Scale Space

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Scale Invariant Image Representation
BLOB DETECTION ~ SCALE SELECTION
RECALL: IMAGE DIFFERENTIATION

\[ I \otimes z = \frac{\partial I}{\partial u} \]

\[
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**RECALL: SOBEL FILTER**

\[
\frac{\partial G}{\partial u} = \frac{G(u + h) - G(u - h)}{2h}
\]

Max at edge
**Recall: Sobel Filter**

\[ \frac{\partial G}{\partial u} = \frac{G(u+h) - G(u-h)}{2h} \]

Max at edge
RECALL: IMAGE SHARPENING

Zero crossing at the edge
Laplacian

\[
\frac{G(u + h) + G(u - h) - 2G(u)}{h}
\]
LAPLACIAN

\[
G(u + h) - G(u) - G(u) + G(u - h) = \frac{G(u + h) - G(u)}{h} - \frac{G(u) - G(u - h)}{h}
\]
Laplacian

\[ \lim_{h \to 0} \left( \frac{G(u+h) - G(u)}{h} - \frac{G(u) - G(u-h)}{h} \right) / h \]
Laplacian

\[ \nabla \cdot \nabla G = \nabla \left( \frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2} \]

Second order derivative of Gaussian, a.k.a., Laplacian of Gaussian
The **Laplacian** is defined as:

\[ \nabla \cdot \nabla G = \nabla \left( \frac{\partial G}{\partial u} \mathbf{i} + \frac{\partial G}{\partial v} \mathbf{j} \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2} \]

This is equivalent to the convolution of the Gaussian function with a specific 3x3 kernel:

\[ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \]

Second order derivative of Gaussian, a.k.a., **Laplacian of Gaussian**
Laplacian

\[ \nabla \cdot \nabla G = \nabla \left( \frac{\partial G}{\partial u} i + \frac{\partial G}{\partial v} j \right) = \frac{\partial^2 G}{\partial u^2} + \frac{\partial^2 G}{\partial v^2} \]

Edge

* 

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix}
\]

= 

Zero crossing at edge
Edge Localization

Zero crossing at edge
STRUCTURED EDGES (e.g., BLOB)

Zero crossing edges
Structured Edges (e.g., Blob)
**Structured Edges (e.g., Blob)**

- Convolution of a step function with a blob kernel results in zero crossing edges at the maxima of the blob.
Structured Edges (e.g., Blob)

Max at center of blob

Zero crossing edges
Structured Edges (e.g., Blob)
STRUCTURED EDGES (E.G., BLOB)
STRUCTURED EDGES (E.G., BLOB)

$\sigma = 4$
Structured Edges (e.g., Blob)
Structured Edges (e.g., Blob)

\[ \sigma = 10 \]
**Structured Edges (e.g., Blob)**
Structured Edges (e.g., Blob)

Characteristic scale

LoG response vs Scale (σ)
Structured Edges (e.g., Blob)

x3 bigger sunflower

Characteristic scale

Log response

Scale (σ)
**SCALE SELECTION**

- The scale response (laplacian) is maximized when \( \sigma = \frac{r}{\sqrt{2}} \)

- The characteristic scale can be used to normalize the image, resulting in *scale invariant* image description.
Scale-normalized Laplacian response

Slide credit: Svetlana Lazebnik
Scale Normalization
Scale Normalization