(IP) Solving sparse-dense linear least-squares problems

Miroslav Tůma [1]

Jennifer A. Scott [2]

Large-scale linear least-squares (LS) problems occur widely in practice. They arise both in their own right and as subproblems of more general nonlinear problems. Moreover, the normal equations that are naturally connected to LS problems can shed light on solving the Schur complement systems that are routinely faced in many applications.

Our focus in this talk is on gaining a better understanding of the case when the sparse LS problem contains additional coupling terms represented by one or more dense rows. It has long been recognised that the effectiveness of sparse matrix techniques for directly solving such problems is severely limited by the presence of dense rows. We consider, in particular, the following $m \times n$ $(m > n)$ LS problem

$$\min_x \|Ax - b\|_2 = \min_x \left\| \begin{pmatrix} A_s \\ A_d \end{pmatrix} x - \begin{pmatrix} b_s \\ b_d \end{pmatrix} \right\|_2,$$

in which each row of the $m_d \times n$ block A_d is considered to be dense and A_s is $m_s \times n$ with $m_s \gg m_d \geq 1$; the vector b is partitioned conformally. These LS problems represent a simple motivating case for more general situations that appear in practice where a dense substructure hidden in the problem may prohibit efficient solution.

There are a number of ways to tackle this problem. Classical approaches based on direct methods are summarized in the monograph [2]; see also [4]. More recently, methods based on preconditioned iterative methods [5] or Schur complement reduction [6] have been considered. In this talk, we discuss a number of approaches. One specific approach discussed here is based on matrix stretching in which dense rows are replaced by submatrices with much sparser rows [3, 1]. Experimental problems demonstrate not only the strengths of stretching but also some of its limitations [8]; these point towards future research directions.

References

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[1] Faculty of Mathematics and Physics, Charles University, Prague

[2] The University of Reading and STFC Rutherford Appleton Laboratory, UK

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