3D Point Cloud Segmentation Using Topological Persistence

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The Goal

Segment noisy 3D point clouds by utilizing persistent homology theory.

Introduction

Segmentation algorithms aim to divide a point cloud into constituent clusters that are perceptually meaningful and serve as a vital preprocessing step in robotic systems. The performance of high level tasks such as object localization, feature extraction, and classification are dependent upon the quality of the segmented data. Large datasets produced by low-cost RGB-D sensors have attracted the attention of researchers towards developing efficient algorithms and data structures for point cloud processing. The key contributions of our work are:

- The introduction of *persistent homology* to the area of point cloud processing for 3D perception
- A novel approach for segmenting 3D point clouds based on *topological persistence*

Mathematical Background

The discrete space that we work in uses simplices as building blocks:

- A d-simplex σ is the convex hull of $d+1$ affinely independent vertices $v_0, \dots, v_d \in \mathbb{R}^n$. We denote $\sigma = \text{conv}\{v_0, \dots, v_d\}$ where the dimension of σ is d.
- A face of σ is conv$S$ where $S \subset \{v_0, \dots, v_d\}$ is a subset of the $d+1$ vertices.

Simplices can be joined to form simplicial complexes:

- A simplicial complex K is a finite collection of simplices such that if $\sigma \in K$ and τ is a face of σ, then $\tau \in K$ and if $\sigma, \sigma' \in K$ then $\sigma \cap \sigma'$ is either empty or a face of both σ and σ'.
- Given a set of points $X = \{x_0, \dots, x_m\} \in \mathbb{R}^n$ in Euclidean n-space and a fixed radius ϵ, the Vietoris-Rips complex of X is an abstract simplicial complex whose d-simplices correspond to unordered $(d+1)$-tuples of points that are pairwise within ϵ distance of each other, Figure 1.

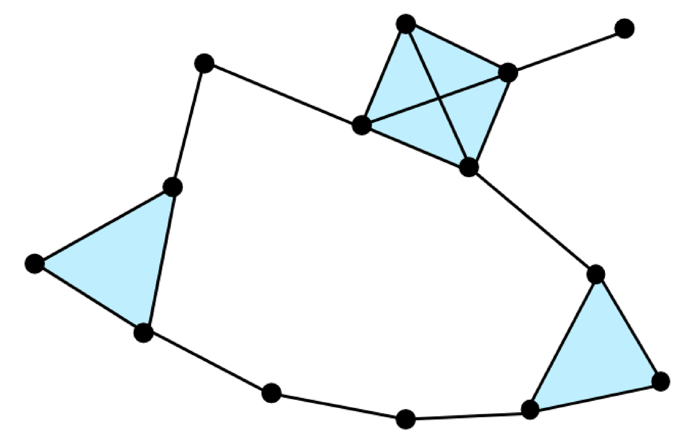

Figure 1: A Vietoris-Rips complex formed by joining together simplices of dimension 0 (vertices), 1 (edges), and 2 (triangles).

Homology is an algebraic means to measure the number of components and holes in a topological space. The dth homology group of K is expressed as

$$H_d(K) = \ker\partial_d / \text{ im } \partial_{d+1},$$

where $\partial_d : C_d(K) \to C_{d-1}(K)$ is a linear mapping of the vector space $C_d(K)$ with basis consisting of oriented d-simplices in K to its oriented faces of dimension $d-1$. Persistent homology is based on the concept that topological features detected over a range of varying scales are more likely to represent true features of the underlying dataset rather than artifacts of noise, poor sampling, or a particular choice of parameters [1, 2].

Problem Statement

Let X be a topological space where $X = \{x_0, \dots, x_m\} \in \mathbb{R}^3$ and $x_0, \dots, x_m$ are the points in a point cloud captured by an RGB-D sensor. To find the persistent homology of X we first represent the topology of the space with a Vietoris-Rips complex. Next, we compute the zero-dimensional homology group $(d = 0)$ of X which corresponds to the number of connected components. Finally, we extract out the connected component clusters.

Simplicial Complex Construction

Given an input point cloud we form the Vietoris-Rips complex, $K = \{\sigma \subset \{x_0, \dots, x_m\} \mid \text{dist}(x_i, x_j) \le \epsilon, \forall x_i \ne x_j \in \sigma\}$, where dist is the Euclidean metric and the vertices of σ are pairwise within distance ϵ. We observe that the 1-skeleton of the Vietoris-Rips complex is sufficient to compute the zeroth homology group of the space.

Computing Persistent Homology

We construct a *filtration* that stores the complexes across the entire range of possible values of a scale parameter. By allowing us to exclude short lived topological features, we can control how long a topological feature has to exist in the filtration before we consider it significant.

Connected Component Extraction

A disjoint-set data structure is initialized by making each 0-simplex its own set (all points are born at time zero). While performing the filtration, connected 0-simplices are merged within the data structure. At the end, we find the connected components based on the sets of points that are joined to 0-dimensional simplices of infinite persistence.

Experimental Results

The experiments are conducted using the Object Segmentation Database [3]. For each point cloud, we show the filtered representation prior to segmentation followed by the color coded clusters extracted after segmentation, Figure 2. The filtration is performed for 10 steps up to the maximum distance value set for the complex. The barcodes show the lifespan of the generators of homology.

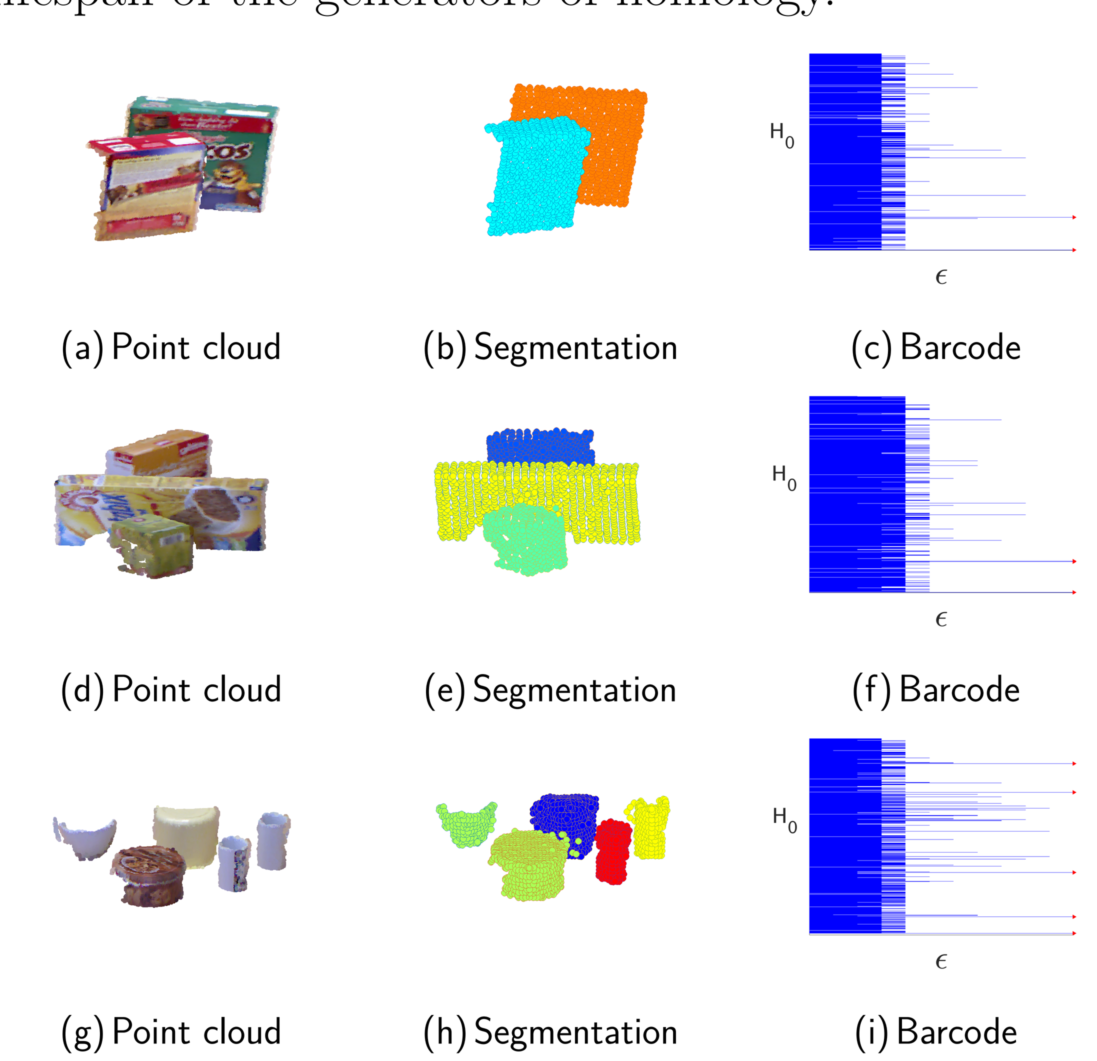

Figure 2: Segmented point clouds using topological persistence.

Conclusion and Future Work

This work develops algorithms and data structures for segmenting 3D point clouds based on computing topological persistence. Our purpose is to not only introduce a new way of performing segmentation, but also to introduce the idea of topological persistence into the area of point cloud processing for 3D perception tasks. Future work includes region-based segmentation, cubical complexes for voxelized datasets, and speeding up of the filtration process.

For Further Information

For the details of our work:

- W.J. Beksi and N. Papanikolopoulos. "3D Point Cloud Segmentation Using Topological Persistence", IEEE International Conference on Robotics and Automation (ICRA), Stockholm, Sweden, 2016.

Preprints and this poster can be downloaded from:

- http://www-users.cs.umn.edu/~beksi

A video visualizing the topological segmentation process can be seen on the Center for Distributed Robotics YouTube channel:

- https://www.youtube.com/user/distrob

References

[1] H. Edelsbrunner, D. Letscher, A. Zomorodian, "Topological Persistence and Simplification", *Discrete and Computational Geometry*, vol. 28, no. 4, pp. 511-533, 2002.

[2] A. Zomorodian, G. Carlsson, "Computing persistent homology", *Discrete and Computational Geometry*, vol. 33, no. 2, pp. 249-274, 2005.

[3] A. Richtsfeld, "The Object Segmentation Database (OSD)", 2012. [Online]. Available: http://www.acin.tuwien.ac.at/?id=289

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