MLPS-St. Paul International Airport

Indoor point at infinity

Parallel lines in 3D converge to a point in the image.
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Vanishing point

Ground plane
3D Parallel Line Projection

Camera plane

Vanishing point

Ground plane
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.
3D Parallel Line Projection

Camera plane

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.

Vanishing point

Ground plane
**Vanishing Point**

1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
3. Multiple vanishing points exist.
Vanishing point

Multiple vanishing point
Vanishing point

Vanishing line for horizon

Vanishing point

Vanishing line: Horizon
What can vanishing line tell us about me?
What can vanishing line tell us about me?

- Horizon
What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)
Parallel 3D planes share the vanishing line.
Different plane produces different vanishing line.
Different plane produces different vanishing line.
How to compute a vanishing point?

Different plane produces different vanishing line.
A 2D line passing through 2D point \((u,v)\): 
\[ au + bv + c = 0 \]

Line parameter: \((a,b,c)\)
A 2D line passing through 2D point \((u,v)\):
\[ au + bv + c = 0 \]

Line parameter: \((a,b,c)\)

\[ au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = I^T x = 0 \]
A 2D line passing through 2D point \((u, v)\):

\[ au + bv + c = 0 \]

Line parameter: \((a, b, c)\)

\[ au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \\ u & v & 1 \end{bmatrix} = l^T x = 0 \]

where \(x = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \) and \(l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \)

2D point \quad Line parameter
A 2D line passing through two 2D points:
\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]

where

\[
\begin{bmatrix}
u_1 \\ v_1 \\ 1
\end{bmatrix}
\begin{bmatrix}
u_2 \\ v_2 \\ 1
\end{bmatrix}
\begin{bmatrix}
a \\ b \\ c
\end{bmatrix}
\]

\[ x_1^T l = 0 \quad x_2^T l = 0 \]
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad \text{and} \quad au_2 + bv_2 + c = 0 \]

\[ \mathbf{x}_1^T \mathbf{l} = 0 \quad \text{and} \quad \mathbf{x}_2^T \mathbf{l} = 0 \]

where

\[ \mathbf{x}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \quad \mathbf{x}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

\[ \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \end{bmatrix} \mathbf{l} = \mathbf{0} \]

\[ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 2 \times 3 \]
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]

\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where

\[ x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix}, \quad l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} x_1^T \\ x_2^T \end{bmatrix} l = 0 \]

\[ \text{null} \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) \]

or

\[ l = x_1 \times x_2 \]
Two 2D lines in an image intersect at a 2D point:

\[ a_1 u + b_1 v + c_1 = 0 \quad a_2 u + b_2 v + c_2 = 0 \]
Two 2D lines in an image intersect at a 2D point:

\[ a_1 u + b_1 \nu + c_1 = 0 \quad a_2 u + b_2 \nu + c_2 = 0 \]

\[ \mathbf{l}_1^T \mathbf{x} = 0 \quad \mathbf{l}_2^T \mathbf{x} = 0 \]

where \( \mathbf{x} = \begin{bmatrix} u \\ \nu \\ 1 \end{bmatrix} \) \( \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \) \( \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \)
Two 2D lines in an image intersect at a 2D point:

\[
a_1 u + b_1 v + c_1 = 0 \quad a_2 u + b_2 v + c_2 = 0
\]

\[
\mathbf{l}_1^T \mathbf{x} = 0 \quad \mathbf{l}_2^T \mathbf{x} = 0
\]

where \( \mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \), \( \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \), \( \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \)

\[
\begin{bmatrix}
\mathbf{l}_1^T \\
\mathbf{l}_2^T
\end{bmatrix} \mathbf{x} = 0
\]

\[
\text{null}\left( \mathbf{A} \right)
\]

\[
0 = \mathbf{l}_1 \times \mathbf{l}_2
\]
**Vanishing Point**

Parallel lines:

\[ l_{11} = u_4 \times u_3 \]

\[ l_{12} = u_1 \times u_2 \]
Vanishing Point

Parallel lines:
\[
\begin{align*}
\mathbf{l}_{11} &= \mathbf{u}_4 \times \mathbf{u}_3 \\
\mathbf{l}_{12} &= \mathbf{u}_1 \times \mathbf{u}_2 \\
\mathbf{l}_{21} &= \mathbf{u}_4 \times \mathbf{u}_1 \\
\mathbf{l}_{22} &= \mathbf{u}_3 \times \mathbf{u}_4
\end{align*}
\]
**Vanishing Point**

Parallel lines:
\[ l_{11} = u_4 \times u_3 \]
\[ l_{12} = u_1 \times u_2 \]
\[ l_{21} = u_4 \times u_1 \]
\[ l_{22} = u_3 \times u_4 \]

Vanishing points:
\[ x_1 = l_{11} \times l_{12} \]
\[ x_2 = l_{21} \times l_{22} \]
**Vanishing Point**

Parallel lines:
\[ \mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \quad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2 \]
\[ \mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \quad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4 \]

Vanishing points:
\[ \mathbf{v}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \quad \mathbf{v}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22} \]

Vanishing line:
\[ \mathbf{l} = \mathbf{v}_1 \times \mathbf{v}_2 \]
GEOMETRIC INTERPRETATION OF VANISHING LINE

[Diagram showing geometric interpretation]

- Height
- Plane of vanishing line
- Ground plane
- Side view
WHERE WAS I?
WHERE WAS I?

Taken from my hotel room (6th floor)  
Taken from beach