IMAGE PYRAMID

HYUN SOO PARK
Salvador Dali, Abraham Lincoln
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RECALL: OBJECT RECOGNITION WITH HOG
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ReCall: Object Recognition with HOG
**Fourier Transform**

Time signal

\[ x(t) \]
**Fourier Transform**

Time signal

\[ x(t) \]

Frequency response

\[ |X(f)| \]

\[ -f \quad f \]
Fourier Transform

Time signal: $x(t)$

Guassian filter: $g(t)$

Frequency response: $|X(f)|$

$|G(f)|$

$x(t) * g(t) = X(f) G(f)$
Fourier Transform

Time signal

\[ x(t) \]

Guassian filter

\[ g(t) \]

\[ X(f) \] \[ G(f) \]

\[ X(f) G(f) \]

Frequency response

\[ |G(f)| \]

\[ |X(f)| \]
**Fourier Transform**

![Graph showing time signal and Gaussian filter with Fourier Transform and Inverse Fourier Transform](image)

- Time signal: $x(t)$
- Gaussian filter: $g(t)$

$$X(f) \cdot G(f)$$

Frequency response:
- $-f$ to $f$
GAUSSIAN FILTERING ~ LOW-PASS FILTERING
GAUSSIAN FILTERING ~ LOW-PASS FILTERING

$g(\sigma = 2)$
GAUSSIAN FILTERING ~ LOW-PASS FILTERING

$g(\sigma = 8)$
GAUSSIAN FILTERING ~ LOW-PASS FILTERING

\[ g(\sigma = 32) \]
SUBSAMPLING

\[ x(t) \]
Subsampling

\[ x(t) \]

Subsampling → \[ \downarrow x(t) \]
SUBSAMPLING WITH G. FILTERING
SUBSAMPLING WITH G. FILTERING

\[ x(t) \ast g(t) \]
Subsampling with G. Filtering

\[ x(t) \]
Naïve Subsampling

\[ x(t) \]
Nyquist’s theorem

\[ f_{\text{sampling}} \geq 2f_{\text{signal}} \]
Aliasing

Naïve subsampling

Smoothing and subsampling: eliminating aliasing effects.
GAUSSIAN FILTERING ~ LOW-PASS FILTERING ~ IMAGE BLURRING
GAUSSIAN FILTERING AND THEN SUBSAMPLING
IMAGE RECONSTRUCTION: UPSAMPLING AND GAUSSIAN BLURRING
Naïve Image Subsampling and Upsampling
MULTI-DIMENSIONAL IMAGE REPRESENTATION
Gaussian Image Pyramid

Memory consumption

\[ |I| \left(1 + \frac{1}{4} + \frac{1}{16} + \cdots\right) = \frac{4}{3} |I| \]
**Composition of Gaussian Filters**

![Diagram showing the composition of Gaussian filters](image)

- Time signal: $x(t)$
- Gaussian filter: $g(t; \sigma_1) * g(t; \sigma_2)$
- Frequency response: $X(f) * G(f)$

**Equations:**

- Fourier Transform (FT): $X(f) = \mathcal{F}\{x(t)\}$
- Inverse Fourier Transform (Inverse FT): $x(t) = \mathcal{F}^{-1}\{X(f)\}$

**Relationship:**

$X(f) = x(t) * g(t; \sigma_1) * g(t; \sigma_2)$
**Composition of Gaussian Filters**

Time signal \( x(t) \)  
Gaussian filter \( g(t; \sqrt{\sigma_1^2 + \sigma_2^2}) \)  

\[ X(f) G(f) G(f) \]
Composition of Gaussian Filters

Time signal \( x(t) \) \(* g(t, \sqrt{\sigma_1^2 + \sigma_2^2}) \) Gaussian filter

\[ X(f) G(f) G(f) \]

Frequency response

\[ X(f)G(f)G(f) \]
Gaussian Image Pyramid

Memory consumption:
\[ |I| (1 + \frac{1}{4} + \frac{1}{16} + \cdots) = \frac{4}{3} |I| \]
\[
I_{1/2} + I_{1/2} + I_{1/2} = \frac{4}{3} |I|
\]
REDUNDANT REPRESENTATION OF GAUSSIAN PYRAMID
Fourier Transform

\[ X(f) = \mathcal{F}\{x(t)\} \]

\[ G(f; \sigma) = \mathcal{F}\{g(t; \sigma)\} \]

\[ g(t; \sigma) \ast x(t) \rightarrow \mathcal{F} \rightarrow G(f; \sigma_1) \rightarrow X(f) \mathcal{F} \rightarrow G(f; \sigma_2) \rightarrow \mathcal{F}^{-1} \rightarrow g(t; \sigma_2) \]

Scale dependent freq. band

Redundant freq. band
**Difference of Gaussian (DoG) ~ Band-pass Filter**

\[
x(t) \ast g(t) \Rightarrow \mathcal{F}\{x(t)g(t)\} = X(f)G(f)
\]
**Difference of Gaussian (DoG) ~ Band-pass Filter**

\[ x(t) \ast (g(t;\sigma_1) - g(t;\sigma_2)) \]

\[ X(f)(G(f;\sigma_1) - G(f;\sigma_2)) \]
**Difference of Gaussian (DOG) ~ Band-pass Filter**

\[ x(t) * (g(t; \sigma_1) - g(t; \sigma_2)) \]

\[ X(f)(G(f; \sigma_1) - G(f; \sigma_2)) \]
**Difference of Gaussian (DoG) ~ Band-pass Filter**

$$x(t) \ast \left( g(t;\sigma_1) - g(t;\sigma_2) \right) = X(f) \left( G(f;\sigma_1) - G(f;\sigma_2) \right)$$
**Laplacian of Gaussian (LOG) ~ DoG**

$$\mathcal{F}(x(t) * (g(t;\sigma_1) - g(t;\sigma_2))) \approx \nabla \cdot \nabla g$$

Laplacian of Gaussian
Laplacian of Gaussian (LoG) ∼ DoG
**IMAGE LAPLACIAN**

$I$  
$I - I \ast G$  
$I \ast G$
**IMAGE LAPLACIAN**

$I \ast G$

$I \ast G - I \ast G \ast G$

$I \ast G \ast G$
**Difference of Gaussian (DoG) ~ Band-pass Filter**

\[ x(t) \ast (g(t; \sigma_1) - g(t; \sigma_2)) \]

\[ X(f)(G(f; \sigma_1) - G(f; \sigma_2)) \]
Signal Reconstruction \(-\text{LoG} + \text{G. Filtering}\)

\[
l(t) = x(t) * \left( g(t; \sigma_1) - g(t; \sigma_2) \right)
\]

\[
x(t) * g(t; \sigma_1) = l(t) + x(t) * g(t; \sigma_2)
\]

Signal reconstruction with laplacian

\[
L(f) = X(f) \left( G(f; \sigma_1) - G(f; \sigma_2) \right)
\]

\[
X(f)G(f; \sigma_1) = L(f) + \frac{X(f)G(f; \sigma_2)}{\text{LoG}}
\]

Smother signal
Image Reconstruction