

Breaking and Provably Fixing Minx

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Abstract. In 2004, Danezis and Laurie proposed *Minx*, an encryption protocol and packet format for relay-based anonymity schemes, such as mix networks and onion routing, with simplicity as a primary design goal. Danezis and Laurie argued informally about the security properties of Minx but left open the problem of proving its security. In this paper, we show that there cannot be such a proof by showing that an active global adversary can decrypt Minx messages in polynomial time. To mitigate this attack, we also prove secure a very simple modification of the Minx protocol.

1 Introduction

In many situations, the ability to communicate anonymously is desirable. Privacy is a valued commodity among internet users, and several cryptographic protocols rely on the existence of anonymous channels. One proposed method of implementing anonymous channels is mix networks. First proposed by Chaum [4], a mix network works by routing messages through a series of mixes. Each mix in the network performs a cryptographic transformation on a received message before resending it, thus making the tracking of messages from mix to mix and sender to receiver very difficult. This process, which is sometimes referred to as onion routing, focuses on routing messages encrypted in a concentric, or layered, fashion, known as *onions* [9]. Each layer of an onion contains routing information for one node, with the goal being that each node in the network can only decrypt enough information to send the encrypted message to the next node in the path [3,5].

Several works have proposed encryption schemes for use in this setting; one scheme of interest is Mixminion, proposed as a successor to the popular Mixmaster scheme [7]. Since the design of Mixminion, several authors [13,3] have proposed schemes with some form of provable security guarantees, although these schemes do not allow for the anonymous replies supported by Mixminion. Additionally, several schemes based on universal reencryption [11] have been proposed; Danezis [6] has shown that several such schemes are not secure when applied to a mixnet.

This paper is concerned primarily with Minx, a packet format and encryption scheme proposed by Danezis and Laurie in 2004 [5]. Minx was designed to provide the same security properties provided by Mixminion but using simplicity

as a key design goal. The authors provide an informal argument for the security of Minx, but leave its formal proof of security as an open question, to be addressed in the future literature.

We resolve this question negatively, by showing that a theoretical algorithm developed by Håstad and Näslund [12] can be used to exploit a subtle flaw in the Minx design and allow a global active adversary to decrypt messages encrypted under Minx in polynomial time. However, we also present a simple modification to Minx that prevents this attack while preserving its anonymous reply functionality, and prove its security under a security definition derived from the notion proposed by Camenisch and Lysyanskaya [3].

The remainder of this paper is organized as follows: Section 2 gives an overview of the Minx protocol; Section 3 presents our attack; Section 4 proposes a simple modification to mitigate the attack; Section 5 introduces a formal security definition for Minx; using this definition, Section 6 proves the security of our modification; and finally Section 7 provides concluding remarks.

2 Minx

The design of Minx is motivated by the desire to provide simpler operation and lower overhead than Mixminion without sacrificing security. Minx’s packet format and associated mix server operation is quite simple, as it removes aspects of Mixminion deemed too complex by Minx’s authors. These removed aspects include integrity checks for sent messages and a “swap step” designed to thwart attacks based upon traffic analysis [7,5].

Instead of attempting to detect tagged messages via integrity checks, Minx nodes process and forward all packets they receive. This prevents an attacker from tagging the end of a message in the hopes of noticing a dropped packet when the modification to the packet format is discovered. Furthermore Minx uses error propagating block ciphers so that tagging packets causes unpredictable changes in routing behavior and destroys the message payload[5].

2.1 Encryption and Decryption of Minx Packets

Minx employs three cryptographic primitives to create packets containing messages of fixed length. These are: RSA encryption [14], a symmetric “error propagating” encryption scheme EP , and a symmetric “bidirectional error propagating” encryption scheme $biEP$. The key properties of these schemes is that changing bit i of the ciphertext causes pseudorandom plaintext for bits $j > i$ when decrypting under EP and for all bits when decrypting under $biEP$. Danezis and Laurie suggest using AES in Infinite Garble Extension mode [8] for EP , and setting $biEP(x) = EP(reverse(EP(x)))$. We will let ℓ denote the length of RSA public keys, and κ the length of symmetric keys used in Minx; Danezis and Laurie recommend using $\ell = 1024$ and $\kappa = 80$.

The Minx packet format implements a simple layered encryption scheme. The layer intended for a node N contains three components: a session key k , a field

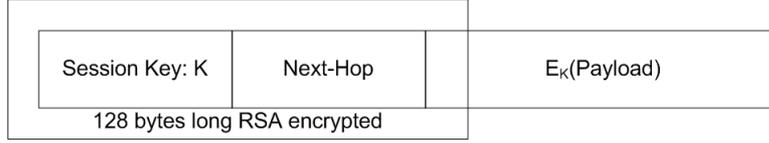


Fig. 1. Minx Packet Format

indicating the packet's next hop, and a payload (the next inner layer) encrypted using EP and session key k . Additionally, the first $\ell/8$ bytes—including the session key, next-hop field, and a portion of the encrypted payload—are RSA encrypted using node N 's public key. This is diagrammed in Figure 1. The innermost layer is encoded slightly differently; the next-hop field is set to a special *final* value to indicate that node N is the intended destination, and the payload (the actual message) is encrypted with the $biEP$ block cipher instead of the EP block cipher.

A Minx packet is created as follows. Suppose sender S wishes to send message M anonymously through $n - 1$ hops to a receiver at mix node N_n . S chooses $n - 1$ intermediate mix nodes $N_1 \dots N_{n-1}$ in the Minx network and n random session keys, $k_1 \dots k_n$. Each mix node N_i has an RSA public key associated with it. Let encryption of a data block D using public key of router N_i be represented as $RSA_{N_i}(D)$, and the encryption of a data block D with EP or $biEP$ keyed with k_i be represented as $EP_{k_i}(D)$ and $biEP_{k_i}(D)$ respectively. Similarly, let $RSA_{N_i}^{-1}(D)$, $EP_{k_i}^{-1}(D)$, and $biEP_{k_i}^{-1}(D)$ be the respective decryption operations. Let $|$ represent concatenation of bit strings, and let $M[i, j]$ denote the byte range from byte i to byte j (inclusive) of bit string M . Finally, let $J(l)$ represent a random string of bits of length l . Figure 2 shows the procedure for a sender to encode a Minx packet as well as the procedure for a Minx node to decode and process a Minx packet. Note that P_i and C_i represent the packet intended for node N_i before and after the header is encrypted with node N_i 's public key. The sender S thus sends the resulting packet C_1 to node N_1 .

When a Minx node receives a packet C_j the decoding process is quite simple. It decodes the first $\ell/8$ bytes of C_j and extracts the session key k_j , the next hop field, and the encrypted payload. To prevent replay attacks the node maintains a table of observed session key hashes, and drops the offending packets. If the next-hop field contains the special *final* value, the packet has reached its final destination so the node decrypts the payload (with $biEP^{-1}$) and processes the enclosed message accordingly. Otherwise the field indicates the next-hop destination, so it decrypts the payload (with EP^{-1}), pads the resulting packet up to the proper size, and forwards it on to its next stop.

2.2 Reply Packets

A packet sender S can choose to include information allowing receiver R to reply without revealing S 's identity. A sender wishing to allow replies creates a special minx packet called a reply block rb_S to allow receiver R to send a message to sender S without knowing the identity of S . The reply block rb_S and the first

Minx Packet Encoding	Minx Packet Decoding
INPUTS: message M node IDs $N_1 \dots N_n$ session keys $k_1 \dots k_n$ PROCEDURE: $P_n = k_n \mid final \mid biEP_{k_n}(M)$ For i from $n - 1$ to 1 : $P_i = k_i \mid N_{i+1} \mid EP_{k_i}(C_{i+1})$ $C_i = RSA_{N_i}(P_i[0, \frac{\ell}{8} - 1]) \mid P_i[\frac{\ell}{8}, -]$ Pad C_1 up to a set size: $C_1 = C_1 \mid J_l$ Return C_1	INPUTS: packet C_j node ID N_j PROCEDURE: $k_j \mid N_{j+1} \mid encC =$ $RSA_{N_j}^{-1}(C_j[0, \frac{\ell}{8} - 1]) \mid C_j[\frac{\ell}{8}, -]$ Check and store $H_{id}(k_j)$. if N_{j+1} is not <i>final</i> : $C_{j+1} = EP_{k_j}^{-1}(encC)$ Send padded message $C_{j+1} \mid J_l$ to N_{j+1} else: Process message $biEP_{k_j}^{-1}(encC[0, (l - 1)])$

Fig. 2. On the left is the procedure for encode a message M in a Minx packet that will travel the path $N_1 \dots N_n$. On the right is the procedure for a Minx node N_j to decode and process an incoming packet.

node in the path specified by rb_S are included in the anonymous message sent to the receiver. The receiver can create a reply by encrypting a message M' with a globally fixed key λ and prefixing the encrypted message with the packet rb_S . The ciphertext $rb_S \mid biEP_\lambda(M') \mid J_l$ is then sent to the specified first node.

As the reply block routes the packet through the network back to S , the appended message M' gains layers of encryption. In order for S to recover this appended message, she includes some extra information – the path and session keys – inside of the original reply block rb_S .

2.3 Minx's Claimed Security Properties

The stated goal of Minx is to provide the same security properties as Mixminion. These security goals are:

- Anonymity given the presence of a global passive adversary which controls all but one node on the message path and can perform active attacks against honest mix servers.
- The ability to generate secure anonymous replies.
- Mix servers, given a message, cannot determine either the total length the message will be routed or their position along the message's path.
- Tagging attacks are totally ineffective. Tagging attacks are defined as attacks which modify a correct, encrypted message in an attempt to recognize the result of the modifications at a later point during routing.

Of particular note is the claim that tagging attacks are ineffective. This claim is based on the assumption that the cryptographic transformations above, when used to decrypt tagged ciphertext, decrypt to something unpredictable and thus do not allow useful information to be gained. In the next section, we show how an active attack that carefully submits many modified messages can exploit Minx's use of “vanilla RSA” to recover plaintexts.

3 Attack on Minx

As mentioned previously, Minx does not meet its stated security goals. Specifically, it is vulnerable to a chosen ciphertext attack that allows an active adversary to successively unwrap the layers of encryption from a packet and eventually extract the enclosed message using a bit oracle constructed from the next-hop portion of the packet header.

Our attack relies on recent theoretical work by Håstad and Nåsland [12]. The main theorem from this work is that all individual plaintext bits of an RSA ciphertext are hard core bits: unless RSA can be inverted in probabilistic polynomial time, no single bit of the plaintext can be predicted in polynomial time with non-negligible advantage.

The theorem is proven in the contrapositive, by showing that an adversary can decrypt an RSA ciphertext if they have the ability to predict a single bit of an arbitrary RSA plaintext when given the corresponding RSA ciphertext. More formally, consider an oracle \mathcal{O}_i , which when given RSA ciphertext $E(x)$ outputs x_i , the i^{th} bit of x , with probability $\frac{1}{2} + \frac{1}{p(\ell)}$, for some polynomial p . Håstad and Nåsland describe a probabilistic polynomial time algorithm that uses this oracle \mathcal{O} to decrypt an RSA ciphertext. The description of the algorithm is outside the scope of this paper, but we note that it requires only a polynomial number of queries to \mathcal{O} on randomly sampled ciphertexts, and runs in expected time $O(\ell^{13})$ where ℓ is the bit-length of the RSA modulus.

3.1 Constructing the Bit Oracle

Generally, obtaining an RSA bit oracle is difficult. However, the current design of Minx allows an adversary to construct such an oracle. As described in Section 2.1, when a Minx node receives a packet, it decrypts the first $\frac{\ell}{8}$ bytes using its RSA private key and then examines the session key and next-hop fields. If the next-hop field specifies another node ID (that is, it doesn't contain the special value *final*), the Minx node uses the session key to decrypt the rest of the packet and forwards it to the specified next-hop destination.

Consider a Minx node that performs no mixing, so that packets are output sequentially in the order they are received. An adversary observing the node's traffic can watch a packet P_j enter and watch the processed packet P_{j+1} leave the node. Since the adversary can observe the destination of P_{j+1} , she knows the corresponding bits of the next-hop field that were in the RSA encrypted header of P_j . This gives a simple construction of a predictor for any bit in the next-hop field of the header. Specifically, given an RSA ciphertext C the adversary implements the oracle \mathcal{O} as follows:

1. Create a packet P with C as the first $\frac{\ell}{8}$ bytes and arbitrary bits for the remainder of the packet (so C is the encrypted header).
2. Send the packet P through the target Minx node.
3. Observe the outgoing packet and record its next-hop destination.
4. Look up the value that corresponds to the next-hop destination.
5. Return the desired bit of the next-hop field.

It is easy to see that with no mixing, the oracle’s bit predictions have 100% accuracy. Since the security of Minx is intended to hold regardless of mixing strategy, this will already be sufficient to prove that no asymptotic security proof is possible.

However, even if a Minx node uses a mixing strategy it is still possible to construct a bit oracle, with slightly reduced accuracy. Suppose that for a given mixing strategy, the adversary can determine for any input packet P , a set S of k packets such S contains the decryption of P with probability at least $1 - \rho$. To predict a next-hop bit in this case, the adversary can submit a packet P , and observe the set S of k packets and their next-hop destinations. By uniformly picking a packet from S and predicting the appropriate bit of its next-hop field, the adversary can predict the desired bit with probability at least $\frac{1}{2} + \frac{1}{2k}(1 - \rho)$. Thus this implementation of the bit oracle meets the theorem’s requirements.

3.2 Attack Walkthrough

To follow the full attack, consider a target Minx packet P_1 heading towards next-hop Minx node N_1 . The adversary is interested in decoding this packet to determine the enclosed message and its intended destination. Recall that the first $\frac{\ell}{8}$ bytes of the packet P_1 consist of the header encrypted with N_1 ’s RSA public key. The adversary proceeds to run Håstad and Näslund’s algorithm to extract the unencrypted header of packet P_1 . Whenever the algorithm invokes a call to the bit oracle, the adversary follows the implementation of the bit oracle described above. After successfully running the algorithm, the adversary will have obtained the unencrypted header of packet P_1 . From the header the adversary extracts the session key and next-hop value. If the next-hop value indicates that N_1 was the final node in the path, the adversary uses the session key to decrypt the rest of the packet and extracts the original message. Otherwise the next-hop value indicates the next node N_2 , and the session key is used to recover the next Minx packet P_2 . The adversary then repeats the procedure for packet P_2 with destination N_2 . Eventually the adversary will reach the last node in the path and extract the plaintext message and its destination.

In this attack, Minx nodes could occasionally drop packets that contain a previously used session key. Since the oracle queries are chosen from an unbiased pairwise-independent distribution, the probability of this event is negligible in the session key length.

Although this attack is not truly practical, requiring an expected time of $O(\ell^{13})$ for each hop¹, it is sufficient to show that there cannot be a proof of security for Minx in the standard cryptographic security model. Furthermore, it is interesting to note that Minx is somewhat fragile in its security against more practical attacks: if the next-hop portion of the header had been in the most significant bits (before the session key, rather than after it), a simple modification

¹ However, it is interesting to note that in some special cases, Håstad and Näslund’s algorithm actually reveals a node’s RSA private key, thus allowing the adversary to decrypt *all* messages passing through the node.

of Bleichenbacher’s “million message attack” [2] could be used to recover packet plaintexts with only $O(\ell)$ oracle queries for RSA moduli of length ℓ .

3.3 Insecurity of Reply Packets

We note that, in addition to being subject to the same attacks as regular packets, Minx reply packets are subject to an additional attack that distinguishes them from regular packets at the first hop. As outlined in Section 2.2, reply packets are constructed by appending the encryption, under the fixed key λ , of a message M' to a reply packet rb_S , creating $rb_S|E_\lambda(M')|J$. As rb_S is of fixed size and λ is fixed and public, a global passive adversary (or dishonest first-hop mix server) can simply attempt to decrypt the appropriate portion of any packet using key λ . If the result is recognizable as plaintext, the packet corresponds to the first hop of a reply message.

4 Fixing Minx

In this section we propose modifications to Minx. Our attack is possible because routing packets leak information about bits in the packet header plaintext. Our proposed modification is to use a cryptographically secure hash function to obscure the link between the observed behavior and the packet header information, thus removing the bit oracle present in the original Minx specification.

4.1 Details

In our modification, the session key and next-hop field are no longer explicitly encoded in the packet header. When a node processes an incoming packet it computes the hash of the unencrypted header and extracts the session key and next hop from the hash output. The modifications appear in Figure 3. Note that previously only repeated session keys were disallowed, but we now disallow repeated headers. Also note that previously part of the payload was contained in the RSA encrypted portion, but now the payload starts after the $\frac{\ell}{8}$ byte header. The former is done to prevent replay attacks, and the latter to simplify our proof.

The sender is now required to find random headers whose hash indicates the correct next-hop (or *final*). However, this is a minimal burden on the sender as the next-hop field is only 1 byte, and thus the expected number of headers to try before success is only $2^8 = 256$. In an implementation headers could be pulled from a precomputed list, thus reducing the average cost of creating a packet. Furthermore, if the sender does not care about which intermediate nodes are used, they only have to check that the final next-hop value N'_n indicates the correct destination and that none of the intermediate values next-hop values N'_i encode the special *final* value. Also note that the sender no longer explicitly generates session keys, as they are randomly chosen through use of the secure hash function.

Modified Packet Encoding	Modified Packet Decoding
INPUTS: message M node IDs $N_1 \dots N_n$ PROCEDURE: do $Header_n = \text{random } \frac{\ell}{8} \text{ bytes}$ $(k_n N'_n) = H(Header_n)$ until $N'_n = \text{final}$ $C_n = \text{RSA}_{N_n}(Header_n) \text{biEP}_{k_n}(M)$ For j from $n - 1$ to 1 : do $Header_j = \text{random } \frac{\ell}{8} \text{ bytes}$ $(k_j N'_{j+1}) = H(Header_j)$ until $N'_{j+1} = N_{j+1}$ $C_j = \text{RSA}_{N_j}(Header_j) \text{EP}_{k_j}(C_{j+1})$ Pad C_1 up to a set size: $C_1 = C_1 J_i$	INPUTS: packet C_j node ID N_j PROCEDURE: $Header = C_j[0, \frac{\ell}{8} - 1]$ if $Header$ has been seen before drop the packet else cache $Header$ $Pay = C_j[\frac{\ell}{8}, -]$ $(k_j N_{j+1}) = H(\text{RSA}_{N_j}^{-1}(Header))$ if N_{j+1} is not <i>final</i> : $C_{j+1} = \text{EP}_{k_j}^{-1}(Pay)$ Send padded message $C_{j+1} J$ to N_{j+1} else: Process message $\text{biEP}_{k_j}^{-1}(Pay[0, (l - 1)])$

Fig. 3. On the left is our modified procedure for encode a message M in a Minx packet that will travel the path $N_1 \dots N_n$. On the right is our modified procedure for a Minx node N_j to decode and process an incoming packet.

Form Reply Block($SK, N_1 \dots N_n, name, IV$)

```

do
  header_n = random
  (k_n | nextHop_n) = H(header_n)
until nextHop = REPLY
for j = n - 1 to 1
  do
    header_j = random
    (k_j | nextHop_j) = H(header_j)
  until nextHop_j = N_j
secretKey = H'(SK | IV)
M = name | IV | EP_{H'(SK | IV)}(name | Nym | n | header_1 | ... | header_n)
O_n = RSA_{N_n}(header_n) | biEP_{k_n}(M)
for j = n - 1 to 1
  O_j = RSA_{N_j}(header_j) | EP_{k_j}(O_{j+1})
pad O_1 up to size L' (smaller than normal packet size)
return O_1, secretKey

```

Fig. 4. Reply Block formation

4.2 Reply Packets

As described in Section 3.3, the use of a globally fixed key reduces the security of Minx reply packets. If we have the anonymous sender S send a secret key $secretKey$ along with the reply block rb_S and first node destination, then the

receiver can use *secretKey* in place of the fixed key λ . A reply message M' can be created by appending $E_{secretKey}(M')$ to the reply block.

Recall that our modified packet format no longer uses sender specified session keys, and these session keys need to be stored in the reply block so that the sender S can later recover R 's reply message. Thus creating the reply block requires precomputing all the headers so that they can be included in the extra information S sends to herself. The creation of a reply block is shown in Figure 4.

5 Formalization of Minx

In order to prove our modification to Minx is secure, we must first formally define what it means for an onion routing encryption protocol to be “secure.” Our formal definition of security uses a slightly modified version of the onion routing security framework provided by Camenisch and Lysyanskaya [3]. Note that in both this section and the subsequent proof section our discussion is in terms of “onions” and “routers” as used by Camenisch and Lysyanskaya; these correspond exactly to “packets” and “nodes” when discussing Minx.

Camenisch and Lysyanskaya syntactically define an onion routing scheme to consist of two functions, *ProcOnion* and *FormOnion*. When given a private key SK , an onion O_i and a router N_i , *ProcOnion* decrypts O_i and returns the next router in the path N_{i+1} and onion O_{i+1} to be sent to that router. *FormOnion*($m, (N_1, \dots, N_{n+1}), (PK_1, \dots, PK_{n+1})$) creates an onion containing message m and path N_1, \dots, N_{n+1} , using the public keys PK_1, \dots, PK_{n+1} .

Camenisch and Lysyanskaya’s framework for onion security is defined in the adversary and challenger game format. The challenger picks a challenge router and public key PK and gives it to the adversary A , but keeps the corresponding private key secret. The adversary then picks an path index j , sets $PK_j = PK$, picks n other routers and generates public keys $PK_1, \dots, PK_{j-1}, PK_{j+1}, \dots, PK_{n+1}$ (and corresponding private keys). The adversary then submits to the challenger a message m , the index j , the public keys PK_1, \dots, PK_{n+1} . The challenger then forms an onion either containing message m and path $PK_1, \dots, PK_j, \dots, PK_{n+1}$ as requested by the adversary, or a random message and path $PK_1, \dots, PK_j, \dots, PK_{n+1}$. The resulting outer onion O_1 is then given to A . The adversary can then request to have any number of onions $O' \neq O_j$ decrypted by the challenge router using a *procOnion* oracle and can observe the results (A knows all the other private keys so it doesn’t need an oracle for the other routers). Thus without knowing the private key for PK_j , and not being able to process the critical onion O_j , A 's goal is to distinguish which of the two possible onions it was given with nonnegligible advantage over random guessing.

To accommodate Minx in this framework and to simplify our proof of security presented later, we make two simple changes to the framework.

First, in addition to disallowing A from submitting an onion identical to O_j , we add a further restriction and forbid the adversary from submitting any onions with the same header as O_j . This is due to the fact that in Minx modifying the “tail” of a packet doesn’t the next-hop information in the current header (though

it corrupts the message and the rest of the path). Thus in the context of Minx an adversary could submit to the challenge router an onion $O'_j \neq O_j$ but with the same header and easily determine if the challenge router was the last stop. Also this formal restriction is consistent with Minx's policy of dropping packets with previously used headers.

Second, we drop the requirement in [3] that A cannot "re-wrap" O_j . Consider the specified path of the challenge onion, and let N_{j-1} be the router preceding the challenge router N_j . In Camenisch and Lysyanskaya's framework an adversary instantly "wins" the game (thus proving the protocol insecure) if she can construct a different onion O' that goes through a different router $N' \neq N_{j-1}$ yet when processed yields the onion $O'' = O_j$. This is trivial to perform in the context of Minx - just encrypt O_j with a new header using a different node's public RSA key. Again, since Minx nodes are stateful and drop duplicate headers, this event does not correspond to an attack in the Minx setting.

Our modified definition of security is expressed formally as follows:

1. Adversary A receives a (randomly chosen) challenge public key PK and router name N .
2. A can send any number of onions O_i of her choosing to the challenger and observe the output $(N_{i+1}, O_{i+1}) \leftarrow \text{ProcOnion}(SK_i, O_i, N_i)$.
3. A submits a message m , path $N_1 \dots N_{n+1}$, an index j in the path, and public/secret keys for all routers $1 \leq i \leq n+1, i \neq j$. The challenger randomly selects $b \in \{0, 1\}$.

If $b = 0$, the challenger computes:

$$(O_1, \dots, O_{n+1}) \leftarrow \text{FormOnion}(m, (N_1, \dots, N_{n+1}), (PK_1, \dots, PK_{n+1}))$$

If $b = 1$, the challenger randomly selects $r \leftarrow \{0, 1\}^{|m|}$ and computes:

$$(O_1, \dots, O_j) \leftarrow \text{FormOnion}(r, (N_1, \dots, N_j), (PK_1, \dots, PK_j))$$

O_1 is given to adversary A .

4. A can then send any onion O_i whose header differs from O_j and obtain $\text{ProcOnion}(SK_i, O_i, N_i)$.
5. A outputs a guess \hat{b}_A , for the bit b .

We say that an onion routing scheme is secure if for every polynomial time (in the security parameter, e.g. the length of the public key) adversary A , $\Pr[\hat{b}_A = b] - \frac{1}{2}$ is negligible in the security parameter.

Note that as is the case with Minx, Camenisch and Lysyanskaya's framework does not consider the mixing strategy of the onion routers. This allows analysis of the cryptographic aspects of onion routing independently from the mixing strategies used. Under this framework it is easy to see that Minx does not meet this definition of security, since the adversary A can follow the attack described in Section 3 to perfectly distinguish between the $b = 0$ and $b = 1$ cases.

6 Security Proof

In this section we formally prove the security of our modified version of Minx. Recall the game based definition of security in Section 5, in which the adversary

is required to try to guess the value of b . If no polynomial time adversary can get a non-negligible advantage over the random guessing strategy, the protocol is considered secure. Our proof is in the random oracle model [1] and relies on two cryptographic assumptions. First, we assume RSA is a trapdoor one-way permutation: given a randomly chosen RSA modulus N , and $E_N(x)$ for a randomly chosen $x \in \{0, 1\}^{\lceil \log_2 N \rceil}$, no polynomial time algorithm can output x with non-negligible probability. Second, we assume that EP and $biEP$ are implemented using a block cipher (such as AES) in IGE mode, as Danezis and Laurie suggest[5], and that the underlying block cipher is a pseudorandom permutation: given oracle access to a bijection, no polynomial time algorithm can distinguish between an oracle for a uniformly chosen bijection and an oracle for the block cipher with a randomly chosen key. This assumption implies, in particular, that the ciphertexts output by EP and $biEP$, when a key is used only once, are indistinguishable from random bitstrings of the same length [8].

6.1 Outline of Hybrids

Our security proof is similar to standard hybrid arguments, such as appear in [10]. We consider the probability that the adversary outputs 1 in a sequence of four *hybrid* games:

- In Game 0 the challenger always follows the $b = 0$ case from the original game, that is, $O_j = RSA_j(\text{Header}_j)|EP(O_{j+1})$, with the next-hop portion of $H(\text{Header}_j)$ indicating router N_{j+1} as the next hop.
- In Game 1 the challenger acts the same as in Game 0 except when forming O_j . First the challenger forms $O_j = RSA_j(\text{Header}_j)|EP(O_{j+1})$ as he would in Game 0, but then replaces the encrypted header $RSA_j(\text{Header}_j)$ with random bits to yield $O_j = \text{random}|EP(O_{j+1})$. The rest of the outer layers are formed as normal.
- In Game 2 the challenger acts the same as in Game 0 except when forming O_j . Instead of forming O_j as he would in Game 0, the challenger sets all the bits of O_j to be random yielding $O_j = \text{random}$. The rest of the outer layers are formed as normal.
- In Game 3 the challenger follows the $b = 1$ case, except when forming O_j (note that O_j is the innermost onion in the $b = 1$ case). The challenger first forms O_j as he would in the $b = 1$ case, so $O_j = RSA_j(\text{Header}_j)|BiEP(r)$ where the next-hop field of $H(\text{Header}_j)$ encodes the special *Final* value. The challenger then keeps the same encrypted header, but replaces the payload portion, $BiEP(r)$, with random bits to yield $O_j = RSA_j(\text{Header}_j)|\text{random}$.
- In Game 4 the challenger always follows the $b = 1$ case, that is, $O_j = RSA_j(\text{Header}_j)|BiEP(r)$ where the next-hop field of $H(\text{Header}_j)$ indicates *Final*.

We argue that the difference in the probabilities that the adversary outputs 1 in each adjacent pair of games must be negligible. Then by the triangle inequality, we will have that the difference in probabilities between Game 0 and Game 4 must also be negligible.

The adversary, knowing all private keys except the j^{th} private key, can always decrypt O_1 to get O_2 , and then decrypt O_2 to get O_3 , etc., until obtaining O_j . Additionally, in all of the games $O_1 \dots O_{j-1}$ contain no differences, so in essence the adversary is attempting to distinguish between the two cases for O_j .

6.2 Proof of Indistinguishability

We first provide a lemma that ProcOnion is not useful to an adversary. One might expect calling *ProcOnion* would give the adversary an advantage in distinguishing the games, as *ProcOnion* has access to the j^{th} private key. However, since all information returned by *ProcOnion* is a function of the hash function H , we can simulate the information an adversary would gain from calls to *ProcOnion* by replacing H with a random oracle and appropriately responding to its queries to H .

Lemma 1. *An adversary A cannot distinguish between normal outputs from ProcOnion and a third party T simulating ProcOnion and the Oracle. Thus, A gains no information from ProcOnion.*

Proof. T simulates *ProcOnion* and the Oracle by maintaining a table with three columns: x, y , and h , where x is a header plaintext, $y = E(x)$ is the corresponding header ciphertext, and h is our Oracle output for x . A row will always contain a value in the ciphertext and Oracle output columns, but the plaintext value may be empty/unknown.

When A makes an Oracle query x , T calculates $y = E(x)$ and checks if ciphertext y is in the table. If so he returns the corresponding Oracle output listed in the table. Otherwise T returns a random value r , and places x, y, r in the table. When A queries *ProcOnion* with an onion O consisting of header ciphertext y and encrypted payload d , T checks if ciphertext y is already in the table. If so, T looks at the Oracle output $h = \text{nextHop} \parallel \text{sesKey}$. T then decrypts d with *sesKey* and returns the output along with *nextHop*. Otherwise T picks a random value r and makes a table entry with an empty plaintext, y as the ciphertext, and r as the Oracle output. T then takes $h = \text{nextHop} \parallel \text{sesKey}$, decrypts d with *sesKey* and returns the output along with *nextHop*.

Since A has no information about the value of the hash at the plaintext unless he has already queried the oracle there, and we enforce consistency in these cases, the probability of any outcome with our simulation technique is identical to the probability with a random oracle and a correct *ProcOnion* oracle.

We now show that our steps are indistinguishable, beginning with Games 0 and 1. Let $Pr_i[z]$ equal the probability of event z occurring in game i . Suppose that A can distinguish between Game 0 and Game 1 with advantage $\epsilon = Pr_0[\hat{b}_A = 1] - Pr_1[\hat{b}_A = 1]$. Using A , we construct an algorithm $M(y, PK)$ that decrypts RSA ciphertext $y = E(x)$ for a uniform x given the corresponding public key PK . M will simulate A 's calls to ProcOnion and the Random Oracle using a table as explained in the previous lemma. M runs as shown in Figure 5.

$M(y, PK)$

```

m, path  $\leftarrow$   $\mathbf{A}(PK)$ 
o  $\leftarrow$  y || random payload
b  $\leftarrow$   $\mathbf{A}(o, m, \text{path}, \text{public key})$ 
if table contains  $y$  in ciphertext column
    return corresponding plaintext
else
    return random  $x \bmod PK$ 
    
```

Fig. 5. M distinguishing algorithm

Note that if the ciphertext y shows up in the table, it must be as a result of querying the Random Oracle with a value x such that $y = E(x)$, as A is forbidden from querying *ProcOnion* with an onion that has ciphertext header y . Let Q be the event that A queries the Random Oracle on a value x such that $y = E(x)$. Note that when Q does not occur, there is no difference in A 's view of the two games: without querying at x , there is no correlation between the encrypted header and the string $EP(O_{j+1})$. Thus we have $Pr_0[\hat{b}_A = b \mid \bar{Q}] = Pr_1[\hat{b}_A = b \mid \bar{Q}]$ for any bit b . Also note that regardless of what A does, $Pr_1[\hat{b}_A = 1 \mid Q] - Pr_0[\hat{b}_A = 1 \mid Q] \leq 1$. Therefore:

$$\begin{aligned}
 \epsilon &= Pr_1[\hat{b}_A = 1] - Pr_0[\hat{b}_A = 1] \\
 &= Pr_1[\hat{b}_A = 1 \mid Q] \cdot Pr[Q] + Pr_1[\hat{b}_A = 1 \mid \bar{Q}] \cdot Pr[\bar{Q}] \\
 &\quad - \left(Pr_0[\hat{b}_A = 1 \mid Q] \cdot Pr[Q] + Pr_0[\hat{b}_A = 1 \mid \bar{Q}] \cdot Pr[\bar{Q}] \right) \\
 &= Pr[Q] \cdot (Pr_1[\hat{b}_A = 1 \mid Q] - Pr_0[\hat{b}_A = 1 \mid Q]) \\
 &\leq Pr[Q] .
 \end{aligned}$$

Since M succeeds with probability exactly that of Q and runs in time proportional to the running time of A , and it is assumed that RSA cannot be broken with non-negligible probability, A 's advantage must be negligible, and thus Games 0 and 1 are indistinguishable.

Now we argue that Games 1 and 2 are indistinguishable. Suppose that for some adversary A , $Pr_2[\hat{b}_A = 1] - Pr_1[\hat{b}_A = 1]$ is non-negligible. Then this adversary can be used to create an algorithm M to distinguish between the symmetric encryption of a chosen message (under a random, secret key) and random bits, leading to an attack on the block cipher. M simulates the challenger in games 1 and 2, and invokes the adversary A up until the challenge message is prepared. M prepares onion O_{j+1} as the challenger in games 1 and 2 does, and then requests a symmetric-scheme challenge-string that is either the symmetric encryption of O_{j+1} under an unknown secret key or a string of random bits. In either case, M prepends a random header to this string and gives it to A . In case the symmetric-scheme adversary received random bits, he outputs 1 with probability $Pr_2[\hat{b}_A = 1]$, and otherwise he outputs 1 with probability $Pr_1[\hat{b}_A = 1]$. Since we assumed that the symmetric schemes EP and $biEP$ are implemented in a

manner that makes ciphertexts (under a random key) indistinguishable from random bits, Games 1 and 2 are indistinguishable.

The transition between Games 2 and 3 is similar to the transition between Games 0 and 1. The only difference between Game 2 and Game 3 is the header of O_j - Game 2 has a random header and Game 3 has a meaningful header. As in the first transition, an adversary that could distinguish between Games 2 and 3 could decrypt an RSA ciphertext using only the public key. Under the assumption that this is impossible for a polynomially limited adversary, Games 2 and 3 are indistinguishable.

Finally, we argue that Games 3 and 4 are in fact identically distributed. This follows from the fact that with any fixed bijection, $biEP(r)$ for random bits r is uniformly distributed: each block is formed by applying a fixed bijection to a uniformly random string. Thus $\Pr_4[\hat{b}_A = 1] = \Pr_3[\hat{b}_A = 1]$.

Thus we have that for each $i \in \{0, 1, 2, 3\}$, $\epsilon_i = \Pr_{i+1}[\hat{b}_A = 1] - \Pr_i[\hat{b}_A = 1]$ is negligible. Since $\Pr_4[\hat{b}_A = 1] - \Pr_1[\hat{b}_A = 1] = \sum_{i \leq 3} \epsilon_i$, and $\Pr[\hat{b}_A = b] = \frac{1}{2} + \frac{1}{2}(\Pr_4[\hat{b}_A = 1] - \Pr_0[\hat{b}_A = 1])$, we have that any A must have negligible advantage against our modified version of Minx.

6.3 Reply Packets

In order to prove that reply packets are secure, we must first define what security property we want from them. Recall that reply onions under Minx are designed to be indistinguishable from standard onions. A natural question is to ask to whom they should be indistinguishable: the *sender* can clearly differentiate the reply onion until its first hop. And the receiver, if he retains the secret keys, can do so at any hop (e.g. by looking for the encrypted headers). While it is an interesting question whether a reply-onion scheme exists that avoids these issues, we do not address it in this paper. (Nor does the original Minx design)

We define security by a simple game: the challenger picks public keys PK_1, \dots, PK_n and gives them to the adversary, who then picks a message M that could fit in a reply block. The challenger then flips a coin $b \in \{0, 1\}$; if $b = 0$, he encrypts M in a standard onion, and if $b = 1$, he encrypts M in a reply onion. The resulting onion is given to the adversary, who then outputs a guess \hat{b}_A . A reply onion scheme is secure if for every polynomial time adversary, $\Pr[\hat{b}_A = b] - \frac{1}{2}$ is negligible.

Under this definition, it should be clear that our modified reply onions are secure: the only difference between the cases $b = 0$ and $b = 1$ is that for a reply onion, the ‘‘ciphertext’’ after the final header $E(header_n)$ is an encryption under EP of a ‘‘short’’ string followed by an encryption under EP of M using a different key; whereas for a ‘‘standard’’ onion this ciphertext is an encryption under EP of M followed by padding. Since ciphertexts produced by EP are indistinguishable from random bits, the triangle inequality implies the security of our construction.

7 Conclusions and Future Work

Our work here represents two contributions of note. First, we have described a novel attack which demonstrates how leaking one bit in an encrypted message

can have significant security ramifications. We hope that our demonstration of this will influence the design of future protocols. Second, we have described a modification to Minx that prevents this attack, and have provided a formal proof that our solution meets the protocol's original security goals.

One shortcoming of our suggested modification is that it wastes message space by appending a full RSA modulus worth of random bits for every layer of encryption; Minx avoids this by encrypting the key, the next hop, and a portion of the next encrypted onion in the RSA header. The design of a scheme that reduces this overhead is an interesting question for future research.

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