CSci 5511 Spring 2003

Key for 2nd Midterm Exam

1. *15 points*
   Use resolution to prove that $(S \vee R)$ is entailed by the following set of propositional expressions:

   1. $\neg(\neg Q) \wedge Z$
   2. $\neg W$
   3. $(\neg W \wedge Q) \Rightarrow (\neg P)$
   4. $(W \wedge Z) \Rightarrow S$
   5. $Q \Rightarrow (S \vee P)$
   6. $(P \wedge Q) \Rightarrow R$

   Answer: we convert to CNF and add the negated goal

   1a. Q

   1b. Z

   2. $\neg W$

   3. $W \vee \neg Q \vee \neg P$

   4. $\neg W \vee \neg Z \vee S$

   5. $\neg Q \vee S \vee P$

   6. $\neg P \vee \neg Q \vee R$

   $\neg Ga.$ $\neg S$

   $\neg Gb.$ $\neg R$

   We resolve $\neg Gb.$ with 6. and obtain

   7. $\neg P \vee \neg Q$

   then 7. with 5.

   8. $\neg Q \vee S$

   and 8. with 1a.

   9. S

   and, finally, 9. with $\neg Ga$ to obtain the empty clause.

2. *20 points*

(a) Write the following sentences in predicate calculus:

1. Every student has taken at least one computer science course.
   $\forall x\, Student(x) \Rightarrow \exists y\, CScourse(y) \wedge Take(x, y)$
2. A student has taken at most one computer science course.
   $\exists x\, Student(x) \wedge [\forall y\, CScourse(y) \wedge Take(x, y) \Rightarrow [\forall z CScourse(z) \wedge z \neq y \Rightarrow \neg Take(x, z)]]$
   or, equivalently,
   $\exists x\, Student(x) \wedge [\forall y\, CScourse(y) \wedge Take(x, y) \Rightarrow [\forall z CScourse(z) \wedge Take(x, z) \Rightarrow z = y]]$
3. Every student has been in every building on campus.
   $\forall x \forall y\, Student(x) \wedge Building(y) \Rightarrow Visited(x, y)$
4. There is a student who has been in every room of at least one building on campus.
   $\exists x\, Student(x) \wedge \exists y\, Building(y) \wedge [\forall z\, Room(z, y) \Rightarrow Visited(x, z)]$
5. Every student has been in at least one room of every building on campus.
   $\forall x \forall y\, Student(x) \wedge Building(y) \Rightarrow \exists z\, Room(z, y) \wedge Visited(x, z)$

(b) transform the expressions you wrote in part (a) to CNF.

1a $\neg Student(x) \vee CScourse(S(x))$

1b $\neg Student(x) \vee Take(a, S(x))$

2 $Student(S0) \wedge \neg CScourse(y) \vee \neg Take(S0, y) \vee \neg CScourse(z) \vee z = y \vee \neg Take(S0, z)$

3 $\neg Student(x) \vee \neg Building(y) \vee Visited(x, y)$

4a $Student(S1)$

4b $Building(S2)$

4c $\neg Room(z, S2) \vee Visited(S1, z)$

5a $\neg Student(x) \vee \neg Building(y) \vee Room(S3(x, y), y)$

5b $\neg Student(x) \vee \neg Building(y) \vee Visited(x, S3(x, y))$

(c) Does 5 entail 3? If yes, prove it by resolution (adding additional expressions if needed). If not, explain why not.

Answer: to answer we need to negate 3, add it to a knowledge base which contains 5 and any other expression we would have to add, and prove a contraddiction.

Given 3. $\forall x \forall y\, Student(x) \land Building(y) \Rightarrow Visited(x, y)$
we negate it and obtain
3a $Student(S4)$
3b $Building(S5)$
3c $\neg Visited(S4, S5)$
We need to add a new expression
6. $\forall x \forall y \forall z\, Room(z, y) \land Visited(x, z) \Rightarrow Visited(x, y)$
which in CNF is
6 $\neg Room(z, y) \lor \neg Visited(x, z) \lor Visited(x, y)$
We resolve 5a with 3a unifying {x/S4} and obtain
7 $\neg Building(y) \lor Room(S3(S4, y), y)$
7 is resolved with 6 using unification {z/S3(S4,y)} and we obtain
8 $\neg Visited(x, S3(S4, y)) \lor Visited(x, y) \lor \neg Building(y)$
which is resolved against 3c with unification {x/S4, y/S5} to obtain
9 $\neg Visited(S4, S3(S4, S5)) \lor \neg Building(S5)$
which we resolve with 5b unifying {x/S4, y/S5} to obtain
10 $\neg Student(S4) \lor \neg Building(S5)$
We resolve it with 3a to obtain
11 $\neg Building(S5)$
which, finally, resolved with 3b produces
12 NIL

3. *10 points*
Specify if each of the following expressions represents correctly the corresponding English statement. If not explain why not and correct it.

   1. Every cat owner loves all animals.
   $\forall x \forall z\, [\exists y Cat(y) \Rightarrow Owns(x, y)] \Rightarrow Loves(x, z)$
   Answer: The sentence says if someone owns a cat, that person loves all things. We need $\land$ instead of $\Rightarrow$ and we need to specify that z is an animal. The correct sentence is:
   $\forall x \forall z\, [\exists y\, Cat(y) \land Owns(x, y) \land Animal(z)] \Rightarrow Loves(x, z)$
   2. No person would harm a cat.
   $\forall x \forall y\, Person(x) \land Cat(y) \land \neg Harm(x, y)$

Answer: we need ⇒ instead of ∧
$\forall x \forall y \, Person(x) \wedge Cat(y) \Rightarrow \neg Harm(x, y)$

4. *15 points*
   You are given the following pairs of clauses where upper case letters indicate constants, lower case letters indicate variables, functions, or predicates. Consider each pair independently of the others. In each pair variables with the same name are meant to be the same variable. For each of the pairs specify if they can be resolved. If yes show the results of the unification process, if not explain why.

   1. p(B, C, x, z, f(A, z, B)) and not p(y, z, y, C, w)
      Answer: {y/B,z/C,x/B,w/f(A,C,B)}
   2. r(f(y), y, x) and not r(x, f(A), f(v))
      Answer: {x/f(y),y/f(A),v/f(A)}
   3. q(f(A,x), x) and not q(f(z,f(z,D)), z)
      Answer: {z/A,x/f(A,D), FAIL}

5. *15 points*
   Show the backed-up up values for all the nodes in the following game tree and show the branches that are pruned by alpha-beta. For each branch pruned, explain briefly why alpha-beta can prune it. Follow the convention used in the textbook to examine the branches in the tree from left to right.