Key for Midterm 2
Wednesday November 17
75 minutes == 75 points
open book and notes

1. 10 points
Two sentences in propositional calculus can be shown to be equivalent by proving that one entails the other and viceversa. Show that \( \neg(p \land q) \equiv \neg p \lor \neg q \) by doing the following steps:

(a) Prove by contradiction using resolution

\[ \neg(p \land q) \models \neg p \lor \neg q \]

We put \( \neg(p \land q) \) in the KB, add the negation of the goal \( \neg(\neg p \lor \neg q) \), transform them to CNF, and prove a contradiction.
1. \( \neg(p \land q) \) becomes \( \neg p \lor \neg q \)
2. \( \neg(\neg p \lor \neg q) \) becomes \( p \land q \) which produces
2a. \( p \)
2b. \( q \)
We resolve 1. \( \neg p \lor \neg q \) with 2a. \( p \) and obtain 3. \( \neg q \)
We resolve 3. \( \neg q \) with 2b. \( q \) and produce a contradiction.

(b) Prove by contradiction using resolution

\[ \neg p \lor \neg q \models \neg(p \land q) \]

We put \( \neg p \lor \neg q \) in the KB, add the negation of the goal \( \neg(\neg(p \land q)) \), transform them to CNF, and prove a contradiction.
1. \( \neg p \lor \neg q \) 2. \( \neg(\neg(p \land q)) \) becomes \( p \land q \) which produces
2a. \( p \)
2b. \( q \)
We resolve 1. \( \neg p \lor \neg q \) with 2a. \( p \) and obtain 3. \( \neg q \)
We resolve 3. \( \neg q \) with 2b. \( q \) and produce a contradiction.

2. 15 points
You are given the following sentence "Heads I win, tails you lose."

(a) Represent it in propositional calculus using the following propositions Head, Tail, IWIn, YouLose.
1. \( Head \Rightarrow IWIn \)
2. \( Tail \Rightarrow YouLose \)

(b) Suppose that you are told "Head". Prove, using any method you like, that "You lose". To do the proof you might need to represent additional knowledge.
We are given
1. $\text{Head} \Rightarrow \text{IWin}$
2. $\text{Tail} \Rightarrow \text{YouLose}$
3. $\text{Head}$

and the additional knowledge
4. $\text{IWin} \Rightarrow \text{YouLose}$

We can use Modus Ponens.
From 3 and 1 we infer 5. $\text{IWin}$.
From 5 and 4 we infer 6. $\text{YouLose}$

(c) Suppose that you are now told "Tail". Can you prove that "I do not win"? Do you need any additional knowledge? Comment briefly on your choice of additional knowledge.

This cannot be proven. Let’s see why. We are given
1. $\text{Head} \Rightarrow \text{IWin}$
2. $\text{Tail} \Rightarrow \text{YouLose}$
3. $\text{Tail}$

and the additional knowledge
5. $\text{YouLose} \Rightarrow \text{IWin}$

using Modus Ponens from 3. and 2. we can derive 6. $\text{YouLose}$.
Using 6. and 5. we can derive $\text{IWin}$.
Since $\text{IWin}$ is satisfiable, its negation $\neg \text{IWin}$ is unsatisfiable.

3. 10 points

Convert the following sentences into a form in which all the quantifiers are as far to the left as possible (without changing the meaning of the sentence).

(a) $\forall x[\exists y \text{Loves}(x, y) \lor \text{Loves}(y, x)] \Rightarrow \text{Happy}(x)$

We can move the existential quantifier $\exists y$ out of the premise of the implication by replacing it with the universal quantifier $\forall x$ which quantifies across the entire expression.

$\forall x \forall y [\text{Loves}(x, y) \lor \text{Loves}(y, x)] \Rightarrow \text{Happy}(x)$

To understand why this is correct, we can transform to CNF and back:

$\forall x [\neg \exists y \text{Loves}(x, y) \lor \text{Loves}(y, x)] \lor \text{Happy}(x)]$

$\forall x [\forall y \neg \text{Loves}(x, y) \land \neg \text{Loves}(y, x)] \lor \text{Happy}(x)]$

$\forall x \forall y [\text{Loves}(x, y) \lor \text{Loves}(y, x)] \Rightarrow \text{Happy}(x)]$

(b) $\forall x [\text{Happy}(x) \Rightarrow [\exists y \text{Loves}(x, y)]]$

We can move the existential quantifier $\exists y$ from the conclusion of the implication and move it to quantify over the entire expression.

$\forall x \exists y [\text{Happy}(x) \Rightarrow \text{Loves}(x, y)]$

To understand why this is correct, we can transform to CNF and back:

$\forall x \neg \text{Happy}(x) \lor \exists y \text{Loves}(x, y)$

$\forall x \exists y \neg \text{Happy}(x) \lor \text{Loves}(x, y)$

$\forall x \exists y [\text{Happy}(x) \Rightarrow \text{Loves}(x, y)]$
4. 30 points

Write the following sentences in predicate calculus. Be consistent in your choice of predicates,

(a) "Every city has a dogcatcher who has been bitten by every dog in town."
   \[\forall x \text{City}(x) \Rightarrow [\exists y \text{DogCatcher}(y) \land \forall z \text{Dog}(z) \land \text{LivesIn}(z, x) \Rightarrow \text{BittenBy}(y, z)]\]

(b) "All mushrooms are either purple or poisonous but not both." This sentence requires an exclusive or.
   \[\forall x \text{Mushroom}(x) \Rightarrow [\text{Purple}(x) \lor \text{Poisonous}(x)] \land \neg [\text{Purple}(x) \land \text{Poisonous}(x)]\]
   or, equivalently,
   \[\forall x \text{Mushroom}(x) \Rightarrow [\text{Purple}(x) \land \neg \text{Poisonous}(x)] \lor [\neg \text{Purple}(x) \land \text{Poisonous}(x)]\]

(c) "All purple mushrooms except one are poisonous."
   \[\exists x \text{Mushroom}(x) \land \text{Purple}(x) \land \neg \text{Poisonous}(x) \land \forall y [\text{Mushroom}(y) \land \text{Purple}(y) \land \neg \text{Poisonous}(y) \Rightarrow x = y]\]

(d) "Rich people have big houses." This sentence has more than one interpretation. The most obvious is that if rich people have a house it is big.
   \[\forall x \forall y \text{Rich}(x) \land \text{HasHouse}(x, y) \Rightarrow \text{Big}(y)\]
   which could also be written as
   \[\forall x \text{Rich}(x) \Rightarrow [\forall y \text{HasHouse}(x, y) \Rightarrow \text{Big}(y)]\]
   Another possible interpretation is that rich people have at least a big house:
   \[\forall x \text{Rich}(x) \Rightarrow \exists y \text{HasHouse}(x, y) \land \text{Big}(y)\]

(e) "Big houses require work unless they have a house keeper and no garden."
   \[\forall x \text{House}(x) \land \text{Big}(x) \Rightarrow \text{Work}(x) \lor [\exists y \text{Keeper}(y, x) \land \neg \exists z \text{Garden}(z, x)]\]
   or, equivalently,
   \[\forall x \text{House}(x) \land \text{Big}(x) \land [\neg \exists y \text{Keeper}(y, x) \lor \exists z \text{Garden}(z, x)] \Rightarrow \text{Work}(x)\]

(f) "If Bill does not have a big house, Bill is not rich."
   \[\neg [\exists x \text{HasHouse}(Bill, x) \land \text{Big}(x)] \Rightarrow \neg \text{Rich}(Bill)\]
   or, equivalently
   \[\forall x \neg \text{HasHouse}(Bill, x) \lor \neg \text{Big}(x) \Rightarrow \neg \text{Rich}(Bill)\]
   Another possible interpretation: if Bill has a house and the house is not big then Bill is not rich.
   \[\forall x \text{HasHouse}(Bill, x) \land \neg \text{Big}(x) \Rightarrow \neg \text{Rich}(Bill)\]

5. 10 points

Show the backed-up values for all the nodes in the following game tree and show the branches that are pruned by alpha-beta. For each branch pruned, explain briefly why alpha-beta prunes it. Follow the convention used in the textbook to examine the branches in the tree from left to right.

The pruning is shown in the figure. The first pruning can be done because -3 (which is the value for min) is less than 0 (which is the value for Max). The next pruning can be done because 3 (which is the current value for Max) is greater than 0 (the current
best choice for the top min node). No additional pruning can be done. After the last node (with a value of -3) is reached, the top min node selects -3.

\[
\begin{align*}
\text{min} & \quad \text{Max} \\
& \begin{array}{c}
\text{min} \\
0 \\
-3 \\
3 \\
-3 \\
3 \\
0 \\
-2 \\
3 \\
2 \\
-3
\end{array}
\end{align*}
\]

because $-3 < 0$ \hspace{1cm} because $0 < 3$