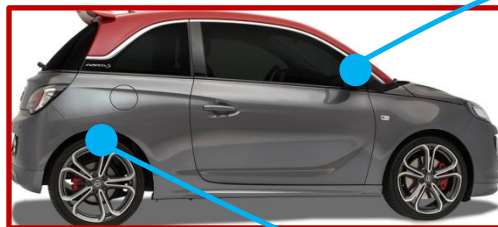
Side view



PARAMETRIC TRANSFORMATION



Different depth/orientation



$$\mathbf{x}' = W(\mathbf{x}; p)$$

Unknowns: p

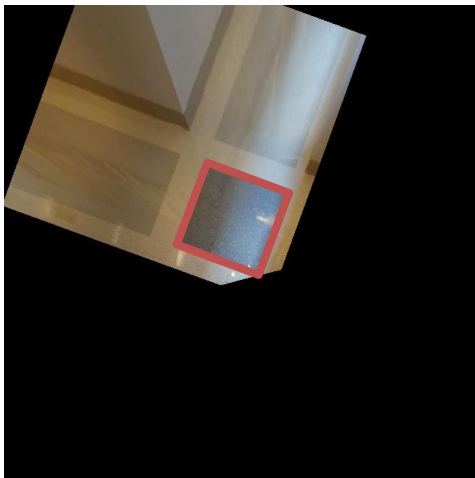
RECALL: PARAMETRIC TRANSFORMATIONS



Euclidean (3 dof)

- Length
- Angle
- Area

$$\begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ & & 1 \end{bmatrix}$$



Similarity (4 dof)

- Length ratio
- Angle

$$\begin{bmatrix} \alpha \cos\theta & -\alpha \sin\theta & t_x \\ \alpha \sin\theta & \alpha \cos\theta & t_y \\ & & 1 \end{bmatrix}$$



Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$$



Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}$$

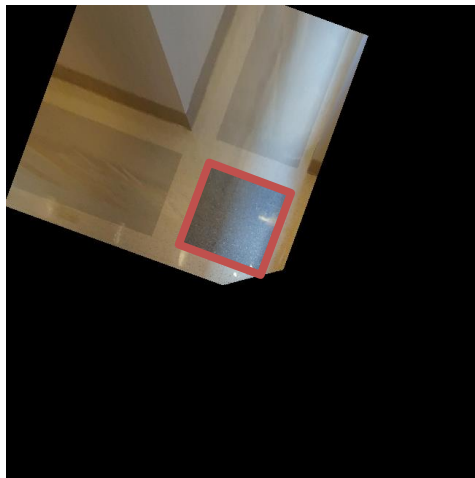
RECALL: PARAMETRIC TRANSFORMATIONS



Euclidean (3 dof)

- Length
- Angle
- Area

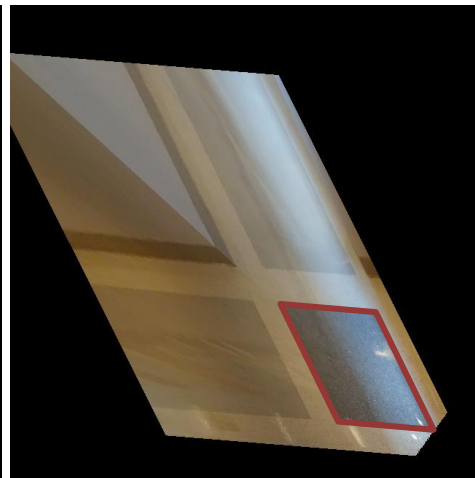
Ex) Aerial images



Similarity (4 dof)

- Length ratio
- Angle

Change of depth



Affine (6 dof)

- Parallelism
- Ratio of area
- Ratio of length

Far objects



Projective (8 dof)

- Cross ratio
- Concurrency
- Colinearity

Planar objects

OPTICAL FLOW



Template
 $T(x)$



Target image
 $I(x)$

Brightness constancy

$$I(x + \Delta x) \approx T(x)$$

IMAGE ALIGNMENT



Template
 $T(x)$



Target image
 $I(x)$

Brightness constancy

$$I(W(x; p)) \approx T(x)$$

ex) affine transform

$$W(x; p) = \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

IMAGE ALIGNMENT



Template
 $T(x)$



Target image
 $I(x)$

Brightness constancy

$$I(W(x; p)) \approx T(x)$$

ex) affine transform

$$\begin{aligned} W(x; p) &= \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \\ &= \begin{bmatrix} (p_1 + 1)u + p_2 v + p_3 \\ p_4 u + (p_5 + 1)v + p_6 \end{bmatrix} \end{aligned}$$

IMAGE ALIGNMENT



Template
 $T(x)$



Target image
 $I(x)$

Brightness constancy

$$I(W(x; p)) \approx T(x)$$

Objective: to find the optimal warping parameter p that minimizes warping error.

$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - \underbrace{T(x)}_{\text{Template}})^2$$

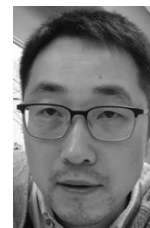


IMAGE ALIGNMENT



Template
 $T(x)$



Target image
 $I(x)$

Brightness constancy

$$I(W(x; p)) \approx T(x)$$

Objective: to find the optimal warping parameter p that minimizes warping error.

$$p^* = \underset{p}{\text{minimize}} \sum_x \underbrace{(I(W(x; p)) - T(x))^2}_{\text{Warped image} \quad \text{Template}}$$



IMAGE ALIGNMENT



Template
 $T(x)$



Target image
 $I(x)$

Brightness constancy

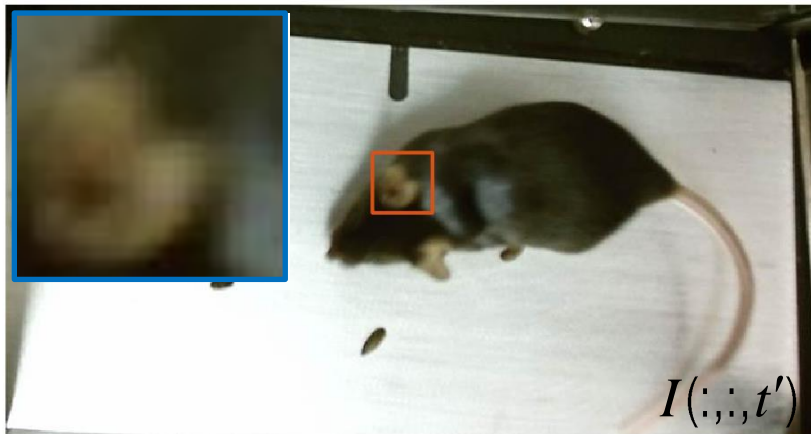
$$I(W(x; p)) \approx T(x)$$

Objective: to find the optimal warping parameter p that minimizes warping error.

$$p^* = \underset{p}{\text{minimize}} \sum_x \frac{(I(W(x; p)) - T(x))^2}{\text{Error image}}$$



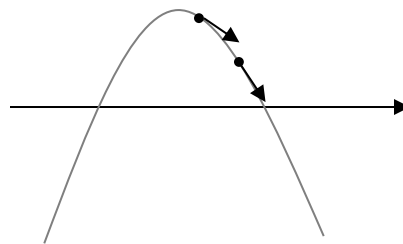
RECALL: LOCAL PATCH TRACKING



$$x = (A^T A)^{-1} A^T b \quad x = \begin{bmatrix} -2.76 \\ 1.27 \end{bmatrix}$$

First order approximation

$$I(x + u\delta t, y + v\delta t, t + \delta t) \\ \approx I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t$$



Gauss-Newton's method

IMAGE ALIGNMENT OBJECTIVE

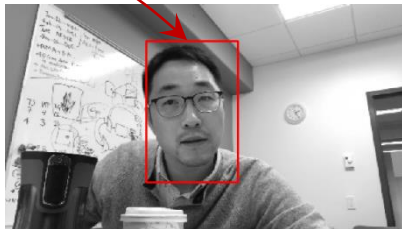
$$W(x; p)$$



$T(x)$



$I(W(x; p))$



$I(x)$

$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \frac{\partial I}{\partial p} \Delta p$$

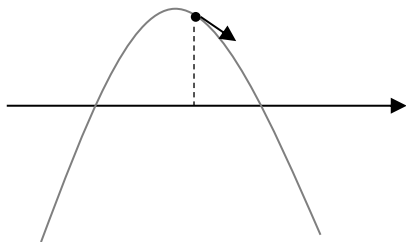
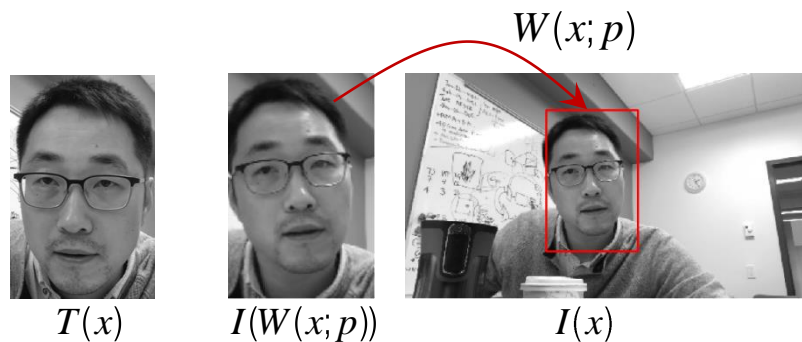


IMAGE ALIGNMENT OBJECTIVE



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \frac{\partial I}{\partial p} \Delta p$$

Ex) optical flow (translation)

$$W(x; p) = x + \Delta x$$

$$\rightarrow \frac{\partial I}{\partial p} \Delta p = \frac{\partial I}{\partial x} \Delta x = \nabla I \Delta x$$

$$= \text{Image} \Delta u + \text{Image} \Delta v$$

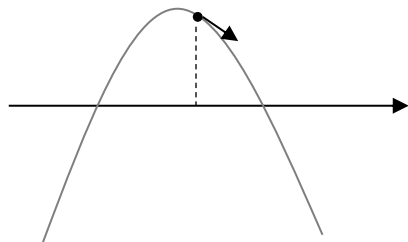
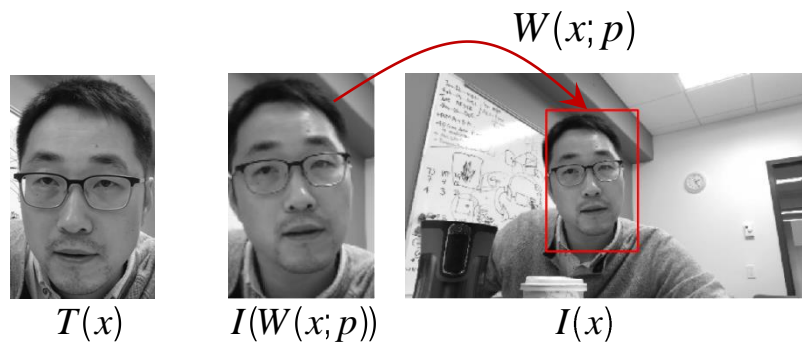


IMAGE ALIGNMENT OBJECTIVE



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \frac{\partial I}{\partial p} \Delta p$$

Ex) affine transform

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p} \Delta p$$

What does the gradient image
w.r.t. the affine parameters mean?

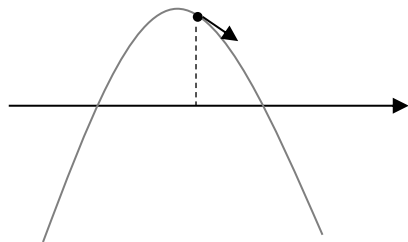
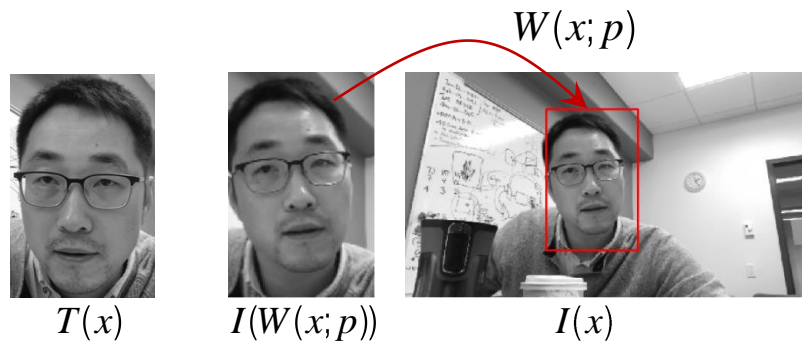


IMAGE ALIGNMENT OBJECTIVE



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \frac{\partial I}{\partial p} \Delta p$$

Ex) affine transform

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p} \Delta p = \frac{\partial I}{\partial x} \frac{\partial W}{\partial p} \Delta p$$

Chain rule

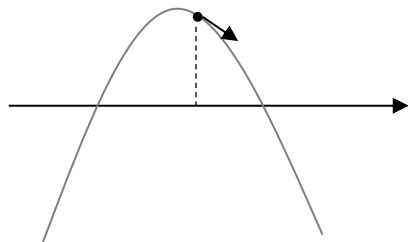
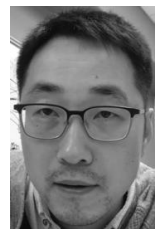


IMAGE ALIGNMENT OBJECTIVE

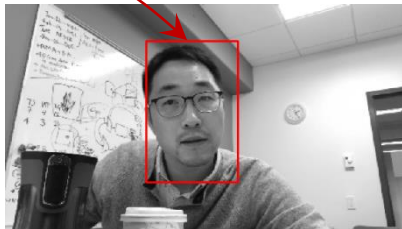
$$W(x; p)$$



$T(x)$



$I(W(x; p))$



$I(x)$

$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \frac{\partial I}{\partial p} \Delta p$$

Ex) affine transform

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p} \Delta p = \frac{\partial I}{\partial x} \frac{\partial W}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

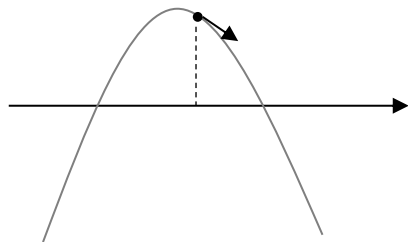
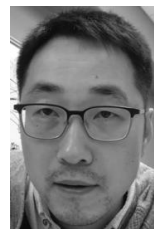


IMAGE ALIGNMENT OBJECTIVE

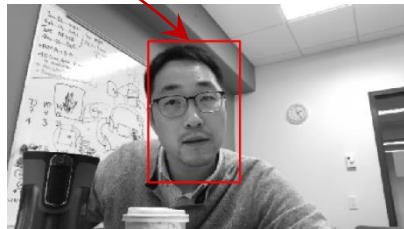
$$W(x; p)$$



$T(x)$



$I(W(x; p))$



$I(x)$

$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \frac{\partial I}{\partial p} \Delta p$$

Ex) affine transform

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

$$\rightarrow \frac{\partial I}{\partial p} \Delta p = \frac{\partial I}{\partial x} \frac{\partial W}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\text{Jacobian: } \frac{\partial W}{\partial p} = \begin{bmatrix} \frac{\partial u}{\partial p_1} & \dots & \frac{\partial u}{\partial p_6} \\ \frac{\partial v}{\partial p_1} & \dots & \frac{\partial v}{\partial p_6} \end{bmatrix}$$

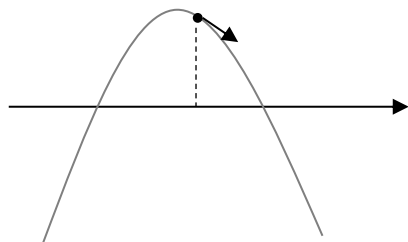


IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

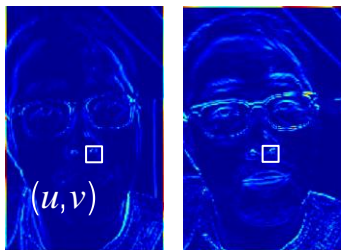
1 x 2

$$\begin{bmatrix} \frac{\partial u}{\partial p_1} & \dots & \frac{\partial u}{\partial p_6} \\ \frac{\partial v}{\partial p_1} & \dots & \frac{\partial v}{\partial p_6} \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6

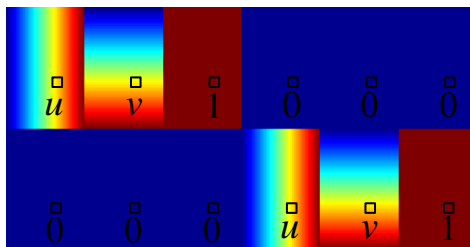


IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

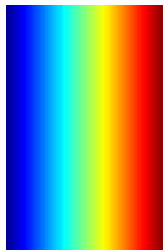
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

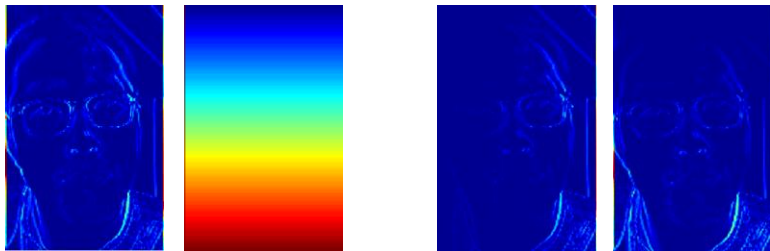
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

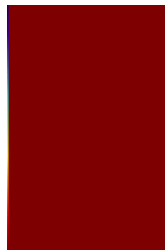
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6

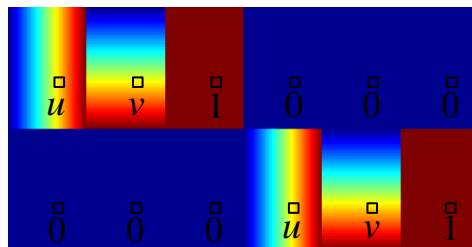
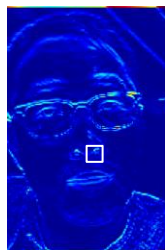


IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

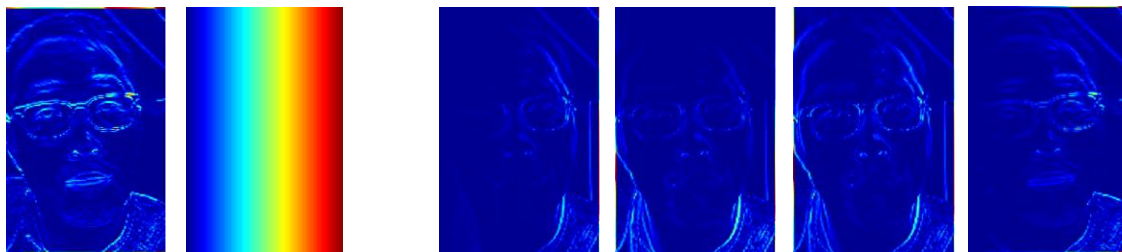
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

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$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

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$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

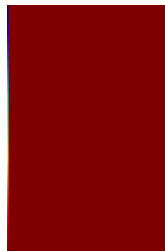
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

1 x 6



$$\nabla I \frac{\partial W}{\partial p}$$

IMAGE JACOBIAN (STEEPEST DESCENT IMAGE)

$$W(x; p) = \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} p_1 u + p_2 v + p_3 \\ p_4 u + p_5 v + p_6 \end{bmatrix} \rightarrow \frac{\partial I}{\partial p} \Delta p = \nabla I \frac{\partial W}{\partial p} \Delta p$$

$$\begin{bmatrix} I_x & I_y \end{bmatrix}$$

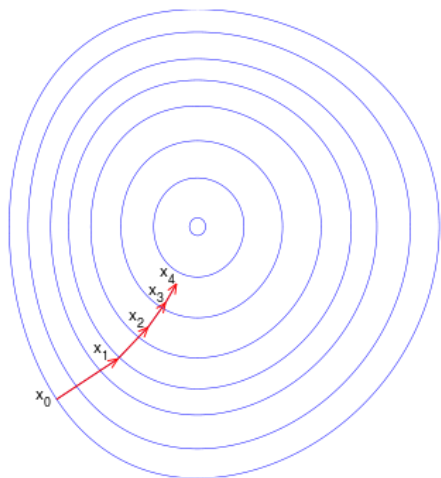
1 x 2

$$\begin{bmatrix} u & v & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & u & v & 1 \end{bmatrix}$$

2 x 6

$$\begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_6 \end{bmatrix}$$

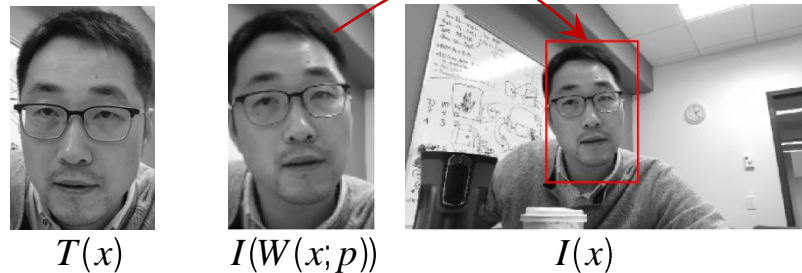
1 x 6



$\nabla I \frac{\partial W}{\partial p}$: steepest decent images

IMAGE ALIGNMENT OBJECTIVE

$$W(x; p)$$



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



2. Find Δp that minimizes the obj. function at p

$$I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \quad \longrightarrow \quad \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x; p))$$

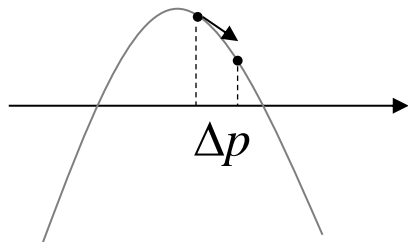
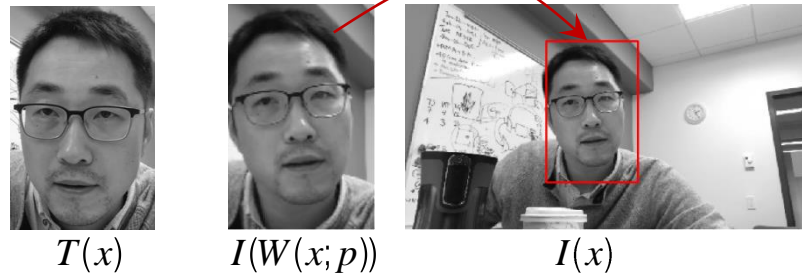


IMAGE ALIGNMENT OBJECTIVE

$$W(x; p)$$



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



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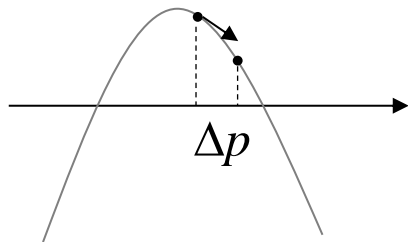
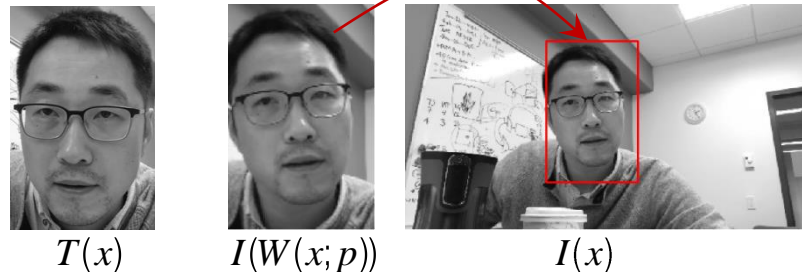


IMAGE ALIGNMENT OBJECTIVE

$$W(x; p)$$



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

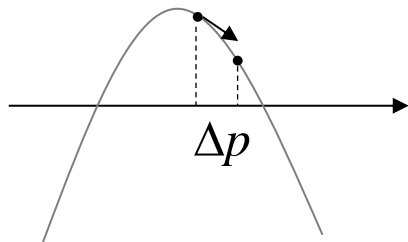
1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



2. Find Δp that minimizes the obj. function at p

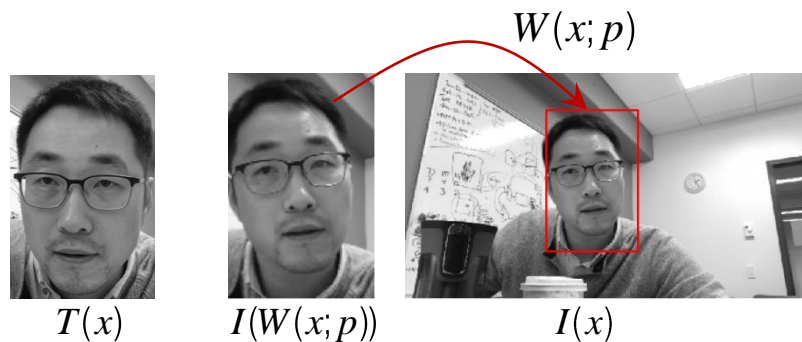
$$I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x; p))$$



$$\begin{bmatrix} \nabla I \frac{\partial W}{\partial p} \Big|_{x_1} \\ \vdots \\ \nabla I \frac{\partial W}{\partial p} \Big|_{x_n} \end{bmatrix} \Delta p = - \begin{bmatrix} I(W(x_1; p)) - T(x_1) \\ \vdots \\ I(W(x_n; p)) - T(x_n) \end{bmatrix}$$



IMAGE ALIGNMENT OBJECTIVE



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

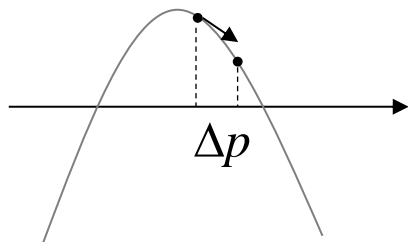
1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



2. Find Δp that minimizes the obj. function at p

$$I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x; p))$$



$$\begin{bmatrix} \nabla I \frac{\partial W}{\partial p} \Big|_{x_1} \\ \vdots \\ \nabla I \frac{\partial W}{\partial p} \Big|_{x_n} \end{bmatrix} \mathbf{x} = \begin{bmatrix} I(W(x_1; p)) - T(x_1) \\ \vdots \\ I(W(x_n; p)) - T(x_n) \end{bmatrix}$$

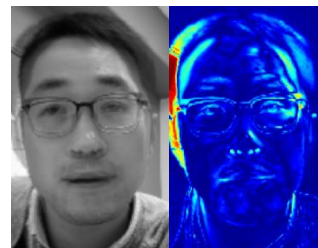
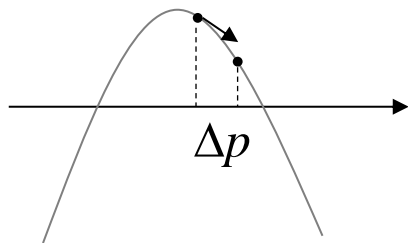
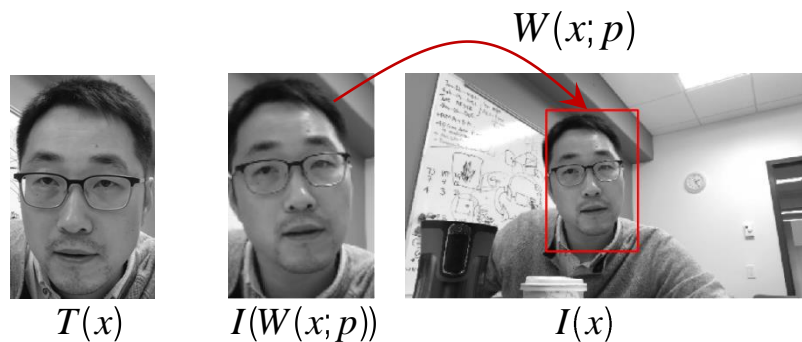


IMAGE ALIGNMENT OBJECTIVE



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



2. Find Δp that minimizes the obj. function at p

$$I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p - T(x) = 0 \longrightarrow \nabla I \frac{\partial W}{\partial p} \Delta p = T(x) - I(W(x; p))$$

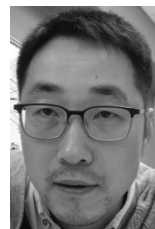
$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T \left(\nabla I \frac{\partial W}{\partial p} \right)$$



IMAGE ALIGNMENT OBJECTIVE

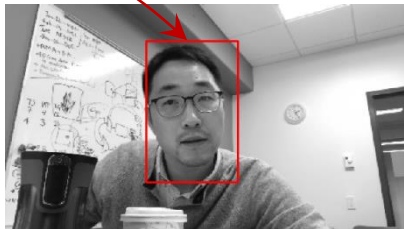
$$W(x; p)$$



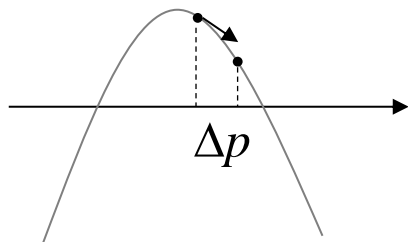
$T(x)$



$I(W(x; p))$



$I(x)$



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



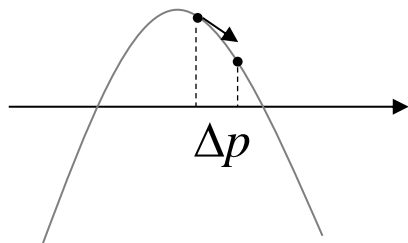
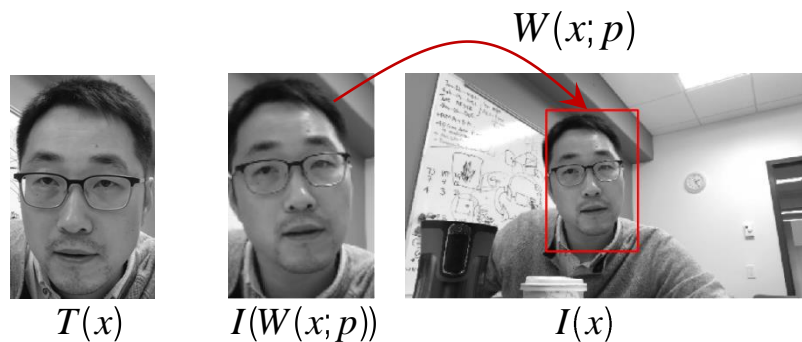
2. Find Δp that minimizes the obj. function at p

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

3. Update $p \leftarrow p + \Delta p$



OPTICAL FLOW DERIVATION



$$\Delta x^* = \underset{\Delta x}{\text{minimize}} \sum_x (I(x; \Delta x) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at x

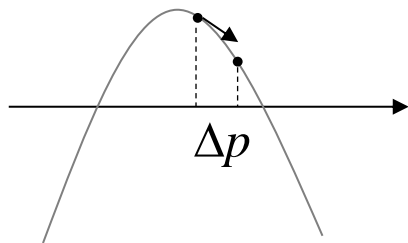
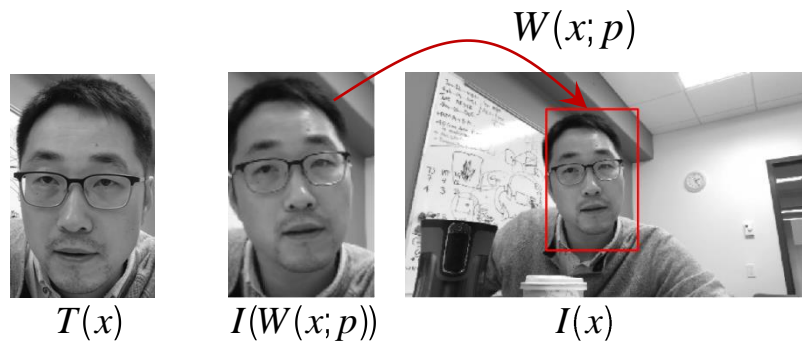
$$I(x + \Delta x) \approx I(x) + \nabla I \Delta x$$



2. Find Δx that minimizes the obj. function at x

$$\nabla I \Delta x = T(x) - I(x) = -(I(x) - T(x)) \quad \triangleq \quad I_x \Delta u + I_y \Delta v = -I_t$$

OPTICAL FLOW DERIVATION



$$\Delta x^* = \underset{\Delta x}{\text{minimize}} \sum_x (I(x; \Delta x) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at x

$$I(x + \Delta x) \approx I(x) + \nabla I \Delta x$$



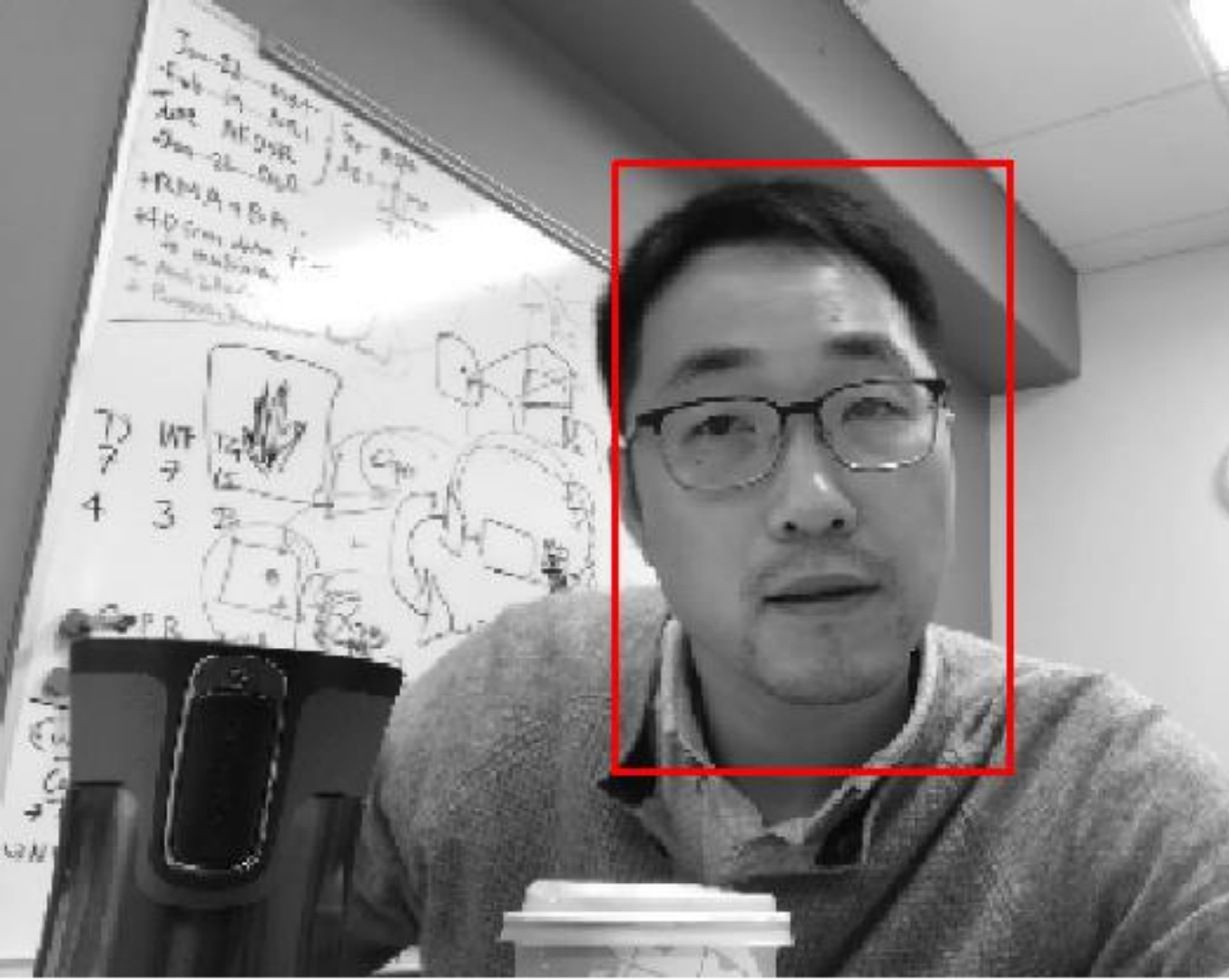
2. Find Δx that minimizes the obj. function at x

$$\nabla I \Delta x = T(x) - I(x) = -(I(x) - T(x)) \quad \triangleq \quad I_x \Delta u + I_y \Delta v = -I_t$$

$$\Delta x = H^{-1} \sum_x (\nabla I)^T (T(x) - I(W(x; p)))$$

$$\text{where } H = \sum_x \nabla I^T \nabla I$$

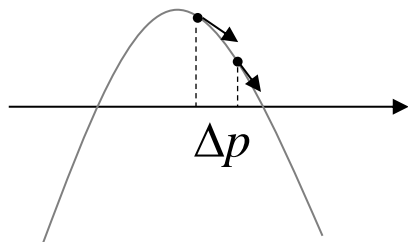
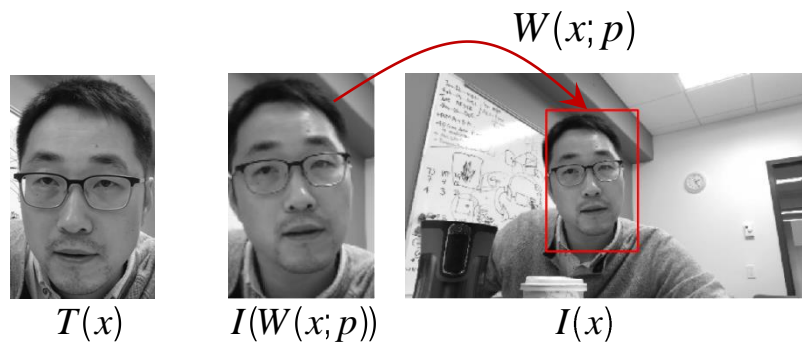
p



$$p \leftarrow p + \Delta p$$



IMAGE ALIGNMENT



$$p^* = \underset{p}{\text{minimize}} \sum_x (I(W(x; p)) - T(x))^2$$

Guass-Newton's method

1. Linearize the obj. function at p

$$I(W(x; p + \Delta p)) \approx I(W(x; p)) + \nabla I \frac{\partial W}{\partial p} \Delta p$$



2. Find Δp that minimizes the obj. function at p

$$\Delta p = H^{-1} \sum_x \left(\nabla I \frac{\partial W}{\partial p} \right)^T (T(x) - I(W(x; p)))$$

3. Update $p \leftarrow p + \Delta p$

4. Goto 1





$I(x)$



$T(x)$



$I(W(x; p))$



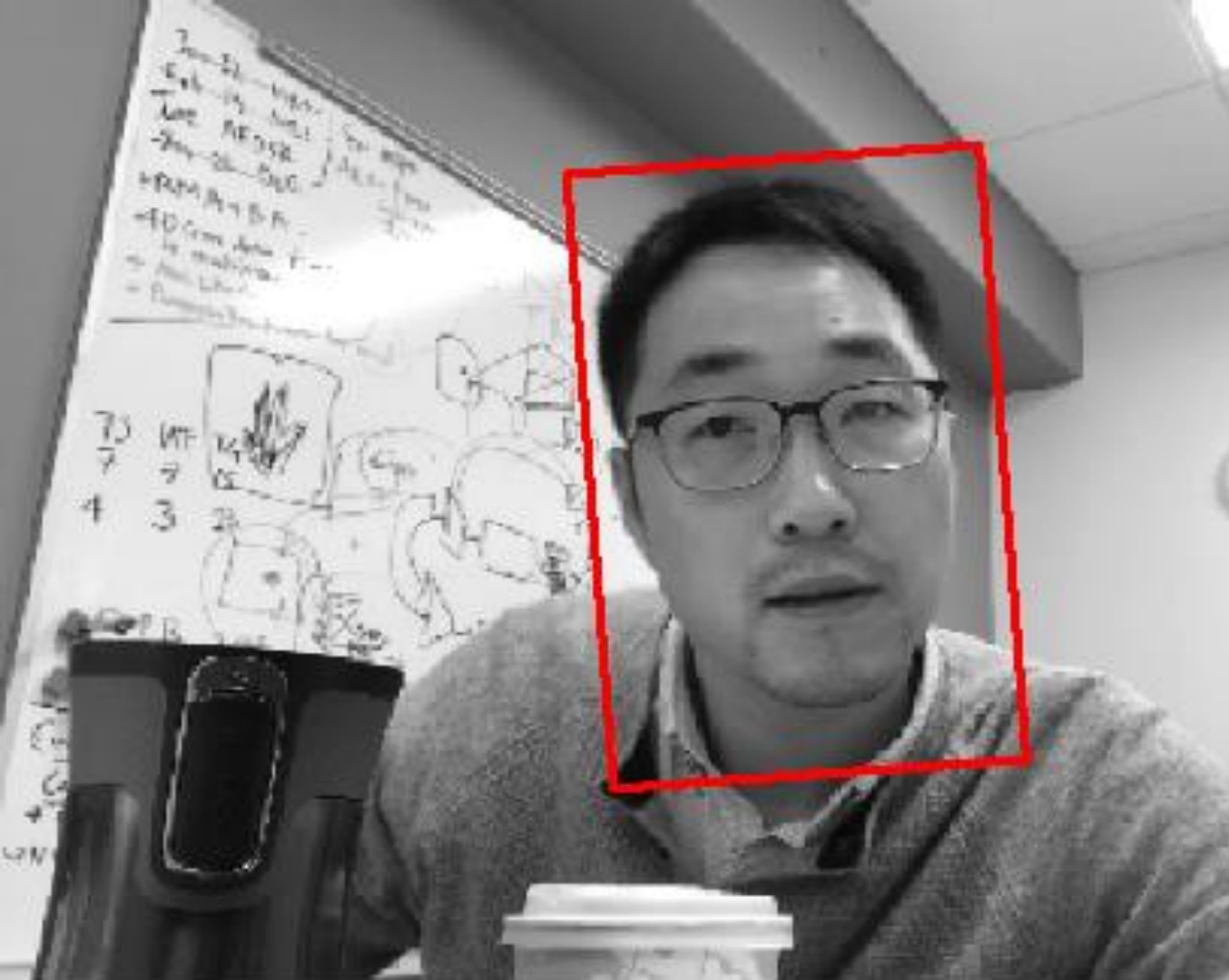
$0.5T(x) + 0.5I(W(x; p))$



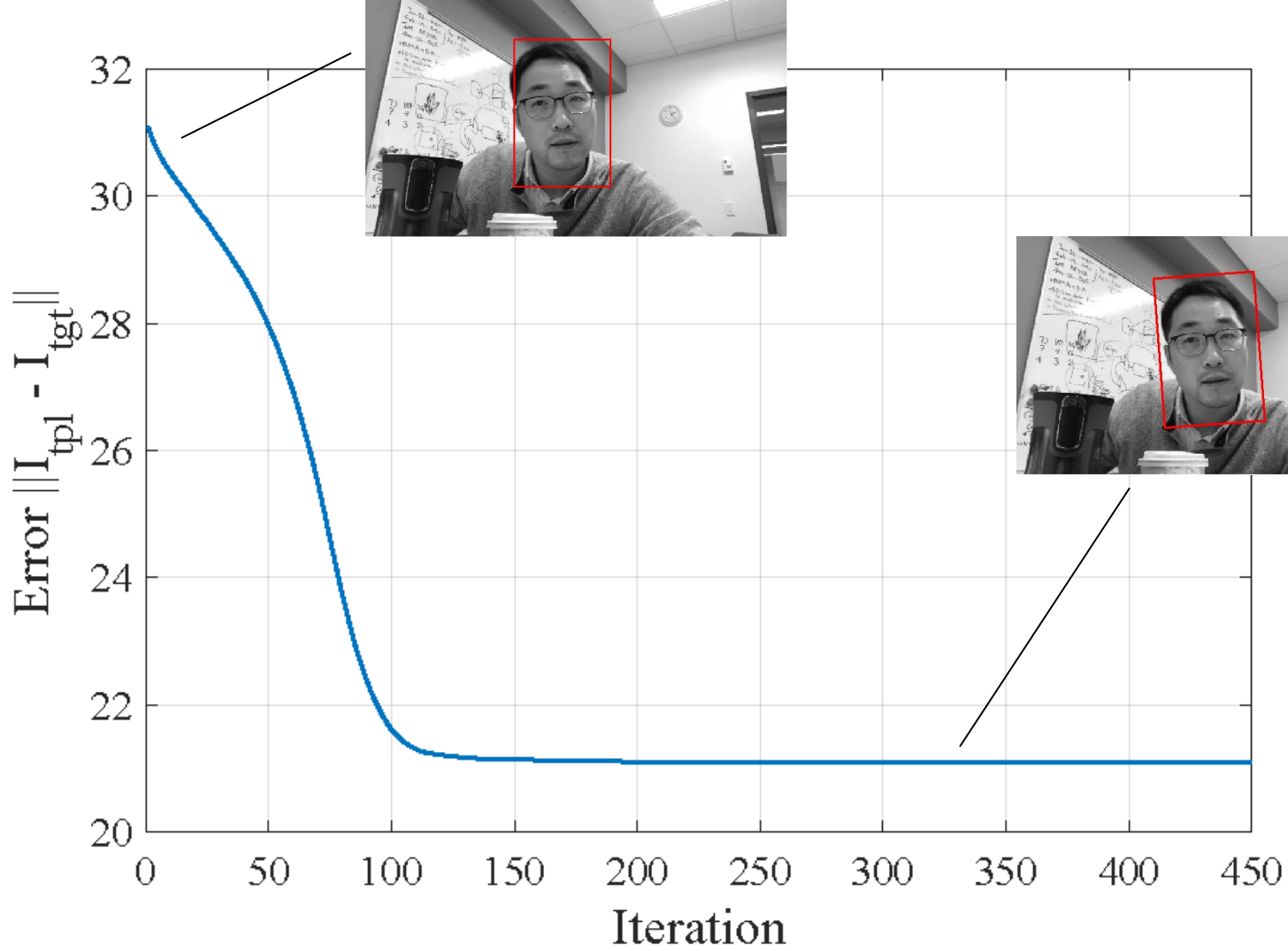
$|T(x) - I(W(x; p))|$

$$p \leftarrow p + \Delta p$$

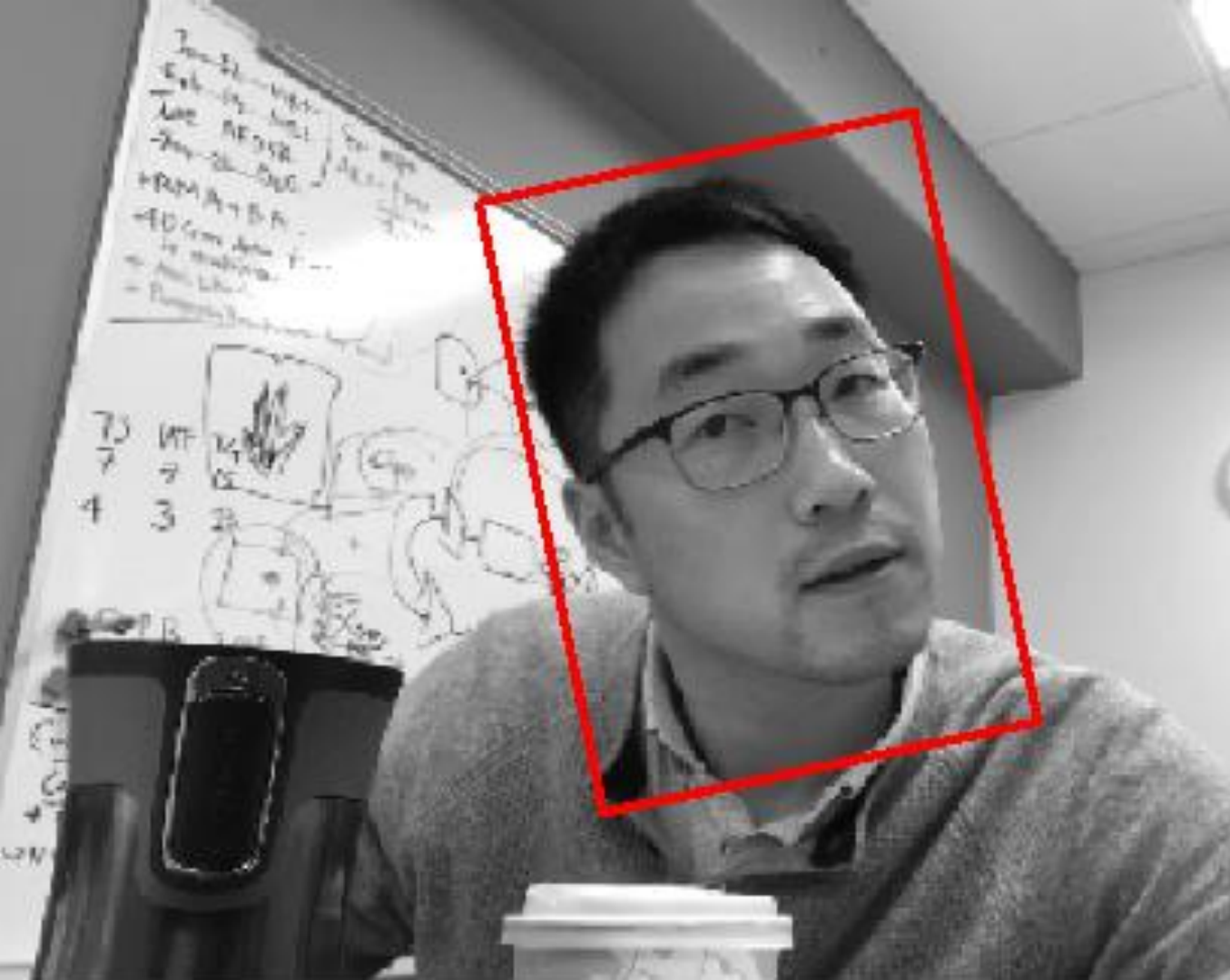




Error: 8.18





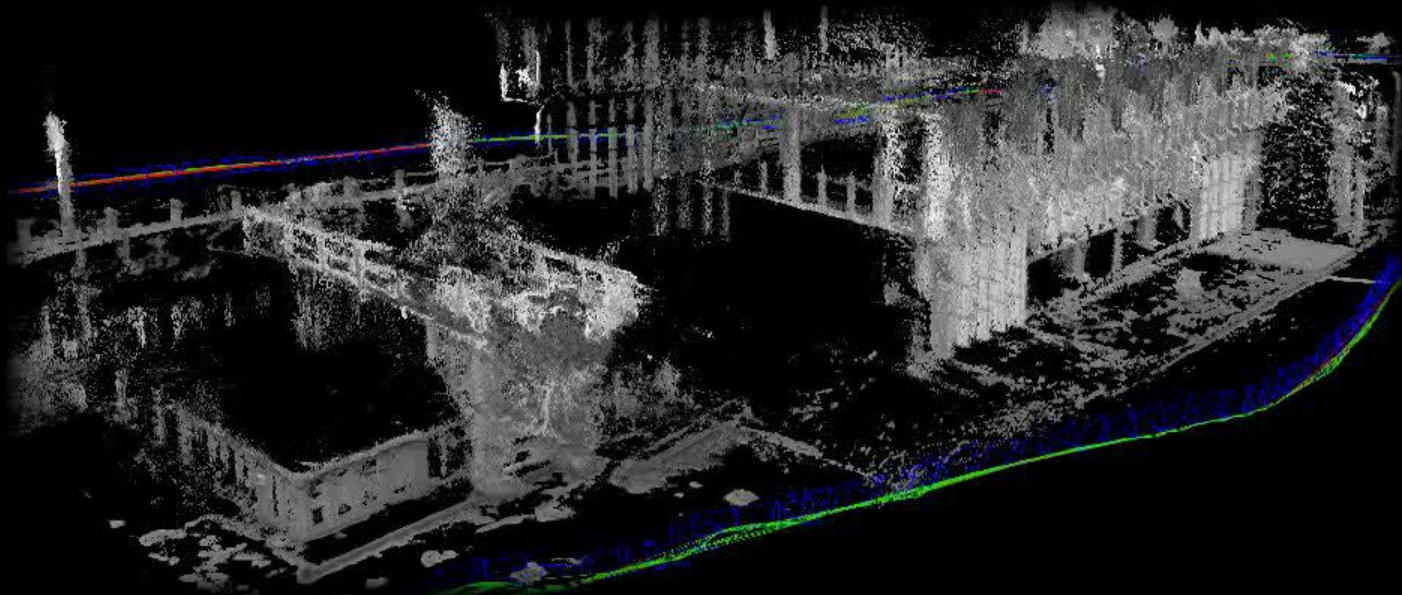




Monkey interest point tracking

LSD-SLAM: Large-Scale Direct Monocular SLAM

Jakob Engel, Thomas Schöps, Daniel Cremers
ECCV 2014, Zurich



Computer Vision Group
Department of Computer Science
Technical University of Munich

