THE MAKING OF
CHEMICAL BROTHERS ‘WIDE OPEN’
Parallel lines in 3D converge to a point in the image.

Indoor point at infinity

MLPS - St. Paul International Airport
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Ground plane

Camera plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

Camera plane

Ground plane
3D Parallel Line Projection

- Ground plane
- Camera plane
- Vanishing point
3D Parallel Line Projection

Vanishing point

Camera plane

Ground plane
3D Parallel Line Projection

1. Parallel lines in 3D meet at the same vanishing point in image.
1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
1. Parallel lines in 3D meet at the same vanishing point in image.
2. The 3D ray passing camera center and the vanishing point is parallel to the lines.
3. Multiple vanishing points exist.
Vanishing point

Multiple vanishing point
Vanishing point

Vanishing line for horizon

Vanishing point

Vanishing line: Horizon
What can vanishing line tell us about me?
What can vanishing line tell us about me?

- Horizon
What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
What can vanishing line tell us about me?

- Horizon
- Camera pitch angle (looking down)
- Camera roll angle (tilted toward right)
Celestial Navigation

Two points at infinity (vanishing points) tells us about where I am.
Parallel 3D planes share the vanishing line.
Different plane produces different vanishing line.
Different plane produces different vanishing line.
How to compute a vanishing point?

Different plane produces different vanishing line.
A 2D line passing through 2D point \((u,v)\): 

\[ au + bv + c = 0 \]

Line parameter: \((a, b, c)\)
Point-Line in Image

A 2D line passing through 2D point \((u, v)\):

\[ au + bv + c = 0 \]

Line parameter: \((a, b, c)\)

\[ au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = l^T x = 0 \]
Point-Line in Image

A 2D line passing through 2D point \((u,v)\):
\[ au + bv + c = 0 \]

Line parameter: \((a,b,c)\)

\[ au + bv + c = 0 \rightarrow \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = l^T x = 0 \]

where \( x = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \) and \( l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \)

2D point \hspace{2cm} \text{Line parameter}
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \]

\[ au_2 + bv_2 + c = 0 \]
Point-Point in Image

A 2D line passing through two 2D points:
\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]
\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where \( x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \quad l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \)
Point-Point in Image

A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]

\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where

\[ x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \quad l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \]

\[ \begin{bmatrix} x_1^T \\ x_2^T \end{bmatrix} l = 0 \]

\[
\begin{bmatrix} a & b & c \\ 2 & 3 & 0 \end{bmatrix} = 0
\]
A 2D line passing through two 2D points:

\[ au_1 + bv_1 + c = 0 \quad au_2 + bv_2 + c = 0 \]

\[ x_1^T l = 0 \quad x_2^T l = 0 \]

where \( x_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} \quad x_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} \quad l = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \)

\[ \rightarrow \begin{bmatrix} x_1^T \\ x_2^T \end{bmatrix} l = 0 \]

\[
\begin{bmatrix}
A & \mid & 0 \\
0 & \mid & 0
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix} = \text{null} \left( \begin{bmatrix} A \end{bmatrix} \right) \quad \text{or} \quad l = x_1 \times x_2
\]
Point-Point in Image

\[ x_1 = [1804;934;1]; \]
\[ x_2 = [1052;1323;1]; \]
Point-Point in Image

GetLineFromTwoPoints.m

\[ x_1 = [1804;934;1] \]
\[ x_2 = [1052;1323;1] \]

\[ l = \text{Vec2Skew}(x_1) \times x_2 \]

Cross product
Point-Point in Image

GetLineFromTwoPoints.m

\[ x_1 = [1804;934;1]; \]
\[ x_2 = [1052;1323;1]; \]
\[ l = \text{Vec2Skew}(x_1) \times x_2; \]

Cross product

\[ l = \begin{bmatrix} -389 \\ -752 \\ 1404124 \end{bmatrix} \]
Point-Point in Image

Cross product with skew-symmetric matrix representation:

\[
\begin{align*}
    \mathbf{a} \times \mathbf{b} &= \begin{bmatrix}
    a_2 b_3 - a_3 b_2 \\
    a_3 b_1 - a_1 b_3 \\
    a_1 b_2 - a_2 b_1
    \end{bmatrix} \\
    &= \begin{bmatrix}
    0 & -a_3 & a_2 \\
    a_3 & 0 & -a_1 \\
    -a_2 & a_1 & 0
    \end{bmatrix} \begin{bmatrix}
    b_1 \\
    b_2 \\
    b_3
    \end{bmatrix} = \begin{bmatrix} a \end{bmatrix} \begin{bmatrix} b \end{bmatrix}
\end{align*}
\]

GetLineFromTwoPoints.m

\[
\begin{align*}
x1 &= [1804;934;1]; \\
x2 &= [1052;1323;1]; \\

l &= \text{Vec2Skew}(x1) \times x2; \\
\text{Cross product}
\end{align*}
\]

Cross product

\[
\begin{align*}
    l &= \\
    &=-389 \\
    &=-752 \\
    &=1404124
\end{align*}
\]
Line-Line in Image

Two 2D lines in an image intersect at a 2D point:

\[ a_1 u + b_1 v + c_1 = 0 \quad a_2 u + b_2 v + c_2 = 0 \]
Two 2D lines in an image intersect at a 2D point:

\[ a_1 u_1 + b_1 v_1 + c_1 = 0 \quad a_2 u_2 + b_2 v_2 + c_2 = 0 \]

\[ l_1^T x = 0 \quad l_2^T x = 0 \]

where \( x = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad l_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \quad l_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \)
Two 2D lines in an image intersect at a 2D point:

\[ a_1u + b_1v + c_1 = 0 \quad a_2u + b_2v + c_2 = 0 \]

\[ \mathbf{l}_1^Tx = 0 \quad \mathbf{l}_2^Tx = 0 \]

where \( \mathbf{x} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \) \( \mathbf{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \) \( \mathbf{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \)

\[ \rightarrow \begin{bmatrix} \mathbf{l}_1^T \\ \mathbf{l}_2^T \end{bmatrix} \mathbf{x} = 0 \]

\[ \begin{bmatrix} A \\ 2x3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \text{null}(A) \]

or \( \mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 \)
Line-Line in Image

GetPointFromTwoLines.m

\[ l_1 = [-398; -752; 1404124]; \]
\[ l_2 = [310; -924; 303790]; \]
\[ x = \text{Vec2Skew}(l_1) \times l_2; \]
\[ x = \frac{x}{x(3)} \]
\[ x = \begin{bmatrix} 1779.0 \\ 925.6 \\ 1 \end{bmatrix} \text{ similar to } (1804, 934) \]
2D Point and Line Duality

The 2D line joining two points:

\[ l = x_1 \times x_2 \]

The intersection between two lines:

\[ x = l_1 \times l_2 \]

Given any formula, we can switch the meaning of point and line to get another formula.
2D Point and Line Duality

The 2D line joining two points:
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Given any formula, we can switch the meaning of point and line to get another formula.
\[ x_2 = T x_1 \leftrightarrow l_2 = T^T l_1 \quad T: \text{Transformation} \]
The 2D line joining two points:

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The intersection between two lines:

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Given any formula, we can switch the meaning of point and line to get another formula.

\[ x_2 = T x_1 \leftrightarrow l_2 = T^T l_1 \quad \text{T: Transformation} \]

\[ \therefore l_1^T x_1 = 0 \]
The 2D line joining two points:

\[ \mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2 \]

The intersection between two lines:

\[ \mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 \]

Given any formula, we can switch the meaning of point and line to get another formula.

\[ \mathbf{x}_2 = T\mathbf{x}_1 \leftrightarrow \mathbf{l}_2 = \mathbf{T}^\top\mathbf{l}_1 \quad T: \text{Transformation} \]

\[ \therefore \mathbf{l}_1^\top\mathbf{x}_1 = (\mathbf{l}_1^\top\mathbf{T}^\top)(T\mathbf{x}_1) = 0 \]
2D Point and Line Duality

The 2D line joining two points:

\[ l = x_1 \times x_2 \]

The intersection between two lines:

\[ x = l_1 \times l_2 \]

Given any formula, we can switch the meaning of point and line to get another formula.

\[ x_2 = T x_1 \leftrightarrow l_2 = T^\top l_1 \quad T: \text{Transformation} \]

\[ \therefore l_1^\top x_1 = (l_1^\top T^\top)(T x_1) = (T^\top l_1)^\top (T x_1) = l_2^\top x_2 \]
The 2D line joining two points:

\[ l = x_1 \times x_2 \]

The intersection between two lines:

\[ x = l_1 \times l_2 \]

Given any formula, we can switch the meaning of point and line to get another formula.

\[ x_2 = T x_1 \leftrightarrow l_2 = T^T l_1 \quad T: \text{Transformation} \]

\[ \therefore l_1^T x_1 = (l_1^T T^1)(T x_1) = (T^T l_1)^T (T x_1) = l_2^T x_2 \]

\[
\begin{bmatrix}
\cos\theta & \sin\theta & 0 \\
-sin\theta & \cos\theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]
2D Point and Line Duality

The 2D line joining two points:
\[ l = x_1 \times x_2 \]

The intersection between two lines:
\[ x = l_1 \times l_2 \]

Given any formula, we can switch the meaning of point and line to get another formula.
\[ x_2 = T x_1 \iff l_2 = T^T l_1 \quad T: \text{Transformation} \]
\[ : l_1^T x_1 = (l_1^T T^1)(T x_1) = (T^T l_1)^T (T x_1) = l_2^T x_2 \]

\[
 x_2 = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} x_1 \longleftrightarrow l_2 = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}^T l_1 = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} l_1
\]
Geometric Interpretation (Point)
Geometric Interpretation (Point)
Geometric Interpretation (Point)

\[ \hat{u} = K^{-1}u \]

Normalized coordinate:
Geometric Interpretation (Point)

Normalized coordinate:

\[ \hat{u}_1 = K^{-1}u_1 \quad \hat{u}_2 = K^{-1}u_2 \]
Geometric Interpretation (Line)

Normalized coordinate:

\[ \lambda \hat{u}_1 = \lambda K^{-1}u_1 \]
\[ \lambda \hat{u}_2 = \lambda K^{-1}u_2 \]

\[ \hat{\mathbf{i}} = \hat{u}_1 \times \hat{u}_2 \]

where \( \hat{\mathbf{i}} = \) ?
Geometric Interpretation (Line)

Normalized coordinate:

\[ \hat{u}_1 = K^{-1}u_1 \quad \hat{u}_2 = K^{-1}u_2 \]

\[ \hat{l} = \hat{u}_1 \times \hat{u}_2 \]

where \( \hat{l} = (K^{-1})^{-T}l = K^Tl \) due to duality
Geometric Interpretation (Line)

Normalized coordinate:

\[ \hat{u}_1 = K^{-1}u_1 \quad \hat{u}_2 = K^{-1}u_2 \]

\[ \hat{I} = \hat{u}_1 \times \hat{u}_2 \]

where \( \hat{I} = \left(K^{-1}\right)^{-T}I = K^Tl \) due to duality

A 2D line in an image defines to a 3D plane passing the camera center:

\[ \hat{I} \rightarrow \pi \]
Geometric Interpretation (Line)

Normalized coordinate:
\[ \hat{u}_1 = K^{-1}u_1 \quad \hat{u}_2 = K^{-1}u_2 \]
\[ \longrightarrow \quad \hat{l} = \hat{u}_1 \times \hat{u}_2 \]
where \( \hat{l} = (K^{-1})^{-T}l = K^Tl \) due to duality

A 2D line in an image defines a 3D plane passing the camera center:
\[ \hat{l} \rightarrow \pi \]

Plane normal:
\[ ? = \lambda \hat{l} \]
Geometric Interpretation (Line)

A 2D line in an image defines to a 3D plane passing the camera center:

\[ \hat{l} \rightarrow \pi \]

Plane normal:

\[ (\lambda_1 \hat{u}_1) \times (\lambda_2 \hat{u}_2) = \lambda \hat{l} \]

Normalized coordinate:

\[ \hat{u}_1 = K^{-1}u_1 \quad \hat{u}_2 = K^{-1}u_2 \]

\[ \hat{l} = \hat{u}_1 \times \hat{u}_2 \]

where \( \hat{l} = \left( K^{-1} \right)^T l = K^Tl \) due to duality
Geometric Interpretation (Line)

3D plane $\pi = \begin{bmatrix} \hat{i} \\ 0 \end{bmatrix}$

Normalized coordinate:

$\hat{u}_1 = K^{-1}u_1 \quad \hat{u}_2 = K^{-1}u_2$

$\vec{\hat{i}} = \hat{u}_1 \times \hat{u}_2$

where $\vec{\hat{i}} = (K^{-1})^{-T}I = K^{T}I$ due to duality

A 2D line in an image defines to a 3D plane passing the camera center:

$\vec{\hat{i}} \rightarrow \pi$

Plane normal:

$(\lambda_1\hat{u}_1) \times (\lambda_2\hat{u}_2) = \lambda\vec{\hat{i}}$

$\therefore \pi = \begin{bmatrix} \vec{\hat{i}} \\ 0 \end{bmatrix}$
Geometric Interpretation (Line-Line)
Geometric Interpretation (Line-Line)
Geometric Interpretation (Line-Line)

2D lines in an image intersect a 2D point corresponding to a 3D ray:

\[ \hat{u} = \hat{l}_1 \times \hat{l}_2 \]
Geometric Interpretation (Line-Line)

2D lines in an image intersect a 2D point corresponding to a 3D ray:

\[ \hat{u} = \hat{l}_1 \times \hat{l}_2 \]

: the 3D ray is perpendicular to two plane normals.
Vanishing Point

Parallel lines:

\[ l_{11} = u_4 \times u_3 \quad l_{12} = u_1 \times u_2 \]
Vanishing Point

Parallel lines:

\[ l_{11} = u_4 \times u_3 \]
\[ l_{12} = u_1 \times u_2 \]
\[ l_{21} = u_4 \times u_1 \]
\[ l_{22} = u_3 \times u_4 \]

Ground plane
Vanishing Point

Parallel lines:

\[ l_{11} = u_4 \times u_3 \]
\[ l_{12} = u_1 \times u_2 \]
\[ l_{21} = u_4 \times u_1 \]
\[ l_{22} = u_3 \times u_4 \]

Vanishing points:

\[ \mathbf{x}_1 = l_{11} \times l_{12} \]
\[ \mathbf{x}_2 = l_{21} \times l_{22} \]
Vanishing Point

Parallel lines:
\[ \mathbf{l}_{11} = \mathbf{u}_4 \times \mathbf{u}_3 \quad \mathbf{l}_{12} = \mathbf{u}_1 \times \mathbf{u}_2 \]
\[ \mathbf{l}_{21} = \mathbf{u}_4 \times \mathbf{u}_1 \quad \mathbf{l}_{22} = \mathbf{u}_3 \times \mathbf{u}_4 \]

Vanishing points:
\[ \mathbf{v}_1 = \mathbf{l}_{11} \times \mathbf{l}_{12} \quad \mathbf{v}_2 = \mathbf{l}_{21} \times \mathbf{l}_{22} \]

Vanishing line:
\[ \mathbf{l} = \mathbf{v}_1 \times \mathbf{v}_2 \]
Vanishing Point

\[ l_{11} = \text{GetLineFromTwoPoints}(m_{11}, m_{12}); \]
\[ l_{12} = \text{GetLineFromTwoPoints}(m_{13}, m_{14}); \]
\[ l_{21} = \text{GetLineFromTwoPoints}(m_{21}, m_{22}); \]
\[ l_{22} = \text{GetLineFromTwoPoints}(m_{23}, m_{24}); \]

\[ v_1 = \text{GetPointFromTwoLines}(l_{11}, l_{12}); \]
\[ v_2 = \text{GetPointFromTwoLines}(l_{21}, l_{22}); \]

\[ \text{vanishing\_line} = \text{GetLineFromTwoPoints}(x_1, x_2); \]

\text{ComputeVanishingLine.m}
Geometric Interpretation of Vanishing Line
Geometric Interpretation of Vanishing Line

Camera plane

Ground plane
Geometric Interpretation of Vanishing Line

Camera plane

Ground plane
Geometric Interpretation of Vanishing Line

Camera plane

Plane of vanishing line

Ground plane

Plane of vanishing line

Ground plane

Side view
Geometric Interpretation of Vanishing Line

- Side view
- Ground plane
- Plane of vanishing line
- Height
- Side view
- Ground plane
Where was I (how high)?
Where was I (how high)?

Taken from my hotel room (6th floor)  
Taken from beach
Where was I (how high)?

Taken from my hotel room (6th floor)

Taken from beach
First person video

Cylindrical projection

Height of the camera wearer