(IP) Block Preconditioning for Implicit Runge-Kutta Methods for Time-Dependent PDE Problems

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Many important engineering and scientific systems require the solution of time-dependent PDE systems. Many of these sytems have specific stability needs in order to compute realistic solutions such as needing A-stable or L-stable methods. For example, the Eddy Currents Equation is a stiff parabolic PDE with a possible rapid transient, and benefits from an L-stable method. Similarly, the incompressible Navier–Stokes equations for fluid flow are differential algebraic equations (DAE), and can be viewed as infinitely stiff. Therefore, L-stable time stepping methods can be beneficial there as well.

Certain classes of implicit Runge-Kutta (IRK) time-stepping methods, such as the Radau I and Radau II methods provide L-stability, but one price of using an IRK method is needing to solve large linear systems at each time step. Suppose, for example, our PDE has been linearized and discretized with N degrees of freedom. Using an s-stage IRK method leads to an $sN \times sN$ linear system that must be solved at each time step. These systems are block-$(s \times s)$ systems, where each block is $N \times N$.

In this talk, we investigate preconditioners for such systems. In some cases, we can take advantage of known structure in the subblocks. For example, in the time-dependent incompressible Navier–Stokes equations, each subblock is related to a linear system from the steady-state fluid flow equations, for which there are several effective preconditioners.

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