RANSAC

Hyun Soo Park
RECALL: LOCAL FEATURE MATCHING
RECALL: ROBUST FILTERING
**Line Fitting**

Given points: \((u_1, v_1), \ldots, (u_n, v_n)\)

Find the best line:

\[
\begin{align*}
v_1 & \approx mu_1 + d \\
\vdots & \vdots \\
v_n & \approx mu_n + d
\end{align*}
\]

\[
\begin{bmatrix}
u_1 \\
\vdots \\
u_n
\end{bmatrix}
\begin{bmatrix}
m \\
d
\end{bmatrix}
= 
\begin{bmatrix}
v_1 \\
\vdots \\
v_n
\end{bmatrix}
\]
**Line Fitting**

Given points: \((u_1, v_1), \ldots, (u_n, v_n)\)

Find the best line: \[ v_1 \approx m u_1 + d \]

\[ \vdots \]

\[ v_n \approx m u_n + d \]

Least squares solution: \[ x = (A^T A)^{-1} A^T b \]
Outlier

Data
Ground truth
Least squares

\[
\begin{bmatrix}
u_1 & 1 \\
u_n & 1 \\
\end{bmatrix}
\begin{bmatrix}
m \\
d \\
\end{bmatrix}
= 
\begin{bmatrix}
v_1 \\
v_n \\
\end{bmatrix}
\]
Outlier

\[
\begin{bmatrix}
  u_1 & 1 \\
  u_n & 1 \\
\end{bmatrix}
\begin{bmatrix}
  m \\
  d \\
\end{bmatrix} =
\begin{bmatrix}
  v_1 \\
  v_n \\
\end{bmatrix}
\]
Outlier Sensitivity

\[ y = mx + d \]

\[ (u, v_{1}) \]

\[ |v_{1} - mu_{1} - d| \]

Quadratic magnification of error of outliers

Outlier

Least squares solution:

\[ x = \left( A^{T}A \right)^{-1} A^{T}b \]
**Outlier Rejection Strategy**

Outlier rejection strategy:
To find the best line that explains the maximum number of points.

**Assumptions:**
1. Majority of good samples agree with the underlying model.
2. Bad samples does not consistently agree with a single model.
RANSAC: RAndom SAmple Consensus
1. Random sampling

RANSAC: RRandom SAmple Consensus
RANSAC: RAndom SAmple Consensus

1. Random sampling
2. Model building
RANSAC: RRandom SAmple Consensus

1. Random sampling
2. Model building
3. Thresholding
RANSAC: RAndom SAmple Consensus

1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

# of inliers: 7
RANSAC: RAdom SAmple Consensus

1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: RAramd SAmple Consensus
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: RAndom SAmple Consensus
RANSAC: RAndom SAmple Consensus

1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

# of inliers: 10
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

RANSAC: RAndom SAMple Consensus
RANSAC: RRandom SAmple Consensus

1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting
RANSAC: RRandom SAmple Consensus

1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting
1. Random sampling
2. Model building
3. Thresholding
4. Inlier counting

# of inliers: 23
Maximum number of inliers

**RANSAC: Random Sample Consensus**
Required number of iterations with $p$ success rate:

$\text{Prob. of success} > \text{Prob. of desired success} \quad p$
Required number of iterations with $p$ success rate:

Prob. of success > Prob. of desired success $P$

Prob. of success: $1 - (1 - \text{prob. of success per trial})^k$
Required number of iterations with $p$ success rate:

\[ \text{Prob. of success} > \text{Prob. of desired success} \quad p \]

\[ \text{Prob. of success: } 1 - (1 - \text{prob. of success per trial})^k \]

\[ \text{Prob. of success per trial: } w^n \]

where \[ w = \frac{\# \text{ of inliers}}{\# \text{ of samples}} \]

and $n$ is the number of samples to build a model.
Required number of iterations with $p$ success rate:

Prob. of success > Prob. of desired success $P$

Prob. of success: $1 - (1 - \text{prob. of success per trial})^k$

Prob. of success per trial: $w^n$

where $w = \frac{\# \text{ of inliers}}{\# \text{ of samples}}$

and $n$ is the number of samples to build a model.

Prob. of success: $1 - (1 - w^n)^k$
Required number of iterations with $p$ success rate:

Prob. of success > Prob. of desired success $P$

Prob. of success: $1- (1-$prob. of success per trial$)^k$

Prob. of success per trial: $w^n$

where $w = \frac{\text{# of inliers}}{\text{# of samples}}$

and $n$ is the number of samples to build a model.

Prob. of success: $1- \left(1- w^n\right)^k$  

$$k = \frac{\log(1- \rho)}{\log\left(1-w^n\right)}$$
Recall: Homography Computation

\[ \begin{bmatrix}
  u_1 & v_1 & 1 & 0 & 0 & 0 & -u_1u'_1 & -v_1u'_1 \\
  0 & 0 & 0 & u_1 & v_1 & 1 & -u_1v'_1 & -v_1v'_1 \\
  u_4 & v_4 & 1 & 0 & 0 & 0 & -u_4u'_4 & -v_4u'_4 \\
  0 & 0 & 0 & u_4 & v_4 & 1 & -u_4v'_4 & -v_4v'_4 \\
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  1 \\
\end{bmatrix} = \begin{bmatrix}
  u'_1 \\
  v'_1 \\
  u'_4 \\
  v'_4 \\
\end{bmatrix} \]

\[ Ax = b \quad \rightarrow \quad x = \left( A^T A \right)^{-1} A^T b \]
Homography from 4 random correspondences
Inlier counting

Number of inliers: 5
Homography from 4 random correspondences
Inlier counting

Number of inliers: 8
Homography from 4 random correspondences
Inlier counting

Number of inliers: 25
Homography from 4 random correspondences
Inlier counting

Number of inliers: 76
Homography from 4 random correspondences
Inlier counting

Number of inliers: 83
RECALL: ROBUST FILTERING
RECALL: PARAMETRIC MODEL
RECALL: IMAGE WARPGING