3D Localization for a Mars Rover Prototype*

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Abstract

In this paper we consider the problem of localizing a mobile robot on uneven terrain. The localization problem is decomposed into two stages; attitude estimation followed by position estimation. The innovation of our method is the use of a smoother, in the attitude estimation loop that outperforms other Kalman filter based techniques in estimate accuracy. The smoother exploits the special nature of the data fused; high frequency inertial sensor (gyroscope) data and low frequency absolute orientation data (from a compass or sun sensor). Two Kalman filters form the smoother. During each time interval one of them propagates the attitude estimate forward in time until it is updated by an absolute orientation sensor. At this time, the second filter propagates the recently renewed estimate back in time. The smoother optimally exploits the limited observability of the system by combining the outcome of the two filters. The system model uses gyro modeling which relies on integrating the kinematic equations to propagate the attitude estimates and obviates the need for complex dynamic modeling. The Indirect (error state) form of the Kalman filter is developed for both parts of the smoother. The proposed approach is independent of the robot structure and the morphology of the ground. It can easily be transfered to another robot which has an equivalent set of sensors. Quaternions are used for the 3D attitude representation, mainly for practical reasons discussed in the paper. The proposed innovative algorithm is tested in simulation and the overall improvement in position estimation is demonstrated.

1 Introduction

Future missions to Mars will demand long traverses (several km) of rovers to sites of scientific interest. In order to autonomously perform their scientific tasks, these rovers need to know their position precisely. The focus of our research effort is to localize an experimental rover capable of navigating in a 3D environment. Specifically, we are motivated by the problem of localizing the next generation of robot rovers [10] on the surface of Mars. Localization is the problem of determining the position of a mobile with respect to a global or local frame of reference in the presence of sensor noise, uncertainties and potential failures. The basic idea behind many mobile robot localization techniques is to combine sensor data with a priori knowledge about the specifications of these sensors, the structure of the mobile platform, and the environment the vehicle travels in. For example, it is often assumed that a detailed map of the area is known. In this case, the problem of identifying the position of the robot is the problem of finding an area within the map such that the expected sensor values are at all times in accordance with the actual readings.

The assumptions made hereafter are that 1)*No prior maps* of the environment are available and 2) Global Positioning System (GPS) signals are *not detectable* on the surface of Mars. In this case absolute positioning is not feasible. The robot is not capable of determining its position directly by sensing its surroundings (absolute localization). Instead, relative positioning techniques have to be involved. The rover must track its position starting from the landing site through every point of its trajectory.

Many current localization efforts have focused on supporting high quality position tracking. Different sensing devices and odometric techniques have been exploited for this purpose. The common characteristic of these approaches is that they rely on the integration of some kinetic quantity. The main drawbacks of any form of odometry are: 1) Every sensor monitoring the motion of the vehicle has a certain type and level of noise contaminating its signal. Integration of the noisy components causes gradual error accumulation and makes the estimates untrustworthy. 2) The kinematic model of the vehicle is never accurate. For example, we do not know with infinite precision the distance between the wheel axes of the vehicle. 3) The sensor models also suffer from inaccuracies and can become very complicated. For example, the use of complicated models to describe the gyroscope drift. 4) The motion of the vehicle involves external sources of error that are not observable by the sensors used. For example,

*This work is supported by JPL, Caltech under contract #959816.

slippage in the direction of motion or in the perpendicular direction is many times not detected by the motion sensors. Externally provided or extracted information is necessary from time to time if we wish to keep the error bounded. This group of approaches is also referred to as "dead-reckoning".

Global (absolute) orientation measurements can drastically increase the accuracy of the position tracking estimate and reduce the rate of growth of the associated uncertainty. In the case of a rover such as Rocky 7, its attitude can be estimated (relative) in real time by integrating the rotational velocity of the vehicle as this is measured by 3 gyroscopes on-board. The problem with this approach is that while the robot is in motion, the rates of roll, pitch and yaw available from gyroscopes are subject to drift and noise. The orientation estimates drift away from their real values and thus they become un-trustworthy. Even small errors in the orientation fast produce large errors in position. As explained in later sections, the absolute (global) orientation of the vehicle can be measured but only intermittently. The focus of this research effort is to provide the best attitude and position estimates when the absolute orientation measurements are not available continuously.

In this paper we address the problem of 3D localization for mobile robots in the absence of absolute positioning information. We concentrate on bounding the attitude uncertainty through periodic use of absolute attitude measurements. As a consequence the position estimate degrades slowly compared to the case when no absolute orientation information is available. The attitude estimate relies on the gyros when the vehicle is in motion while a tri-axial accelerometer is used as an absolute orientation measuring device (roll and pitch) in conjunction with a sun sensor (yaw) when the vehicle is at stop. At the end of each interval of motion a smoother is used which propagates the new absolute orientation information backwards using the previously acquired gyro information. This lowers the uncertainty of the attitude estimate throughout the interval of smoothing; that is when the vehicle was in motion. Both the forward and backward estimators are Indirect (error state) Kalman filters and gyro modeling is used instead of a dynamic model of the robot. Smoothing is being applied here to the mobile robot localization problem for the first time. The proposed smoother based localization algorithm subject to the aforementioned constraints, generalizes across different mobile robot platforms with varying kinematics and dynamics.

In the next section we survey previous work in robot localization. Section 3 examines the dependence of the position estimate on the attitude estimate. We discuss the various attitude measuring devices used, the rationale behind dynamic model replacement and the Indirect Kalman filter and a basic gyro model. Section 4 contains a derivation of the error state equations for the 3-D case using unit quaternions. The linear time-variant equations of the system model and the non-linear equations of the observation model are derived. An Indirect Kalman filter based on these models is developed. The improvement due to the smoother is demonstrated. Section 5 shows how the position is updated using the improved attitude estimates and section 6 summarizes the contributions of this work and discusses future avenues of research.

2 Previous Work

In order to deal with systematic errors in indoor applications, a calibration technique called the UMBmark test is given in [3]. [4] discusses a technique called gyrodometry, which uses odometry data most of the time, while substituting gyro data only during brief instances (e.g. when the vehicle goes over a bump) during which gyro and odometry data differ drastically. This way the system is kept largely free of the drift associated with the gyroscope. A complementary Kalman filter [6] is used in [9] to estimate the robot's attitude from the accelerometer signal during low frequency motion and the gyro signal during high frequency motion. The attitude information is then used to calculate a position increment. In [1] the authors use a low cost INS system (3 gyroscopes, and a triaxial accelerometer) and 2 tilt sensors. Their approach is to incorporate in the system *a priori* information about the error characteristics of the inertial sensors and to use this directly in an Extended Kalman Filter (EKF) to estimate position.

Examples of absolute localization include [13] in which the localization algorithm is formalized as a tracking problem, employing an EKF to match beacon observations to a map in order to maintain an estimate of the position of the mobile robot; [2] in which the authors use an EKF to fuse odometry and angular measurements of known landmarks and [22] in which a Bayesian approach is used to learn useful landmarks for localization.

Most of the above approaches limit themselves to the case of planar motion. In addition, their accuracy depends heavily on the presence of some form of an absolute positioning system. We consider motion on uneven terrain (3D localization) and propose an estimation algorithm that is capable of incorporating absolute position measurements but is also able to provide reliable estimates in the absence of externally provided positioning information. Our method performs attitude estimation using an Indirect Kalman filter that operates on the error state.

3 Localization and Attitude Estimation

In this section we examine the relation between the attitude estimate and the position estimate. We use an experimental Mars rover prototype (Rocky 7 [10]) as the motivating example throughout this paper. The assumption is that the robot has wheel encoders, 3 gyros, 3 accelerometers and a sun sensor. Since there is no device measuring the absolute position of the rover (there is no GPS on Mars), the position can only be estimated through the integration of the accelerometer signal which has bias and noise. Consider also, that the propagation of the position relies upon the attitude estimate. Small errors in orientation fast become large errors in position. Formally speaking, the position is not observable and thus the uncertainty of its estimate will grow unbounded. The most promising course of action with this set of sensors is to focus on gaining a very precise attitude estimate. As a result the position uncertainty will grow at a slower rate:

1. The accelerometer measures both the vehicle's acceleration and the projection of the gravitational acceleration on the accelerometer local frame. The relation between these is described by:

$$\ddot{\vec{p}}(t) = \vec{f}(t)/m = \vec{a}_{acc}(t) - A(q(t))\vec{g} \qquad (1)$$

   where $\ddot{\vec{p}}$ is the vehicle's (non-gravitational) acceleration, $\vec{a}_{acc}$ is the measurement from the 3-axis accelerometer and $\vec{g}$ is the gravitational acceleration. Precise knowledge of the orientation matrix $A(q)$ is mandatory to extract $\ddot{\vec{p}}$ accurately.

2. The next step requires integration of $\ddot{\vec{p}}$ to derive the position. $\ddot{\vec{p}}$ is local (i.e. expressed in a coordinate frame attached to the robot) and in order to calculate the position in global coordinates the attitude information is once again required:

$$\vec{p}(t) = \int_0^t dt' \int_0^{t'} A^T(q(t''))\ddot{\vec{p}}(t'')dt'' \qquad (2)$$

3.1 Attitude Measuring Devices

The on-board gyroscopes can be used to calculate the attitude of the vehicle by integrating their signal. On the other hand, the sun sensor directly measures the values of the two components of a two-dimensional vector. This vector is the projection of the unit vector towards the sun on the sun sensor plane. Another sensory input of the same nature is required in order to satisfy attitude observability requirements. While the accelerometer is mainly used to advance the position estimate (Equations 1,2) it can also be used in an alternative way. An accelerometer can measure the local gravitational acceleration, a three-dimensional vector parallel to the local vertical. This provides another orientation fix independent from the sun and thus makes the vehicle's attitude observable. When the vehicle is stopped the accelerometer measures only the gravitational acceleration namely $\vec{a}_{acc} = A(q)\vec{g}$. The roll and pitch of the vehicle can thus be precisely calculated. The sun sensor provides the yaw measurement and thus the matrix $A(q)$ is observable and precisely known when at stop.

This method fails when the rover is in motion. The gravity vector is then "contaminated" by the non-gravitational acceleration of the vehicle (Equation 1). The gravity vector could be extracted while the vehicle is moving if an independent measurement of its own acceleration was available. Research efforts [23, 9] have tried to address this problem using additional information from odometry. We believe that these approaches are sufficient for indoor applications and can deal with cases of motion over small objects but are not accurate enough for general outdoor environments mainly because of the limited accuracy of the estimates of the non-gravitational acceleration. A more thorough consideration of the problem would require dynamic modeling of the vehicle. An estimator that incorporates a dynamic model of the vehicle [21] could estimate its non-gravitational accelerations.

3.2 Dynamic Model Replacement

In our approach we avoid dynamic modeling and restrict ourselves to **use the accelerometer only when the rover is at stop**. The reasons for avoiding dynamic modeling are: 1. generality, 2. practical estimator size, 3. reported poor payoffs [11] due to dynamic modeling, and 4. complexity. Due to space constraints, we do not discuss these further, the interested reader is referred to [18, 19] for further details.

3.3 The Indirect-feedback Kalman Filter

As mentioned before, Kalman filtering has been widely used for localization purposes. The kinds that usually appear in mobile robot applications are the linear Kalman filter and the Extended Kalman filter (EKF) forms of the full state Kalman filter. In this work we choose to use the error-state form of both the linear Kalman filter and EKF. In the error-state (indirect) formulation, the *errors* in orientation are among the estimated variables, and each measurement presented to the filter is the difference between the INS and the external source data (i.e from absolute orientation sensors). In the following section we derive the equations needed for such a formulation. The primary reasons to pick this formulation are 1. No explicit modeling of the vehicle dynamics is needed, 2. The filter runs at a relatively low frequency, and 3. In case the filter fails, integrated estimates of the INS data continue to be available.

In the feedback form of the Indirect-feedback Kalman filter the updated error estimate is actually fed back to the INS to correct its "new" starting point, i.e. the state that the integration for the new time step will start from. The rationale behind the Indirect Kalman filter as well as the feedback form are discussed in further detail in [18, 19].

3.4 Gyro Modeling

A great difficulty in all attitude estimation approaches that use gyros, is the low frequency noise component, also referred to as bias or drift that violates the white noise assumption required for standard Kalman filtering. This problem has attracted the interest of many researchers since the early days of the space program [15]. Inclusion of the gyro noise model in a Kalman filter by suitably augmenting the state vector has the potential to provide estimates of the sensor bias when the observability requirement is satisfied. Early implementations of gyro noise models in Kalman filters can be found in [16].

An estimate of the attitude would imply the derivation of the dynamics of the robot, which we wish to avoid for the reasons listed in the previous section. In order to do so we relate the gyro output signal to the bias and the angular velocity of the vehicle using the simple and realistic model [8]. In this model the angular velocity about a particular axis $\omega = \dot{\theta}$ is related to the gyro output ω_m according to the equation:

$$\dot{\theta} = \omega_m + b + n_r \tag{3}$$

where b is the drift-rate bias and n_r is the drift-rate noise. n_r is assumed to be a Gaussian white-noise process with covariance N_r. The drift-rate bias b is not a static quantity but is driven by a second Gaussian white-noise process, the gyro drift-rate ramp noise n_w. Thus $\dot{b} = n_w$ with covariance N_w. The two noise processes are assumed to be uncorrelated.

4 3-D Attitude Estimation

The proposed method in the 3D case is summarized in Figure 1. It should be noted that only the forward filter estimate is available in real-time. The smoother runs off-line (during the times that the robot is halted). This technique is not limited to robots used for Mars exploration. It can be applied to any other autonomous vehicle equipped with an equivalent set of sensors. The mixing of high frequency inertial sensors with low frequency absolute (position or orientation) sensors is becoming common in mobile robotics. Robots equipped with GPS or landmark tracking devices, usually carry additional sensors that can be used for localization when the GPS signal degrades or the landmarks are occluded. Our framework could be used to combine the data from such sensor sets as well.

4.1 Attitude kinematics

We use quaternions to parameterize the robot's attitude for three practical reasons. First, the prediction equations are treated linearly, secondly the representation is free of singularities and finally the attitude matrix is algebraic in the quaternion components, thus eliminating the need for transcendental functions. The reader is referred to [5] for a review on quaternions.

The physical counterparts of quaternions are the rotational axis $\hat{n}$ and the rotational angle θ that are used in the Euler theorem regarding finite rotations. By taking the vector part of a quaternion and normalizing it, we can find the rotational axis, and from the last parameter we can obtain the angle of rotation [7]. Following the notation in [12], a unit quaternion is defined as:

$$q = [q_1 \quad q_2 \quad q_3 \quad q_4]^T \qquad q^T\ q = 1 \tag{4}$$

where the first three elements of the quaternion can be

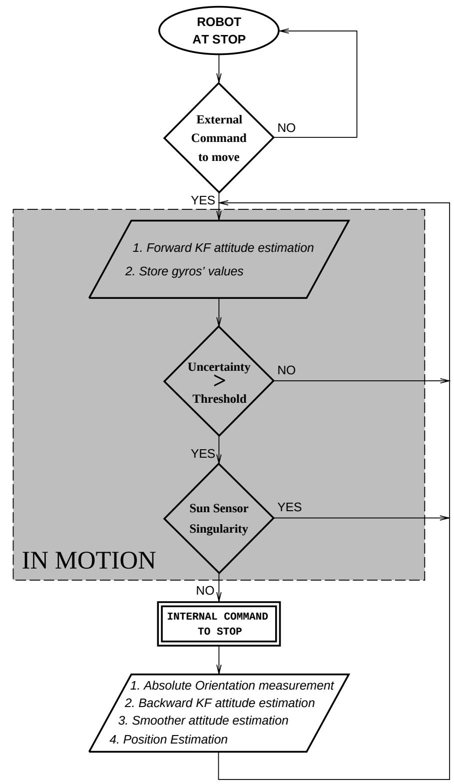

Figure 1: Algorithm Flow Chart: While the robot is in motion the forward Kalman filter uses gyro data to produce (in real-time) a first approximation of the attitude estimate. When the covariance of this estimate exceeds a preset threshold the robot is stopped. An absolute orientation measurement is made using the sun sensor and the three-axis accelerometer. A backward estimate is computed (off-line) and its results are combined (off-line) with the estimate from the forward filter using a smoother. Finally, the position is estimated (off-line) using the (smoothed) attitude estimate for each instant of the trajectory.

written in a compact form as:

$$\vec{q} = \hat{n} sin(\theta/2) \tag{5}$$

The attitude matrix is obtained from the quaternion according to the relation:

$$A(q) = (|q_4|^2 - |\vec{q}|^2) I_{3\times3} + 2\vec{q}\vec{q}^T + 2q_4 [[\ \vec{q}\ ]] \tag{6}$$

where

$$[[\ \vec{q}\ ]] = \begin{bmatrix} 0 & q_3 & -q_2 \\ -q_3 & 0 & q_1 \\ q_2 & -q_1 & 0 \end{bmatrix} \tag{7}$$

is a 3×3 skew symmetric matrix generated by the 3×1 vector $\vec{q}$. The matrix $A(q)$ transforms representations of vectors in the reference coordinate system to representations in the body fixed coordinate system. The rate of change of the attitude matrix with time is given by:

$$\frac{d}{dt} A(t) = [[\ \vec{\omega}(t)\ ]]\, A(t) \tag{8}$$

where the corresponding rate for the quaternion is:

$$\frac{d}{dt}q(t) = \frac{1}{2}\Omega(\vec{\omega}(t))q(t) \tag{9}$$

with

$$\Omega(\vec{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \tag{10}$$

At this point we present an approximate body-referenced representation of the error state vector and covariance matrix. The error state includes the bias error and the quaternion error. The bias error is defined as the difference between the true and estimated bias.

$$\Delta\vec{b} = \vec{b}_{true} - \vec{b}_i \tag{11}$$

The quaternion error is not the arithmetic difference between the true and estimated but it is expressed as the quaternion which must be composed with the estimated quaternion in order to obtain the true quaternion.

$$\delta q = q_{true} \otimes q_i^{-1} \quad or \quad q_{true} = \delta q \otimes q_i \tag{12}$$

The advantage of this representation is that since the incremental quaternion corresponds very closely to a small rotation, the fourth component will be close to unity and thus the attitude information of interest is contained in the three vector component $\delta\vec{q}$ where

$$\delta q \simeq [\delta\vec{q} \;\; 1]^T \tag{13}$$

Starting from equations:

$$\frac{d}{dt}q_{true} = \frac{1}{2}\Omega(\vec{\dot{\theta}}_{true})q_{true} \tag{14}$$

and

$$\frac{d}{dt}q_i = \frac{1}{2}\Omega(\vec{\dot{\theta}}_i)q_i \tag{15}$$

where $\vec{\dot{\theta}}_{true}$ is the true rate of change of the attitude and $\vec{\dot{\theta}}_i$ is estimated from the measurements provided by the gyros, it can be shown [18] that

$$\frac{d}{dt}\delta\vec{q} = [[\;\vec{\omega}_m\;]]\,\delta\vec{q} - \frac{1}{2}(\Delta\vec{b} + \vec{n}_r) \quad \frac{d}{dt}\delta q_4 = 0 \tag{16}$$

where $\vec{\omega}_m$ is the output of the gyros. Using the infinitesimal angle assumption in Equation 5, $\delta\vec{q}$ can be written as $\delta\vec{q} = \frac{1}{2}\delta\vec{\theta}$. Thus Equation 16 can be rewritten as:

$$\frac{d}{dt}\delta\vec{\theta} = [[\;\vec{\omega}_m\;]]\,\delta\vec{\theta} - (\Delta\vec{b} + \vec{n}_r) \tag{17}$$

Differentiating Equation 11 and assuming $\vec{\dot{b}}_{true} = \vec{n}_w$ and $\vec{\dot{b}}_i = 0$, the bias error dynamic equation is $\frac{d}{dt}\Delta\vec{b} = \vec{n}_w$ which when combined with Equation 17 yields the error state equation:

$$\frac{d}{dt}\begin{bmatrix} \delta\vec{\theta} \\ \Delta\vec{b} \end{bmatrix} = \begin{bmatrix} [[\;\vec{\omega}_m\;]] & -I_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix}\begin{bmatrix} \delta\vec{\theta} \\ \Delta\vec{b} \end{bmatrix} + \begin{bmatrix} -I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & I_{3\times 3} \end{bmatrix}\begin{bmatrix} \vec{n}_r \\ \vec{n}_w \end{bmatrix} \tag{18}$$

In a more compact form Equation 18 is:

$$\frac{d}{dt}\Delta x = F\Delta x + Gn \tag{19}$$

4.2 Discrete system: Indirect forward Kalman filter equations

4.2.1 Propagation

At this point we define $q_{k/k}$ ($\vec{b}_{k/k}$) as the quaternion (bias) estimate at time t_k based on data up to and including $z(t_k)$, $q_{k/k-1}$ ($\vec{b}_{k/k-1}$) the quaternion (bias) estimate at time time t_{k-1} propagated to t_k, right before the measurement update at t_k. The estimated angular velocity is defined (before and after the update) as:

$$\vec{\omega}_{k/k-1} = \vec{\omega}_m(t_k) - \vec{b}_{k/k-1} \quad \vec{\omega}_{k/k} = \vec{\omega}_m(t_k) - \vec{b}_{k/k} \tag{20}$$

Following [24], the full estimated quaternion is propagated over the interval $\Delta t_k = t_k - t_{k-1}$ as follows:

$$q_{k/k-1} = \left\{ \begin{array}{c} exp[\frac{1}{2}\Omega(\vec{\omega}_{avg})\Delta t_k] + [\Omega(\vec{\omega}_{k/k-1})\Omega(\vec{\omega}_{k-1/k-1}) \\ -\Omega(\vec{\omega}_{k-1/k-1})\Omega(\vec{\omega}_{k/k-1})]\Delta t_k^2/48 \end{array} \right\} q_{k-1/k-1}$$

where the average angular velocity for this interval is approximately

$$\vec{\omega}_{avg} = \frac{\vec{\omega}_{k/k-1} + \vec{\omega}_{k-1/k-1}}{2} \tag{21}$$

The bias estimate is constant over the propagation interval $\vec{b}_{k/k-1} = \vec{b}_{k-1/k-1}$. The propagation equation for the error state covariance is

$$P_{k/k-1} = \Phi(k,k-1)P_{k-1/k-1}\Phi^T(k,k-1) + Q_k \tag{22}$$

If the average angular velocity $\vec{\omega}_{avg}$ is constant over the interval Δt_k, with magnitude ω_{avg} then the discrete system transition matrix $\Phi(k,k-1)$ can be easily calculated from Equation 18 ([18, 20]).

4.2.2 Update

When the rover stops, an absolute orientation measurement is available from the sun sensor and the accelerometer. This is used to update the estimated error state and the covariance [14]. The Kalman gain matrix is given by:

$$K_k = P_{k/k-1}H_k^T\;(H_kP_{k/k-1}H_k^T + R_k)^{-1} \tag{23}$$

The updated covariance and error state equations are:

$$P_{k/k} = P_{k/k-1} - K_kH_kP_{k/k-1} \tag{24}$$

$$\Delta x_{k/k} = \Delta x_{k/k-1} + K_k\Delta z(t_k) \tag{25}$$

or

$$\begin{bmatrix} \delta\vec{\theta}_{k/k} & \Delta\vec{b}_{k/k} \end{bmatrix}^T = K_k\Delta z(t_k) \tag{26}$$

where $\Delta z(t_k)$ is the measurement residual. The propagated error $\Delta x_{k/k-1}$ is zero because we have implemented the feedback formulation of the Indirect Kalman filter. Every time we have a measurement the update is included in the full state and thus the next estimate of the error state $\Delta x_{k/k-1}$ is assumed to be zero. This update is:

$$q_{k/k} = \delta q_{k/k} \otimes q_{k/k-1} = \left[\delta \vec{q}_{k/k} \quad 1\right]^T \otimes q_{k/k-1} \qquad (27)$$

where

$$\delta \vec{q}_{k/k} = (1/2)\delta\vec{\theta}_{k/k} \quad \vec{b}_{k/k} = \vec{b}_{k/k-1} + \Delta\vec{b}_{k/k} \qquad (28)$$

4.2.3 Observation model

Due to space limitations we omit the equations for the observation model. For a detailed derivation the interested reader is referred to [18, 20].

4.3 Backward filter

In the flow chart shown in Figure 1 we see that the robot stops every time the uncertainty grows over a preset threshold. Then the backward filter is engaged and the last attitude estimate is propagated back in time. This last estimate is very precise because it is heavily based on the absolute orientation measurements acquired when the robot stopped. While the backward filter is close to its starting point it is able to provide estimates of higher confidence than those of the forward filter. In order to derive the equations for the backward Indirect Kalman filter we start from the equations of the system for the forward case:

$$\dot{x} = Fx + Gw \quad and \quad z = Hx + v \qquad (29)$$

By defining $\tau = T - t$, where τ is the backward time variable and $T = t_k - t_{k-M}$ is the time interval of smoothing, the backward system equation can be derived from:

$$\frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = -\dot{x} \quad \frac{dx_b}{d\tau} = -Fx_b - Gw \qquad (30)$$

Making the appropriate substitutions we get the following equation for the quaternion estimate propagation:

$$q_{b,k-1/k-1} = \left\{ \begin{array}{c} exp[\frac{1}{2}\Omega(\vec{\omega}_{avg})\Delta t_k] + [\Omega(\vec{\omega}_{k/k-1})\Omega(\vec{\omega}_{k-1/k-1}) - \\ \Omega(\vec{\omega}_{k-1/k-1})\Omega(\vec{\omega}_{k/k-1})]\Delta t_k^2/48 \end{array} \right\}^T q_{b,k/k-1}$$

The bias propagation remains the same as before since the direction of propagation does not affect an assumed constant variable. The backward propagation equation for the covariance is now:

$$P_{b,k-1/k-1} = \Phi_b(k-1,k)P_{b,k/k-1}\Phi_b^T(k-1,k) + Q_{b,k} \quad (31)$$

No new absolute measurements are collected during the backward propagation of the filter and thus, the update equations and the observation model for the backward filter are not considered.

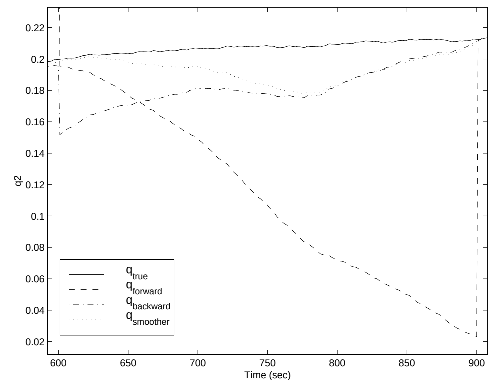

Figure 2: This is the usual outcome due to the bias estimation. The forward filter estimate drifts to the right because it has underestimated the gyro bias. The backward filter overestimates and thus drifts to the left (in the opposite direction). The smoothed estimate outperforms both filters minimizing the average estimation error.

4.4 Smoother

The smoother constructs the best estimate of the state of the system over a time period using all the measurements in that time interval [17]. In our case, the time for which the robot stops to get an absolute orientation measurement allows for post-processing and therefore application of the smoother. In order to calculate the total (smoothed) estimate we use the following equation[1]:

$$P_{total}^{-1}\hat{x}_{total} = P_f^{-1}\hat{x}_f + P_b^{-1}\hat{x}_b \qquad (32)$$

Each covariance matrix P_f, P_b and P_{total} represents the uncertainty of the corresponding estimate. The higher the uncertainty, the larger the covariance matrix. Equation 32 weighs each of the available estimates (from the forward and the backward filter) according to their certainty. The result is the optimal estimate possible, if all the measurements of the time interval of smoothing were available at once. The significant improvement in the quality of the 3D estimate is shown in Figure 4. Different estimated quantities calculated in a representative trial are depicted in Figures 2 and 3. The overal improvement in attitude estimation is presented in Figure 5.

5 From Attitude Estimates to Position Estimates

The accuracy of the position estimate depends heavily on the accuracy of the attitude estimate. Though the position can be calculated in real-time using the output of the forward Kalman filter we choose not to do that. Instead in our algorithm the position estimation takes place off-line as described in Figure 1. After the vehicle stops to collect

[1] Applying this in 3D is somewhat involved because of the particular form of the error quaternion used. The interested reader is referred to [18] for the technical details

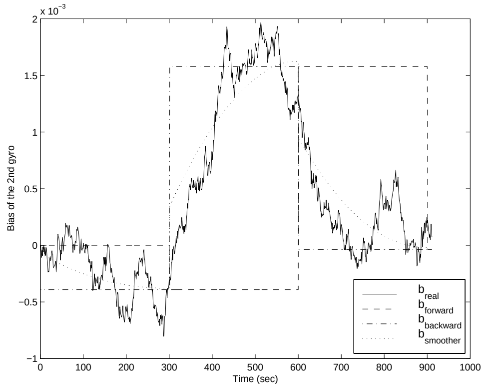

Figure 3: For the second gyro, we show the true bias value, the forward filter's estimate, the backward filter's estimate and the smoothed estimate of the bias. The smoothed (total) estimate stays close to the backward filter estimate for the second half of each smoothing interval while for the first part it depends on both the forward filter's estimate and the backward filter's estimate. This is due to the fact that the initial bias value for the backward filter is more trustworthy for this time interval than the initial value of the forward filter. The asymmetry is due to the fact that the backward filter works with an "initial" estimate which is actually computed after the motion.

an absolute orientation measurement the off-line smoothing of the attitude estimation is performed. The resulting estimate is accurate and is used to compute the current position. As we mentioned before the attitude estimate is an input to Equations 1 and 2. If the integration step is small, we can simplify this calculation as follows. First the increase in position is calculated due to the sensed acceleration and the current velocity:

$$ {}^{L}\Delta p(t_k) = {}^{L}v(t_k)\ \Delta T + {}^{L}a(t_k)\ \Delta T^2/2 \qquad (33)$$

this increment is then transformed to global coordinates using ${}^{G}\Delta p(t_k) = {}^{G}_{L}A(q(t_k))\ {}^{L}\Delta p(t_k)$, before it can be used to compute the next position using

$$ {}^{G}p(t_{k+1}) = {}^{G}p(t_k) + {}^{G}\Delta p(t_k) \qquad (34)$$

The velocity increment during every measurement cycle is ${}^{L}\ \Delta v(t_k) = {}^{L}a(t_k)\ \Delta T$. In global coordinates, we have ${}^{G}\ \Delta v(t_k) = {}^{G}_{L}A(q(t_k))\ {}^{L}\ \Delta v(t_k)$. The new velocity is

$$ {}^{G}\ v(t_{k+1}) = {}^{G}\ v(t_k) + {}^{G}\ \Delta v(t_{k+1}) \qquad (35)$$

This result has to be transformed to local coordinates before it is fed back for the next position update:

$$ {}^{L}\ v(t_{k+1}) = {}^{L}_{G}A(q(t_{k+1}))\ {}^{G}\ v(t_{k+1}) \qquad (36)$$

The resulting improvement in the estimation of the vehicle's position is shown in Figure 6.

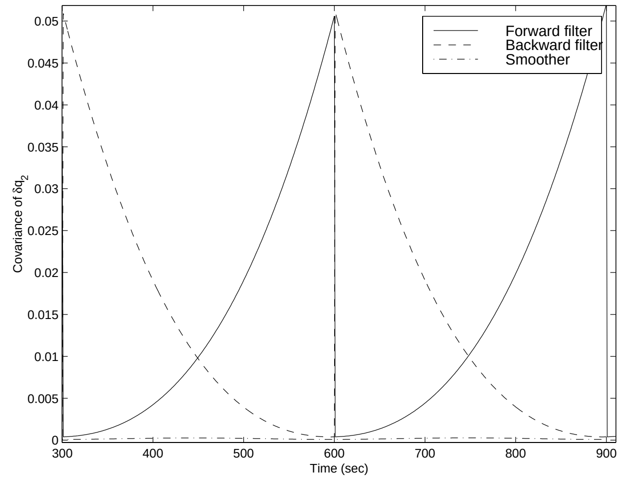

Figure 4: The covariance related to q_2 from the forward filter, backward filter and smoother is shown. At all times the total covariance is lower than either of the corresponding ones calculated from the two filters. Its value remains bounded and varies slightly during the smoothing interval.

6 Conclusion

In this paper we decomposed the localization algorithm into attitude estimation and, subsequently, position estimation. A novel approach that incorporates a smoother was presented. An Indirect (error-state) Kalman filter that incorporates inertial navigation and absolute measurements was developed for this purpose. The dynamic model was replaced by gyro modeling which relies on the integration of the kinematic equations. The error state equations for the three dimensional case were derived and used to formulate the filter's time-variant system model and non-linear observation model. Quaternions were selected for the three dimensional attitude representation. Finally, the improvement due to the proposed method was demonstrated in simulation. Uniformly smaller values of the covariance of the estimate were sustained throughout each of the trials. It should be noted that due to the lack of vehicle specific dynamic modeling the proposed approach is general and may be used on any vehicle chassis with an equivalent set of sensors. Future directions of research include applications (extensions) of this method to cases where the INS sensors are fused with other absolute sensors that measure position (e.g. vision cues, star sensors, beacons etc.)

Acknowledgments

The authors would like to thank Dr. D. Bayard for several illuminating discussions. We also thank Dr. B. Balaram and Dr. G. Rodriguez for technical discussions.

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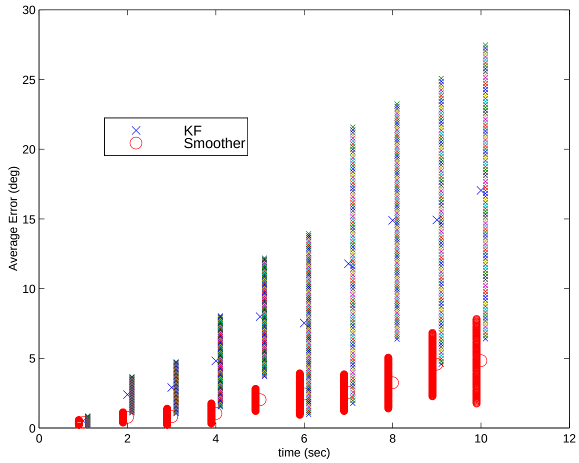

Figure 5: The average orientation error when absolute orientation measurements are available to the robot every $1, 2, .., 10$ *seconds*, for the case of Kalman filter ($\times$) and smoother (o). The vertical bars represent the 3σ intervals of statistical confidence.

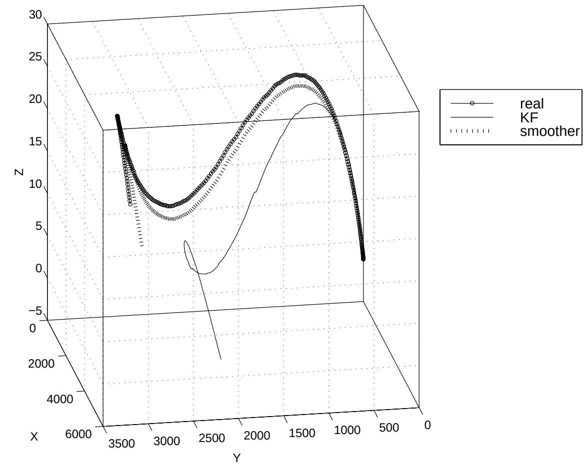

Figure 6: This is an example trajectory in 3D. The thick line (circles) represents the real motion of the vehicle. The thin line is the computed based on the attitude information from the Kalman filter, and the thick dashed is the computed using the attitude information provided by the smoother.

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