PROJECTIVE LINE

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Parallel lines in 3D converge to a point in the image.
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

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Ground plane
3D Parallel Line Projection

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Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D PARALLEL LINE PROJECTION

Camera plane

Vanishing point

Ground plane
3D Parallel Line Projection

Camera plane

Vanishing point

Ground plane
3D PARALLEL LINE PROJECTION

1. Parallel lines in 3D meet at the same vanishing point in image.
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
**Vanishing Point**

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
3. Multiple vanishing points exist.
Vanishing point

Multiple vanishing point
Vanishing point

Vanishing line: Horizon
Parallel 3D planes share the vanishing line.
Different plane produces different vanishing line.
Different plane produces different vanishing line.
How to compute a vanishing point?

Different plane produces different vanishing line.
A 2D line passing through 2D point \((u, v)\): 

\[ au + bv + c = 0 \]

Line parameter: \((a, b, c)\)
A 2D line passing through 2D point \((u,v)\):
\[ au + bv + c = 0 \]

Line parameter: \((a,b,c)\)

\[ au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \\ u & v & 1 \end{bmatrix} \mathbf{x} = 0 \]
Point-Line

A 2D line passing through 2D point \((u,v)\):
\[ au + bv + c = 0 \]

Line parameter: \((a,b,c)\)
\[ au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = l^T \mathbf{x} = 0 \]

where \(\mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}\) and \(l = \begin{bmatrix} a \\ b \\ c \end{bmatrix}\)

2D point  Line parameter
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad \text{and} \quad au_2 + bv_2 + c = 0 \]
**Point-Line**

A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]

\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where

\[ x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}, \quad l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]
A 2D line passing through two 2D points:
\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]
\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where \( x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \)
\( x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \)
\( l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \)

\[
\begin{bmatrix}
    x_1^T \\
    x_2^T
\end{bmatrix}
\begin{bmatrix}
    l
\end{bmatrix} = 0
\]

\[
\begin{bmatrix}
    A \\
    2x3
\end{bmatrix}
\begin{bmatrix}
    \begin{bmatrix}
        0 \\
        0
    \end{bmatrix}
\end{bmatrix} = 0
\]
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]

\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where \( x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \), \( x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \), and \( l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \)

\[ \begin{bmatrix} x_1^T \\ x_2^T \end{bmatrix} l = 0 \]

\[ \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \text{null} \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) \]

or \( l = x_1 \times x_2 \)
Two 2D lines in an image intersect at a 2D point:

\[ a_1 u + b_1 v + c_1 = 0 \quad a_2 u + b_2 v + c_2 = 0 \]
Two 2D lines in an image intersect at a 2D point:

\[ a_1u + b_1v + c_1 = 0 \quad a_2u + b_2v + c_2 = 0 \]

\[ \mathbf{l}_1^\top \mathbf{x} = 0 \quad \mathbf{l}_2^\top \mathbf{x} = 0 \]

where \( \mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \) \( \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \) \( \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \)
Two 2D lines in an image intersect at a 2D point:

\[ a_1 u + b_1 v + c_1 = 0 \quad a_2 u + b_2 v + c_2 = 0 \]

\[ l_1^T x = 0 \quad l_2^T x = 0 \]

where

\[ x = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad l_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad l_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \]

\[ \begin{bmatrix} l_1^T \\ l_2^T \end{bmatrix} x = 0 \]

\[ \begin{bmatrix} A \\ 0 \\ 0 \end{bmatrix} = 0 \Rightarrow \text{null}(A) \]

or

\[ x = l_1 \times l_2 \]
Parallel lines:
\[ l_{11} = u_4 \times u_3 \quad l_{12} = u_1 \times u_2 \]
Parallel lines:
\[ l_{11} = u_4 \times u_3 \]
\[ l_{12} = u_1 \times u_2 \]
\[ l_{21} = u_4 \times u_1 \]
\[ l_{22} = u_3 \times u_4 \]
**Vanishing Point**

Parallel lines:

\[ l_{11} = u_4 \times u_3 \quad l_{12} = u_1 \times u_2 \]

\[ l_{21} = u_4 \times u_1 \quad l_{22} = u_3 \times u_4 \]

Vanishing points:

\[ x_1 = l_{11} \times l_{12} \quad x_2 = l_{21} \times l_{22} \]
Vanishing Point

Parallel lines:
\[ l_{11} = u_4 \times u_3 \quad l_{12} = u_1 \times u_2 \]
\[ l_{21} = u_4 \times u_1 \quad l_{22} = u_3 \times u_4 \]

Vanishing points:
\[ v_1 = l_{11} \times l_{12} \quad v_2 = l_{21} \times l_{22} \]

Vanishing line:
\[ l = v_1 \times v_2 \]
GEOMETRIC INTERPRETATION OF VANISHING LINE
WHERE WAS I?
WHERE WAS I?

Taken from my hotel room (6th floor)  

Taken from beach