2D Correspondence

Left image (Bob)  Right image (Alice)
2D Correspondence

Left image (Bob)  Right image (Alice)
**Epipolar constraint** between two images:

1. A point, $u$, in Bob’s image corresponds to an epipolar line $l_u$ in Alice’s image.
Epipolar constraint between two images:

1. A point, \( \mathbf{u} \), in Bob’s image corresponds to an epipolar line \( \mathbf{l_u} \) in Alice’s image.
2. The epipolar line passes the corresponding point in Alice’s image, \( \mathbf{v} : \mathbf{v} \mathbf{l_u} = 0 \)
Epipolar constraint between two images:

1. A point, \( u \), in Bob’s image corresponds to an epipolar line \( I_u \) in Alice’s image.
2. The epipolar line passes the corresponding point in Alice’s image, \( v \): \( v^\top I_u = 0 \)
3. Any point along the epipolar line can be a candidate of correspondences.
1. Bob’s view corresponds to an epipolar line $l_u$.
2. The epipolar line passes the corresponding point in Alice’s image, $v$.
3. Any point along the epipolar line can be a candidate of correspondences.
Epipolar constraint between two images:

1. A point, $u$, in Bob’s image corresponds to an epipolar line $l_u$ in Alice’s image.
2. The epipolar line passes the corresponding point in Alice’s image, $v$: $v^\top l_u = 0$
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Epipolar constraint between two images:

1. A point, $\mathbf{u}$, in Bob’s image corresponds to an epipolar line $\mathbf{l}_u$ in Alice’s image.
2. The epipolar line passes the corresponding point in Alice’s image, $\mathbf{v}$: $\mathbf{v}^T\mathbf{l}_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
**Epipolar constraint** between two images:

1. A point, $u$, in Bob’s image corresponds to an epipolar line $l_u$ in Alice’s image.
2. The epipolar line passes the corresponding point in Alice’s image, $v$: $v^Tl_u = 0$
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epipolar lines meet at the epipole: $e_{bob}^Tl = 0$, $e_{alice}^Tl = 0$
Epipolar lines

1. Epipolar line in Alice’s image.
2. The epipolar line passes the corresponding point in Alice’s image, $v$.
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epipolar lines meet at the epipole.
Epipolar Line Properties:

1. Each epipolar line in Alice’s image corresponds to an epipolar line in Bob’s image.
2. The epipolar line passes through the corresponding point in Alice’s image, $v$.
3. Any point along the epipolar line can be a candidate of correspondences.
4. Epipolar lines meet at the epipole.
\[ I_v = F_v \]

Fundamental matrix
**EPIPOLAR LINE**

\[ \mathbf{I}_v = \mathbf{Fv} \]

Fundamental matrix

\[ \mathbf{u}^T \mathbf{Fv} = 0 \]

where \( \mathbf{F} = \mathbf{K}^{-T} \begin{bmatrix} \mathbf{t} \end{bmatrix} \mathbf{RK}^{-1} \)

Fundamental matrix
**Fundamental Matrix**

Bob's image

\[ v^T I_u = v^T K^{-T} \begin{bmatrix} t \end{bmatrix} R K^{-1} u = 0 \]

Common for all points

\[ = v^T F u = 0 \]

\[ = v^T (F u) = u^T (F^T v) = 0 \]

Alice's image
**Fundamental Matrix**

Bob's image

\[ v^T I_u = v^T K^{-T} \begin{bmatrix} t \end{bmatrix} R K^{-1} u = 0 \]

Common for all points

\[ = v^T F u = 0 \]

\[ = v^T (F u) = u^T \left( F^T v \right) = 0 \]

Alice's image
**Fundamental Matrix**

Bob: \[ P_{\text{bob}} = K \begin{bmatrix} I_3 & 0_{3 \times 1} \end{bmatrix} \]

Alice: \[ P_{\text{alice}} = K \begin{bmatrix} R & t \end{bmatrix} \]

\[ v^T I_u = v^T K^{-T} \begin{bmatrix} t \end{bmatrix} R K^{-1} u = 0 \]

Common for all points:

\[ = v^T F u = 0 \]

\[ = v^T (F u) = u^T (F^T v) = 0 \]
**Fundamental Matrix**

Bob's image

\[ v^T I_u = v^T K^{-T} [t] R K^{-1} u = 0 \]

Common for all points

\[ = v^T F u = 0 \]

Alice's image

\[ = v^T (Fu) = u^T (F^T v) = 0 \]
**Fundamental Matrix**

Bob's image

Properties of Fundamental Matrix

- Transpose: if $F$ is for $P_{bob}$, $P_{alice}$, then $F^T$ is for $P_{alice}$, $P_{bob}$.

Bob $P_{bob} = K [I_3 \ 0_{3 \times 1}]$

Alice $P_{alice} = K [R \ t]$
Properties of Fundamental Matrix

- Transpose: if $F$ is for $P_{\text{bob}}$, $P_{\text{alice}}$, then $F^T$ is for $P_{\text{alice}}$, $P_{\text{bob}}$.
  \[ I_u = Fu \quad I_v = F^T v \]

- Epipolar line:
**FUNDAMENTAL MATRIX**

Bob's image

Properties of Fundamental Matrix

- Transpose: if $F$ is for $P_{\text{bob}}$, $P_{\text{alice}}$, then $F^T$ is for $P_{\text{alice}}$, $P_{\text{bob}}$. 

  $$l_u^T = F l_u \quad l_v^T = F^T l_v$$

- Epipolar line:

  $$F_{\text{bob}} e_{\text{bob}} = 0 \quad F^T_{\text{alice}} e_{\text{alice}} = 0$$

- Epipole:

  $$v_i^T F e_{\text{bob}} = 0, \quad u_i^T F^T e_{\text{alice}} = 0, \quad \forall i$$

  $$\rightarrow e_{\text{bob}} = \text{null}(F), \quad e_{\text{alice}} = \text{null}(F^T)$$
**Fundamental Matrix**

**Bob’s image**

**Properties of Fundamental Matrix**

- Transpose: if \( F \) is for \( P_{bob} \), \( P_{alice} \), then \( F^T \) is for \( P_{alice} \), \( P_{bob} \).
  
- Epipolar line:
  
  \[
  l_u = F u \quad l_v = F^T v
  \]

- Epipole:
  
  \[
  F e_{bob} = 0 \quad F^T e_{alice} = 0
  \]

- \( \text{rank}(F) = 2 \):
  
  DoF 9 (3x3 matrix) - 1 (scale) - 1 (rank) = 7

**Alice’s image**
CAMERA MOTION
CAMERA MOTION
2D CORRESPONDENCE

Bob’s image

Alice’s image
**2D Correspondence**

\[ \mathbf{v}^T \mathbf{F} \mathbf{u} = 0 \]

Bob’s image  \hspace{1cm} Alice’s image
2D CORRESPONDENCE

\[ F = F(R, t) \]
\[ = K^{-T} \begin{bmatrix} t \\ x \end{bmatrix} RK^{-1} \]

Bob’s image

Alice’s image

\[ v^T Fu = 0 \]

How to compute fundamental matrix?
A computer algorithm for reconstructing a scene from two projections

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A simple algorithm for computing the three-dimensional structure of a scene from a correlated pair of perspective projections is described here, when the spatial relationship between the projections is unknown. This problem is relevant not only to photographic surveying but also to binocular vision, where the non-visual information available to the observer about the scene is only limited by the size of his visual angle.
**FUNDAMENTAL MATRIX ESTIMATION**

\[ v^T F u = 0 \]
**Fundamental Matrix Estimation**

\[ F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \]

Degree of freedom of fundamental matrix:

\[ v^T Fu = 0 \]
**Fundamental Matrix Estimation**

\[
F = \begin{bmatrix}
  f_{11} & f_{12} & f_{13} \\
  f_{21} & f_{22} & f_{23} \\
  f_{31} & f_{32} & f_{33}
\end{bmatrix}
\]

Degree of freedom of fundamental matrix:

\[7 = 9 \text{ (3x3 matrix)} - 1 \text{(scale)} - 1 \text{ (rank 2)}\]

We will estimate fundamental matrix with 8 parameter by ignoring rank constraint and then project onto rank 2 matrix:
**Fundamental Matrix Estimation**

\[ v^T F u = \begin{bmatrix} u^x & u^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix} \]
**FUNDAMENTAL MATRIX ESTIMATION**

\[ v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix} \]

\[ = f_{11} v^x u^x + f_{12} v^y u^x + f_{13} v^x u^y + f_{21} v^x u^y + f_{22} v^y u^y + f_{23} v^y u^y + f_{31} u^x + f_{32} u^y + f_{33} \]

\[ v^T F u = 0 \]
**Fundamental Matrix Estimation**

\[ \mathbf{v}^T \mathbf{u} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ v^x \\ 1 \end{bmatrix} = 0 \]

Linear in \( \mathbf{F} \)

Bob  \( \mathbf{v}^T \mathbf{u} = 0 \)  Alice
**Fundamental Matrix Estimation**

\[
v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix} = \frac{f_{11} u^x v^x + f_{12} u^x v^y + f_{13} u^y v^x + f_{21} u^x v^y + f_{22} u^y v^y + f_{23} v^x + f_{31} u^x + f_{32} u^y + f_{33}}{0}
\]

Linear in \( F \).

\[
\rightarrow \begin{bmatrix} u^x v^x & u^x v^y & v^x & u^y v^x & v^y & u^y & u^y & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0
\]

# of unknowns: 9
# of equations per correspondence: 1
Fundamental Matrix Estimation

\[ v^T F u = \begin{bmatrix} v^x & v^y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u^x \\ u^y \\ 1 \end{bmatrix} = f_{11} u^x v^x + f_{12} u^x v^y + f_{13} u^x + f_{21} u^y v^x + f_{22} u^y v^y + f_{23} u^y + f_{31} v^x + f_{32} v^y + f_{33} \]

\[ = 0 \] Linear in \( F \).

What is minimum \( m \)?
**Fundamental Matrix Estimation**

\[
v^T F u = \begin{bmatrix} u_x & u_y & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_x^T \\ u_y^T \\ 1 \end{bmatrix} = 0
\]

Linear in \( F \).

\[
= f_{11} u_x^2 + f_{12} u_x u_y + f_{13} u_y^2 + f_{21} u_x u_x^T + f_{22} u_x u_y^T + f_{23} u_y u_y^T + f_{31} u_x u_x^T + f_{32} u_x u_y^T + f_{33} u_y u_y^T
\]

What is minimum \( m \)?
The solution is not necessarily satisfy rank 2 conditions.
**Fundamental Matrix Estimation**

The solution is not necessarily satisfy rank 2 constraint.

\[
\begin{bmatrix}
 f_{11} & f_{12} & f_{13} \\
 f_{21} & f_{22} & f_{23} \\
 f_{31} & f_{32} & f_{33}
\end{bmatrix} = \begin{bmatrix} U & D & V^T \end{bmatrix}
\]
**Fundamental Matrix Estimation**

The solution is not necessarily satisfy rank 2 condition.

\[
\begin{bmatrix}
    f_{11} & f_{12} & f_{13} \\
    f_{21} & f_{22} & f_{23} \\
    f_{31} & f_{32} & f_{33}
\end{bmatrix}
= 
\begin{bmatrix}
    U & D \\
    U & \tilde{D}
\end{bmatrix}
\]

\[
\approx F_{\text{rank2}} = 
\begin{bmatrix}
    U & \tilde{D} \\
    U & \tilde{D}
\end{bmatrix}
\]

SVD cleanup