Point and Line at Infinity
Special Case

\[ \hat{I} = K^T I = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \]

Lines pass through the center of image
Special Case

3D plane: $\pi = \begin{bmatrix} a \\ b \\ 0 \\ 0 \end{bmatrix}$

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When the last element of line, \( \mathbf{l} \) is zero:
- the line passes through the center of image.
- the surface normal of the 3D plane is perpendicular to Z axis
- the 3D plane is a plane rotating around Z axis

\[
\begin{bmatrix} a & b & 0 \end{bmatrix} Z = 0
\]

Lines passe through the center of image

\[
\mathbf{l} = K^T \mathbf{l} = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}
\]
Special Case: 2D Point at Infinity

\[ l_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix} \]

\[ l_1 = \begin{bmatrix} a \\ b \\ c_1 \end{bmatrix} \]
Special Case: 2D Point at Infinity

The intersection of two parallel lines in an image:

\[ l_1 = \begin{bmatrix} a \\ b \\ c_1 \end{bmatrix} \]

\[ l_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix} \]

\[ x = l_1 \times l_2 = (c_2 - c_1) \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} \]
Special Case: 2D Point at Infinity

The intersection of two parallel lines in an image:

\[ l_1 = \begin{bmatrix} a \\ b \\ c_1 \end{bmatrix}, \quad l_2 = \begin{bmatrix} a \\ b \\ c_2 \end{bmatrix} \]

\[ x = l_1 \times l_2 = (c_2 - c_1) \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} \]

This point does not correspond to a finite point in the image because:

\[ x = \begin{bmatrix} -b/0 \\ a/0 \\ 0/0 \end{bmatrix} \]
**Special Case: 2D Point at Infinity**

The intersection of two parallel lines in an image:

\[
\begin{bmatrix}
-x_1 \\
-b \\
c_2 - c_1 \\
a \\
0
\end{bmatrix}
\]

This point does not correspond to a finite point in the image because:

\[
x = \begin{bmatrix}
-b/0 \\
a/0 \\
0/0
\end{bmatrix}
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Parallel lines intersect at the point at infinity:

Point at infinity: \( x_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} \)
Point and Line at Infinity

Parallel lines intersect at the point at infinity:

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Point and Line at Infinity

Plane parallel to image plane

Parallel lines intersect at the point at infinity:

Point at infinity: \( x_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} \)

A point at infinity corresponds to a direction.
Point and Line at Infinity

Parallel lines intersect at the point at infinity:

Point at infinity: \( x_\infty = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix} \)

A point at infinity corresponds to a direction.

All points at infinity lie in the line at infinity:

\[ l_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = Z \quad \therefore x_{\text{ideal}}^T l_\infty = 0 \]

where the surface normal is parallel to the Z axis.