Interaction, Competition and Innovation in a Service-Oriented Internet: An Economic Model

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***Abstract*—This paper presents a new economic approach for studying competition and innovation in a complex and highly interactive system of network providers, users, and suppliers of digital goods and services (i.e., service providers). It employs Cournot and Bertrand games to model the competition among service providers and among network providers, respectively, and develops a novel unified model to capture the interaction and competition among these players in a "service-oriented" Internet. Incentives for service and network innovation are studied in this model.**

I. Introduction

As the Internet evolves and diffuses through society, economic factors are often more important than technological ones in determining what services are offered, and how they are priced. But the economics of the Internet is far more mysterious than the underlying technology. That is true even at the level of network providers, where the basic connectivity is reasonably well known, but the business relationships that give rise to observed connections, and the incentives that led to them, are mostly hidden from view. When we come to services offered over the Internet, the complexity increases, and our knowledge drops even further. And the literature in this area is still very limited, in spite of the extensive interest that exists.

In this paper we consider the Internet as a service delivery platform (i.e., we disregard its other roles, such as in providing connectivity among users, etc.), and study the relationships among the entities that provide transport (*network providers*, in our language), those that provide services (*service providers*), and *users*. We assume a certain industry structure (with network providers completely separate from service providers), and several rules imposed by regulators (such as some non-discrimination conditions, and possibly even some price regulation), and some other common market features (such as basic network connectivity being provided to users on a flat rate basis). This constrains the problem enough to provide opportunity to build models that are tractable and yet reflect what is observed in the marketplace, and are rich enough to show interesting dynamics.

We propose a simple economic model of the interaction and competition among service providers, network providers and users. Using this model, we explore how competition affects the network and service providers, and in particular how to maximize the incentives for innovation on the part of network providers and service providers. Our main tools come from the standard economic literature on *Bertrand* and *Cournot* competition (see Section II for a brief overview of these concepts). While both types of competition are well known in economics, *one of the key novelties and contributions of our paper is combining these two different types of games in a single unified framework to capture the co-dependence or interaction between service and network providers.* With our assumptions, we model the competition between service providers using Cournot games, and the competition between network providers using Bertrand games.

The two types of competition (or games) are tied together in a *two-stage Stackelberg game* where service providers determine the optimal (equilibrium) amount of services each produces/offers to meet user demands, and network providers determine what the optimal (equilibrium) prices to charge service providers for transporting the accompanying services (or rather, traffic associated with them). We are able to explicitly solve this unified Cournot-Bertrand model, and thereby study the effects of competition between service providers and competition between network providers on the overall equilibrium market demand/supply and various prices. Furthermore, it enables us to investigate and quantify incentives for service and network providers to innovate and further spur the market demand for services.

Because of the simplifying assumptions, our economic model clearly does not capture the intricate and complex relationships and industry structures that exist in the real Internet, and represents only a *modest* attempt in analyzing and understanding these relationships in a formal economic setting. Nonetheless, our work provides a useful and *tractable* model to generate some *qualitative* insights into these relationships. To our best knowledge, our paper is perhaps the first attempt to explicitly model the interaction, competition and innovation among service providers, network providers and users in a "service-oriented" Internet. As more services–not only content (news, music, videos, etc.), but also software, computing and storage resources (as in *cloud computing*)–are being offered online, we believe that understanding the economic factors that affect the interacting, co-dependent and yet competing relationships among various players in this service-oriented Internet is of critical importance. In light of the recent initiatives in the research community for "clean-slate" designs of future Internet architectures, models for assessing the economic viability of new network architectures are especially needed. We hope our work can inspire more studies to follow.

The remainder of this paper is organized as follows. In Section II we describe the problem setting, additional model assumptions, background and related work. The basic economic model and its equilibrium solution are presented in Section III, and are used in Section IV to study incentives for innovation. Section V provides numerical examples to illustrate the results. Section VI concludes the paper.

II. Problem Setting and Related Work

A. Problem Setting and Basic Assumptions

We consider the Internet as a service delivery platform, and assume three *separate* types of entities: *service providers*, *network providers* and *users*. Fig. 1 schematically depicts the relations among the three entities. Clearly, users are the key drivers in the relations among the three. Users pay a fixed monthly fee to their network providers for basic connectivity, which enables them to access various application services. While users pay for basic connectivity through a flat rate[1], we assume that access to services such as music or video, or cloud computing, is *not* free (and so, in particular, is not paid for through advertising). Instead, users pay for those services to the service providers. The payment might be per unit of service, e.g., $0.99 per song, or might be on a monthly subscription basis for a newspaper or music site. But in some rough sense users' fees to service providers are proportional to those users' volumes of consumption of those services. Service providers derive their revenues from user fees, and have to pay for the costs of creating their offerings, as well as for their transport to the users. Network providers derive their revenues from the flat monthly fees of their users, and the usage-sensitive charges for transport, and have to pay for their infrastructure.

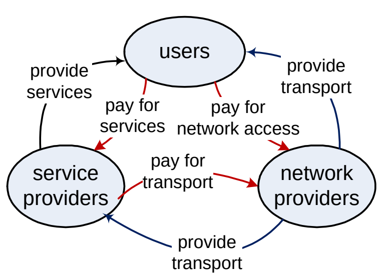

Fig. 1. Relations between network providers, service providers and users.

Without loss of generality, in our model we assume that there are only two service providers who offer the same type of services (in the economic parlance, they are *substitutable goods*) and compete between themselves for user accesses. There are also only two network providers, each connecting their respective users to the two service providers and competing between themselves for transporting services to users. For simplicity, we assume that each user stays with the same network provider throughout (e.g., due to geographical locations and local incumbent monopoly, or other factors)[2]. Fig. 2 depicts the flow of commodities (namely, services and the associated traffic) and prices. Note that this is just a schematic, and omits many of the quantitative relations among network and service providers. Those are given by the key demand relation eq.(1) in the next section. As in the current Internet, the action of any one of the actors, say in improving the transmission infrastructure, or in changing a price, affects all other actors, and leads to interesting dynamics.

B. A Quick Economic Primer and Model Justification

In economics, *demand (function/curve)* is defined as a function between the price of a commodity (*services* in our model) and the quantity of the commodity (the amount of service in our model) that consumers/users are willing and able to purchase at the given price [3]. Likewise, *supply (function/curve)* is defined as a function between the price of a commodity and the quantity of the commodity that a firm (a service provider in our case) is able to produce at the given price. Demand curves are used in economic models to estimate behaviors in *competitive* markets. At the *equilibrium price*, demand and supply are equal, and the *equilibrium quantity* (the amount of service) will be produced and consumed at the equilibrium price. In this paper, we will use this relation between the price that service providers charge users for services and the aggregate volume of services they collectively produce or rather offer to (implicitly) capture the overall user demand for services.

We use two different types of standard economic games in our model: *Cournot competition* between service providers, and *Bertrand competition* between network providers. In economics [4], *Cournot competition/game* is a model of competition in which firms (service providers in our case) *compete on quantity* (i.e., the volume of services) they produce–which they decide independently and at the same time– based on their cost of production, represented by the *marginal cost*, or the cost for producing one additional unit of service. In a Cournot game, *the market price is set at a level such that demand equals the total quantity produced by all firms.* In other words, the service price is determined by the supply and demand for services in the market. In contrast, in a *Bertrand competition/game* firms (network providers in our case) *compete on price*–with each one choosing its price independently (i.e., without collusion) and at the same time–and supply the quantities demanded at those prices.

In our model we use a Cournot game to characterize the competition between two service providers, as we assume they produce *substitutable* services (e.g., music or video downloads), and the price of these services is essentially determined

[1]Although flat rates have been denigrated in the economic literature, they have always been popular with the public, and can be justified as advantageous for network providers using formal models [1].

[2]In other words, users do not change network providers, hence we do not model the competition between network providers for users. We make this simplifying assumption, since our main focus is on interaction between service providers and network providers and competition between service providers as well as between network providers. We note that this competition and the accompanying user dynamics, e.g., joining or leaving the network, are studied [2] in the context of two competing network architectures.

by competition and market supply and demand. In other words, due to competition, different service providers have limited ability to choose their own prices, and in equilibrium, they converge to the same price and consequently determine the equilibrium amount of services each to produce or offer. On the other hand, we use the Bertrand game to characterize the competition between two network providers, as they do *not* directly produce services, but instead transport whatever amount of services produced by service providers to their respective users. They *can* indirectly influence the amount of services produced by service providers, thus the amount (of traffic) they have to transport, by varying the prices they charge them. Hence network providers compete on the network transport prices/fees they charge the service providers to influence or attract the amount of services they transport.

As in standard Cournot/Bertrand games, we assume a *linear demand function* (i.e., the service price is a linear function of the amount of services produced by service providers), which makes both Cournot and Bertrand games easier to solve[3]. In addition, since we do not explicitly take into account the competition between network providers for users, in our model we assume that the user network access charge (denoted as p in our model) is an exogenous variable that can be determined by the market, or a third party, e.g., a policy marker, to maximize certain global/market objective. We do take into account this network access charge in the demand function (cf., eq.(1)), and explore its effect on the equilibrium quantities and prices.

C. Related Work

Network economics has been a very active area of research. While there are many papers on network pricing (see, e.g., [1], [5]–[7] among many others), or more broadly, network economics (see, e.g., [8], [9] for early papers, or the recent proceedings of NetEcon workshops, or recent issues of the *Review of Network Economics* journal), there are relatively few papers focusing on economic models for network architectures and services. Network neutrality is a network architecture-related issue, where various models have been proposed by both economists and computer scientists (see e.g., [10]–[13]). In [14] Gaynor and Braden propose to apply the theory of real option to study flexibility and openness of network architectures. Also worth noting is the recent work [2], where economic models are used to study the competition dynamics between an incumbent technology/architecture and a new entrant technology/architecture. In [15] Scotchmer addresses the broad topic of innovation and incentives, whereas the authors in [16] study the issue of competition and innovation in the specific context of network monitoring and contracting system.

[3]Although more complex demand functions may be used in our model, we believe they do not fundamentally alter the *qualitative* results obtained in the paper.

III. Formal Model and its Solution

A. The Model

As depicted in Fig.2, there are two network providers, denoted by N_i, $i = 1,2$, and two service providers, S^j, $j = 1,2$. Throughout this paper we use the convention that *subscripts* denote *network* providers, whereas *superscripts* denote *service* providers. The user (base), U_i, of network provider N_i is assumed fixed. For conciseness, we will use the singular term "user" to collectively refer to all users of one network provider, and treat the collection of them as if they were a single user. Each user U_i, $i = 1,2$, pays a price of p (e.g., a monthly network access charge) to access the network, and buys services from either one of the two service providers, paying a price of q *per unit of service*. Network provider N_i charges service provider S^j a price of r_i to transport *one unit of service* (or rather the associated traffic) between S^j and U_i. Note that while the (network) transport prices r_1 and r_2 charged by the two network providers may be different, each network provider *does not* charge different prices to different service providers. (Hence a form of *network neutrality* is assumed in our model.) We also note that in our model both q and r_i represent "volume-based" pricing, while p is a flat-rate price.

The overall user demand for services is a function of both the network access charge p and the service price q. Hence when p or q increases, user demands for services decrease. Let $x_i^j, i, j = 1,2$,denote the amount of service produced by service provider S^j and consumed by user U_i. The relation between the service price q, the network access charge p and the service quantities ("supplies"), x_i^j, is assumed to be given by the following (inverse) demand function[4]:

$$q = 1 - \beta p - \sum_{j=1}^{2}\sum_{i=1}^{2}\frac{1}{\gamma_i^j}x_i^j. \tag{1}$$

We first note that eq.(1) implicitly assumes that the service price is normalized to be within the range [0,1]; hence service quantities and other parameters are appropriately normalized. The parameter $\beta(\geq 0)$ in eq. (1) captures the effect of the network access charge p on the overall user demand, viz., the "user demand sensitivity" to p. The parameters $\gamma_i^j(\geq 0), i, j = 1,2$, capture the combined "abilities" of service provider S^j and network provider N_i to offer and transport services to meet the user demand.

The role of γ_i^j's can be intuitively understood as follows: $\gamma_i^j = -\partial x_i^j/\partial q$, so that γ_i^j represents the proportional "market share" of an increase in user demand that service provider S^j and network provider N_i can jointly capture by increasing the service supply (by ∂x_i^j amount) to user U_i of network provider

[4]As mentioned earlier, we assume that at market equilibrium, supply equals demand. Eq. (1) in fact represents the service price q as a function of the supply, i.e., service quantities, x_i^j's, offered by the service providers, S^j, $j = 1,2$. In other words, $X := \sum_{j=1}^{2}\sum_{i=1}^{2} x_i^j$ represents the total market supply, and thus the total market demand. Eq.(1) and its parameters attempt to capture the intricate relation that exists between (market-determined) service price and how service and network providers alike adjust their offering to meet user demand (see the discussion below eq. (1)).

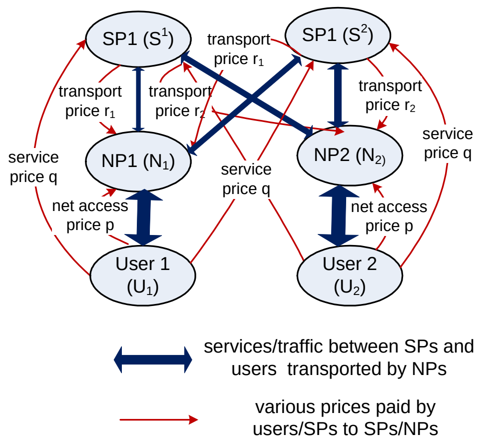

Fig. 2. Model illustration: flows of prices and services/traffic.

N_i when the service price q drops by ∂q. Hence the larger γ_i^j is, the better S^j (and N_i)'s ability to increase its supply and gain market share. In Section IV, we use the parameters γ_i^j to study *incentives for network and service innovation*. For example, by deploying new technology and upgrading its network infrastructure—as reflected by increasing γ_i^j by $\eta_i (> 0)$, a network provider can increase overall user demand (for the same pricess p and q). Likewise, through service innovation, again as reflected by increasing γ_i^j by $\eta^j (> 0)$, a service provider can increase its overall market share of user demand through its improved service supply.

The two service providers engage in a Cournot competition. Given a service price q as in eq. (1), S^1 and S^2 compete to determine the "optimal" amount of services, $x_i^j, i = 1, 2$, to offer users of both networks so as to maximize their (service providers') respective profits. Let s^j denote the *marginal cost* of S^j (for producing one unit of service). The profit generated by S^j for supplying an amount x_i^j of services to user U_i is given by

$$\Pi_i^{S^j} := (q - r_i - s^j) x_i^j, \tag{2}$$

where r_i is the transport price that N_i charges for transporting the traffic associated with *one unit of service*. Let $X^j := x_1^j + x_2^j$ denote the total amount of service produced by S^j. The total profit of S^j for supplying users of both network providers is then

$$\Pi^{S^j} := \Pi_1^{S^j} + \Pi_2^{S^j} = (q - r_1) x_1^j + (q - r_2) x_2^j - s^j X^j. \tag{3}$$

In contrast, network providers engage in a *Bertrand competition*. For $i = 1, 2$, define $Y_i := x_i^1 + x_i^2$, the total amount of services produced by S^1 and S^2 and consumed by user U_i of N_i. In other words, Y_i is the amount of services (or the accompanying traffic) that network provider N_i must transport between its user and the service providers. Network provider N_i does not "produce" Y_i; it only indirectly controls it by adjusting the price r_i it charges the two service providers. Let n_i denote the *marginal cost* of N_i (for transporting the traffic associated with one unit of service). The total profit of N_i is given by

TABLE I
SUMMARY OF MODEL PARAMETERS AND VARIABLES

List of Parameters		
Parameters	β	user demand sensitivity to p
	γ_i^j	(joint) "market share" of S^j & N_i
	$\gamma^j = \gamma_1^j + \gamma_2^j$	total "market share" of S^j
	$\gamma_i = \gamma_i^1 + \gamma_i^2$	total "market share" of N_i
List of Variables		
Quantities	x_i^j	service of S^j consumed by U_i
	$X^j = x_1^j + x_2^j$	total service produced by S^j
	$Y_i = x_i^1 + x_i^2$	total service transported by N_i
Prices	q	user service price
	r_i	network transport price (by N_i)
	p	user network access charge

$$\Pi^{N_i} := (r_i - n_i) Y_i + p, \; i = 1, 2, \tag{4}$$

where the first term in the right hand side represents the net profit for transporting service traffic, and the second term is the price paid by the user for network access. In a Bertrand game, the two network providers, N_i, $i = 1, 2$, compete by determining the "optimal" price r_i each charges the service providers to maximize their respective profit.

Combining the Cournot competition between service providers, and the Bertrand competition between network providers, produces a *Stackelberg* game consisting of a Cournot (sub-)game and a Bertrand (sub-)game. The parameters and variables used in the model are summarized in Table I. In the next subsection we show how the model can be solved.

B. Solving the Model

We solve the model using a two-stage procedure. In the first stage, given the network transport prices r_i's, the service providers S^1 and S^2 compete in a Cournot game. The Nash equilibrium state of the Cournot game yields the "optimal" equilibrium service quantities, x_i^j, $i, j = 1, 2$, and service price q, all as functions of p, r_i's. In the second stage, the network providers N_1 and N_2 compete in a Bertrand game to determine the network transport prices, r_i's, so as to maximize their respective profit. Substituting r_i's into the (optimal) x_i^j's obtained in the Cournot game, we obtain the final optimal equilibrium service quantities, x_i^j's, in terms of p (and the model parameters). Using the resulting expression, we investigate how the network access price p affects the overall demand, and set the optimal price p^* so as to maximize the total *social welfare*. The solution steps are presented in greater details below.

Stage 1: Solving the Cournot Game. Plugging eq. (1) into eq. (2), we have

$$\Pi_i^{S^j} := \sum [(1 - \beta p - \sum_{j=1}^{2} \sum_{i=1}^{2} \frac{1}{\gamma_i^j} x_i^j) - r_i - s^j] x_i^j \tag{5}$$

which is a quadratic function in x_i^j. From standard game theory, *(Nash) equilibrium* is achieved when $\partial \Pi_i^{S^j} / \partial x_i^j = 0$,

$\forall i, j = 1, 2$. This leads to a set of four linear equations, and their solution is expressed in the following lemma.

Lemma 1 (Solution of Cournot Game): The competition of the two service providers modeled as a Cournot game leads to the following equilibrium state: for $j = 1, 2$ and $i = 1, 2$,

$$\hat{x}_i^j = \frac{\gamma_i^j}{5}[(1-\beta p) - 2(r_1 + r_2 + s^1 + s^2) - r_i - s^j]. \quad (6)$$

Furthermore, the equilibrium service price is

$$\hat{q} = \frac{1}{5}(1-\beta p) + \frac{2}{5}(s^1 + s^2 + r_1 + r_2). \quad (7)$$

■

From eq. (6), we can obtain the total (equilibrium) amount of services, $\hat{X}^j = \hat{x}_1^j + \hat{x}_2^j$, produced by service provider S^j, and its corresponding total profit. The total (equilibrium) market demand/supply, $\hat{X} = \sum_{j=1}^2 \hat{X}^j = \sum_{j=1}^2 \sum_{i=1}^2 \hat{x}_i^j$, can also be derived. Due to space limitation, we omit these equations here. From Lemma 1, we see that as a result of the Cournot game, the competition between service providers S^1 and S^2 leads each service provider to determine the "optimal" amount of service to offer–subject to the resulting market-determined service price–so as to maximize their respective profit. These "optimal" amounts of services, $\hat{X}_i^j$, are expressed as functions of r_i's, the prices that the network providers, N_1 and N_2, charge to transport the services (or rather the associated traffic). In other words, given r_1 and r_2, the service providers S^1 and S^2 can determine the optimal X_i^j's (and thus X^1 and X^2).

Stage 2: Solving the Bertrand Game. From Lemma 1, given r_1 and r_2, the total amount of services that each network provider must transport (as the result of the Cournot game played by the service providers) is as follows:
For $i = 1, 2$,

$$\hat{Y}_i = \frac{\gamma_i}{5}[1 - \beta p - 2(r_1 + r_2 + s^1 + s^2) - r_i] - \frac{\gamma_i^1 s^1}{5} - \frac{\gamma_i^2 s^2}{5}, \quad (8)$$

where $\gamma_i := \gamma_i^1 + \gamma_i^2$. Plugging eq. (8) into the network provider profit function eq. (4), again yields a quadratic function in r_i for Π^{N_i}. The Nash equilibrium of the Bertrand game is achieved when $\partial \Pi^{N_i} / \partial r_i = 0$, $i = 1, 2$. The solutions are given in the following lemma.

Lemma 2 (Solution of Bertrand Game): The competition of the two network providers modeled as a Bertrand game leads to the following equilibrium state:

$$\begin{aligned}\tilde{r}_1 &= \tfrac{1}{8}(1-\beta p) - \tfrac{1}{4}(s^1+s^2) + \tfrac{1}{16}(3n_1 - n_2) \\ &\quad - \tfrac{s^1}{16}\left(\tfrac{3\gamma_1^1}{\gamma_1} - \tfrac{\gamma_2^1}{\gamma_2}\right) - \tfrac{s^2}{16}\left(\tfrac{3\gamma_1^2}{\gamma_1} - \tfrac{\gamma_2^2}{\gamma_2}\right) \end{aligned} \quad (9)$$

and

$$\begin{aligned}\tilde{r}_2 &= \tfrac{1}{8}(1-\beta p) - \tfrac{1}{4}(s^1+s^2) + \tfrac{1}{16}(3n_2 - n_1) \\ &\quad - \tfrac{s^1}{16}\left(\tfrac{3\gamma_2^1}{\gamma_2} - \tfrac{\gamma_1^1}{\gamma_1}\right) - \tfrac{s^2}{16}\left(\tfrac{3\gamma_2^2}{\gamma_2} - \tfrac{\gamma_1^2}{\gamma_1}\right) \end{aligned} \quad (10)$$

■

Combining the two lemmas by substituting eqs. (9) and (10) into the equations of Lemma 1 yields the following theorem.

Theorem 1 (Solution of the Model): For a fixed network access charge p, the competitions between service providers and network providers, modeled as a Stackelberg game with two-stage Cournot-Bertrand sub-games, yield the *equilibrium state* $\langle\{\tilde{x}_i^j, i = 1, 2, j = 1, 2\}, \tilde{q}, \tilde{r}_1, \tilde{r}_2\rangle$, where $\tilde{r}_1$ and $\tilde{r}_2$ are given in eqs. (9) and (10) of Lemma 2, and

$$\begin{aligned}\tilde{x}_i^j &= \tfrac{\gamma_i^j}{5}\left[\tfrac{3}{8}(1-\beta p) - \tfrac{3}{4}(s^1+s^2) - \tfrac{7}{16}(n_1+n_2)\right. \\ &\quad \left. + \tfrac{7s^1}{16}\left(\tfrac{\gamma_1^1}{\gamma_1} + \tfrac{\gamma_2^1}{\gamma_2}\right) + \tfrac{7s^2}{16}\left(\tfrac{\gamma_1^2}{\gamma_1} + \tfrac{\gamma_2^2}{\gamma_2}\right)\right] + \tfrac{\gamma_i^j}{5}(w_{-i} - s^j), \end{aligned} \quad (11)$$

where $w_{-i} := \frac{1}{4}[n_{-i} - \frac{1}{\gamma_{-i}}(s^1\gamma_{-i}^1 + s^2\gamma_{-i}^2)]$, and for a given player i, the notation $-i$ indicates the other player (network provider). Furthermore,

$$\begin{aligned}\tilde{q} &= \tfrac{3}{10}(1-\beta p) + \tfrac{1}{5}(s^1+s^2) + \tfrac{1}{20}(n_1+n_2) \\ &\quad - \tfrac{s^1}{20}\left(\tfrac{\gamma_1^1}{\gamma_1} + \tfrac{\gamma_2^1}{\gamma_2}\right) - \tfrac{s^2}{20}\left(\tfrac{\gamma_1^2}{\gamma_1} + \tfrac{\gamma_2^2}{\gamma_2}\right). \end{aligned} \quad (12)$$

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From eq. (11), we can obtain the total user demand (or equivalently, the total service supply) in equilibrium, $\tilde{X} = \sum_{i=1}^2 \sum_{j=1}^2 \tilde{x}_i^j$, which is given below as a function of p.

$$\begin{aligned}\tilde{X} &= \tfrac{\gamma}{5}\left[\tfrac{3}{8}(1-\beta p) - \tfrac{3}{4}(s^1+s^2) - \tfrac{7}{16}(n_1+n_2)\right. \\ &\quad \left. + \textstyle\sum_{j=1}^2 \tfrac{7s^j}{16}\left(\tfrac{\gamma_1^j}{\gamma_1} + \tfrac{\gamma_2^j}{\gamma_2}\right)\right] + \textstyle\sum_{i=1}^2\sum_{j=1}^2\left[\tfrac{\gamma_i^j}{5}(w_{-i} - s^j)\right], \end{aligned} \quad (13)$$

where $\gamma = \gamma_1 + \gamma_2 = \sum_{i=1}^2 \sum_{j=1}^2 \gamma_i^j$.

Ignoring the last term in eqs. (11) and (13), we see that as a result of the Cournot/Bertrand competitions between service/network providers, the amount of service, $\tilde{x}_i^j$'s, offered at equilibrium by S^j to user U_i of network provider N_i is (roughly) in the proportion $\frac{\gamma_i^j}{\gamma}$ of the total market. Thus the "market share" of service provider S^j in network N_i is (roughly) γ_i^j, and its total "market share" in both networks is (roughly) $\gamma^j := \gamma_1^j + \gamma_2^j$. Similarly, network provider N_i carries (roughly) a fraction γ_i/γ of the total traffic generated by the service market, with a market share of (roughly) γ_i.

Effect of Network Access Price p and Its Optimal Choice. Theorem 1 shows that at equilibrium the levels of services, $\tilde{x}_i^j$'s, produced by S^j, $j = 1, 2$ (and consumed by user U_i of network provider N_i, $i = 1, 2$), and therefore the total market supply, $\tilde{X}$, as well as the equilibrium service price, $\tilde{q}$, and network transport prices, $\tilde{r}_i$'s, are all linear functions of the network access charge, p. We assume that the price p is determined by a policy maker with the objective of maximizing *social welfare*. Following standard economic principles, we define the *social welfare function*, denoted by Π^{SW}, as follows:

$$\Pi^{SW}(p) := \Pi^U(p) + \sum_{i=1}^2 \Pi^{N_i}(p) + \sum_{j=1}^2 \Pi^{S^j}(p), \quad (14)$$

where $\Pi^U(p)$ is the *consumer (i.e., user) surplus* when the network access price is p. As the total market demand as a function of p is given by eq. (13), the consumer surplus is therefore given by

$$\Pi^U := \int_p^{p_{max}} \tilde{X}(p) = \frac{1}{2}(p_{max} - p) \cdot \tilde{X}(p). \qquad (15)$$

where the parameter $p_{max}(\leq 1/\beta)$ is the maximal possible network access price such that the total market demand $\tilde{X}(p) = 0$. From eq. (13), it is clear that Π^U is a quadratic function in p of the form $a_1p^2 + b_1p + c_1$, where $a_1 = \frac{3}{2\times 40}\beta\gamma > 0$. Further, it is not hard to see that Π^U is a strictly decreasing function in p for $0 \leq p \leq p_{max}$. In other words, the maximum of Π^U is attained when $p = 0$.

We can re-combine the terms in $\sum_{i=1}^2 \Pi^{N_i}(p) + \sum_{j=1}^2 \Pi^{S^j}(p)$ and re-write the sum of total profits of both service and network providers as follows:

$$\sum_{i=1}^2 \Pi^{N_i}(p) + \sum_{j=1}^2 \Pi^{S^j}(p) = \sum_{j=1}^2 \sum_{i=1}^2 (\tilde{q} - s^j - n_i)\tilde{x}_i^j + 2p. \quad (16)$$

From eqs. (12) and (11), we see that $\sum_{i=1}^2 \Pi^{N_i}(p) + \sum_{j=1}^2 \Pi^{S^j}(p)$ is also a quadratic function of the form $a_2p^2 + b_2p + c_2$, where $a_2 = \frac{3^2}{10\times 40}\beta^2\gamma > 0$. Hence $\sum_{i=1}^2 \Pi^{N_i}(p) + \sum_{j=1}^2 \Pi^{S^j}(p)$ is a convex function. In particular, its maximum is attained when either $p = 0$ or $p = p_{max}$.

Putting everything together, we see that the social welfare function, $\Pi^{SW}(p)$ is a *convex* quadratic function of the form $ap^2 + bp + c$, where $a = a_1 + a_2 > 0$. Hence the social welfare is maximized when either $p = 0$ or $p = p_{max}$. At $p = p_{max}$, $\Pi^U = 0$ and $\sum_{j=1}^2 \Pi^{S^j} = 0$, as $q = 0$ and $x_i^j = 0$, $i, j = 1, 2$, whereas $\sum_{i=1}^2 \Pi^{N_i} = 2p_{max} \leq 2/\beta$. In other words, when $p = p_{max}$, there is no user demand for services. The network providers only profit from network access charges paid by the users. This scenario obviously only makes sense if there is intrinsic value in the network besides the services offered by the service providers. Otherwise, users would have no incentive to use the network and pay a network access charge; the network provider would then lose users and thus receive zero profit in the end. On the other hand, when $p = 0$, the market demand $\tilde{X}(p)$ given in eq.(13) is maximized, and therefore the consumer surplus is also maximized. Further, both the profits of the service providers and the network providers are non-zero. It can be shown that when $\beta\gamma = \beta(\sum_{j=1}^2 \sum_{i=1}^2 \gamma_i^j)$ is sufficiently large, then $p = 0$ maximizes the total social welfare Π^{SW} (instead of $p = p_{max}$). This result can be explained as follows: from eq. (13), $-dX/dp = \frac{3}{40}\beta\gamma$; hence when $\beta\gamma$ is large, a slight drop in p induces a significant increase in user demands. The resulting increase in user demand would then generate enough profit for the network providers (through transporting more services between the service providers and users) to compensate for the slight decrease in network access charge. The overall profit of the network providers would, therefore, increase. Likewise, the large increase in user demand would also generate a net profit for the service providers (as $-dq/dp > -dr/dp$). Hence the overall social welfare increases. In such a scenario, the market regulator (or policy maker) would therefore reduce network access charges as much as possible to spur user demand.

IV. Incentives for Innovations

In this section we consider the following questions: i) under what conditions would either a service provider or network provider have incentives for *service innovation* or *network upgrade*; and ii) how does the interaction and competition among them affect such incentives? For clarity of exposition, we refer to innovation by a service provider as service innovation, while that by a network provider as network upgrade, so as to separate the effects of innovation by service and network providers.

We define *service innovation* as an investment by a service provider which will result in an increase in the quality of the supply or a decrease in the market (service) price, thereby expanding its share of the overall user demand for services. Likewise, we define *network upgrade* as an investment by a network provider which will result in an increase in the quality and/or capability of the network infrastructure or a decrease in the market (transport) price, thereby expanding its share of the market demand for service/traffic transport. In both cases, any innovation or upgrade will increase the overall market demand.

A. Incentives for Service Innovation

Instead of directly considering the investment made by a service provider for service innovation, we indirectly model it by accounting for the *effect* of such service innovation in our model. Consider service provider S^j. Innovation by S^j would lead to an increase in the overall market demand for services, and therefore result in a decrease in the service price q. From the inverse demand function of eq. (1), $\partial x_i^j/(-\partial q) = \gamma_i^j$. Recall that $X^j = x_1^j + x_2^j$ is the total service offering (supply) by service provider S^j. Hence $\partial X^j/(-\partial q) = \partial x_1^j/(-\partial q) + \partial x_2^j/(-\partial q) = \gamma_1^j + \gamma_2^j = \gamma^j$. Namely, *before service innovation by* S^j, a ∂q drop in q would cause a γ^j proportional increase in the supply by service provider S^j. We assume that innovation by S^j would increase its competitiveness, and thus its corresponding "market share" of the overall user demand. Hence we model the *effect* of innovation by S^j by a positive increase in γ^j, and assume that service innovation by S^j has the same effect on its service offerings in both network providers. Namely, after the service innovation by S^j, we have for some $\eta^j > 0$ $\gamma_i^j := \gamma_i^j + \eta^j$, $i = 1, 2$, and $\gamma^j := \gamma^j + 2\eta^j$.

On the other hand, service innovation by S^j may also affect its marginal cost s^j, namely from s^j to $s^j + \sigma^j$. The effect of service innovation on the marginal cost can be either positive (i.e., $\sigma^j > 0$), or negative (i.e., $\sigma^j \leq 0$), or neutral. In other words, investing in service innovation can increase marginal costs when supplying one unit of service, or the accompanying increase in the market demand can in itself result in a decrease in marginal costs, for instance, due to *economies of scale*. Alternatively, it could also have no effect on marginal costs. All three options are accounted for in the model, which captures the effect of service innovation by S^j using two parameters (η^j, σ^j), where $\eta^j > 0$.

Let $\Pi^{j,before}$ denote the overall profit of service provider S^j before service innovation, and $\Pi^{j,after}$ its profit after service innovation. In order for S^j to have an incentive to innovate, its profit after service innovation should be larger than before service innovation. In other words, we must have

$$I^j(\eta^j, \sigma^j) := \Pi^{j,after} - \Pi^{j,before} > 0. \tag{17}$$

Plugging $\gamma^j := \gamma^j + \eta^j$ and $s^j := s^j + \sigma^j$ into the model of Section III-A, we can compute $\Pi^{j,after}$ and express the *incentive function* I^j (of S^j) in terms of (η^j, σ^j). Unfortunately, the resulting formula for $I(\eta^j, \sigma^j)$ is complex; hence we do not include it here. Instead we make the following general observations. First, we note from eqs. (1) and (13) that for a given network access charge, $1 - \beta p$ can be viewed as an "intrinsic" factor that determines the potential market demand for services independently of the parameters γ_j^i's and the marginal costs s^j's and n_i's of service/network providers. Provided that $s^1 + s^2 + n_1 + n_2 << 1 - \beta p$, it can be shown[5] that $I(\eta^j, \sigma^j)$ is an increasing function in η^j and a decreasing function in σ^j. Hence if $\sigma^j < 0$, i.e., service innovation by S^j decreases its marginal cost, then for any $\eta^j > 0$, $I(\eta^j, \sigma^j) > 0$. Therefore when service innovation leads to a decrease in its marginal cost, it always pays for S^j to innovate. On the other hand, when $\sigma^j > 0$, service innovation by S^j would lead to an increase in its marginal cost. In this case, we can show that provided that $s^1 + s^2 + n_1 + n_2 << 1 - \beta p$, there exists a constant $c^j > 0$ such that if $\sigma^j < c^j \frac{\eta^j}{\gamma^j + \eta^j}(1 - \beta p)$, then $I(\eta^j, \sigma^j) > 0$. Hence in this case, it is only when the increase in its marginal cost is upper bounded by an appropriate gain in its market share that service innovation by S^j would then pay off. Otherwise, there is no incentive for S^j to innovate.

B. Incentives for Network Upgrade

As in the case of innovation by service providers, we model the *effect* of network upgrade by network provider N_i using two parameters, (η_i, μ_i), where $\eta_i > 0$ reflects an increase in N_i's market share as a result of its upgrade, i.e, $\gamma_i^1 := \gamma_i^1 + \eta_i$, $\gamma_i^2 := \gamma_i^2 + \eta_i$ and thus $\gamma_i := \gamma_i + 2\eta_i$; whereas μ_i reflects the resulting change in its marginal cost, $n_i := n_i + \mu_i$, which can again be either positive, negative or neutral. The *incentive function* I_i for N_i is therefore given by

$$I_i(\eta_i, \mu_i) := \Pi_{i,after} - \Pi_{i,before}, \tag{18}$$

where $\Pi_{i,before}$ and $\Pi_{i,after}$ denote the profit of N_i before and after network upgrade, respectively. Hence N_i has incentive to upgrade its network if and only if $I_i(\eta_i, \mu_i) > 0$. Again provided that $s^1 + s^2 + n_1 + n_2 << 1 - \beta p$, we can show that $I_i(\eta_i, \mu_i)$ is an increasing function in η_i and a decreasing function in μ_i. Hence when network upgrade leads to a decrease in its marginal cost, namely, $\mu_i < 0$, it always pays for N_i to upgrade its network. On the other hand, when $\mu_i > 0$, we can show that provided that $s^1 + s^2 + n_1 + n_2 << 1 - \beta p$, there exists a constant $c_i > 0$ such that if $\mu_i < c_i \frac{\eta_i}{\gamma_i + \eta_i}(1 - \beta p)$, then $I(\eta_i, \mu_i) > 0$. Hence in the case of $\mu_i > 0$, it is only when the increase in its marginal cost is upper bounded by an appropriate gain in its market share that network upgrade by N_i would pay off. Otherwise, as with service providers, there is no incentive for N_i to upgrade.

[5]This can be informally argued as follows: through algebraic manipulations, we can derive $\partial \Pi_i^{S^j} / \partial \gamma_i^j \approx [C_1(1 - \beta p) - C_2(s^1 + s^2) - C_3(n_1 + n_2)](1 - \beta p) + o(\gamma_1^1, \gamma_2^1, \gamma_1^2, \gamma_2^2, s^1, s^2, n_1, n_2)$, where $C_1 > 0, C_2 > 0$, $C_2 > 0$. Hence when $s^1 + s^2 + n_1 + n_2 << 1 - \beta p$, $\partial \Pi_i^{S^j} / \partial \gamma_i^j > 0$. Similarly, $\partial \Pi_i^{S^j} / \partial s^j \approx -C_1'(1 - \beta p) - C_2'(s^1 + s^2) - C_3'(n_1 + n_2) + o(\gamma_1^1, \gamma_2^1, \gamma_1^2, \gamma_2^2, s^1, s^2, n_1, n_2)$, where $C_1' > 0, C_2'$ and $C_3' > 0$. Hence $\partial \Pi_i^{S^j} / \partial s^j < 0$.

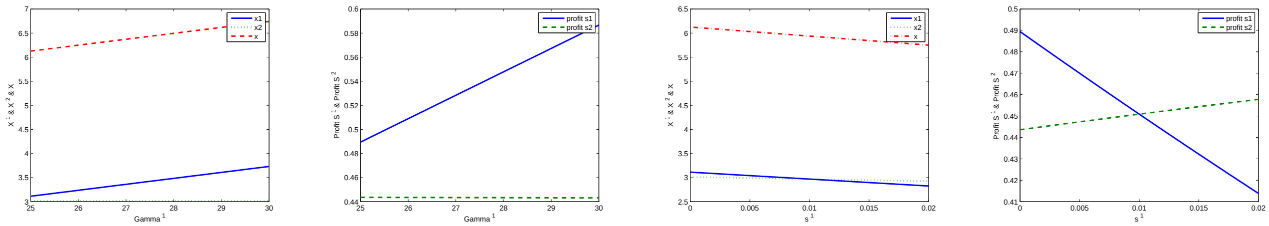
V. Numerical Analysis

In this section, we provide numerical examples to illustrate the insight obtained from the model's solution. In particular, we examine the effect of the model parameters γ_i^j's ("market shares"), s^j's and n_i's (marginal costs), and p (user network access charge) on $\tilde{x}_i^j$'s (the equilibrium quantities or amounts of services produced), profits of service/network providers, user surplus and social welfare. We also illustrate how increases in γ_i^j's and the accompanying change in the marginal costs s^j's and n_i's affect the incentives for service innovation and network upgrade. Throughout the section, unless otherwise stated, default values for the parameters are $\gamma_i^j = 25$, $i, j = 1, 2$, $s^j = 1/100$, $j = 1, 2$ $n_i = 1/50$, $i = 1, 2$ and $p = \frac{1}{2}$. When varying one or more of these parameters, all others are set to these default values.

Effect of γ_i^j's on Service Production and Profits. We use service provider S^1 as an example to examine the effect of γ_i^j's on the equilibrium values of individual service productions $\tilde{x}_i^j$'s, and overall service production $\tilde{X}$. We set $\gamma_1^1 = \gamma_2^1 = \frac{1}{2}\gamma^1$ and vary γ^1 from 25 to 30. Fig. 3(a) shows $\tilde{X}^1 (= \tilde{x}_1^1 + \tilde{x}_2^1)$ of S^1 vs. $\tilde{X}^2$ of S^2 as well as $\tilde{X} = \tilde{X}^1 + \tilde{X}^2$, as γ^1 varies. Fig. 3(b) shows the resulting profits of service providers S^1 and S^2. We see that as γ^1 increases from 25 to 30, both the service production X^1 of service provider S^1 and the overall service production $\tilde{X}$ increase (linearly) with γ^1. In contrast, γ^1 has no visible effect on the service production of S^2. Hence γ^j indeed determines a service provider's proportional share of the overall service production, and reflects its ability to capture "market share" at equilibrium.

In economic terms, increasing γ_j^i's corresponds to a demand curve shift (cf. eq. (1)), and therefore increasing user demands leads to an increased service supply in market equilibrium. Increasing γ^1 (thus γ_1^1 and γ_2^1) while keeping γ^2 the same reflects the ability of S^1 to expand its service production to meet the increased demand. This expansion of service production by S^1 does not affect that of S^2, while increasing the overall service production in the market. Consequently, the profit of S^1 increases while the profit of S^2 stays the same, as shown in 3(b). Similar results are obtained when we vary γ_1 of network provider N_1. Due to space limitation, we do not present them here.

Effect of s^j's on Service Production and Profits. We again use service provider S^1 as an example, and examine the effect of its marginal cost s^1 on the equilibrium service production $\tilde{X}^j$'s of both service providers and the overall equilibrium total service production $\tilde{X}$, as well as their respective profit.



(a) Effect of γ^1 on service production

(b) Effect of γ^1 on profits of service providers

(c) Effect of s^1 on service production

(d) Effect of s^1 on profits of service providers

Fig. 3. Effects of γ^1 and s^1 respectively on service production and profits of service providers.

We vary s^1 from 0 to 0.02, while setting all other parameters to the default values. Fig. 3(c) shows $\tilde{X}^1$, of S^1, $\tilde{X}^2$ of S^2, and $\tilde{X} = \tilde{X}^1 + \tilde{X}^2$, as γ^1 varies. Fig. 3(d) shows the resulting profits of service providers S^1 and S^2. We see that as s^1 is increased from 0 to 0.02, the service production X^1 of service provider S^1 decreases, while the service production (X^2) of S^2 is largely unaffected. The resulting overall service production in the market also decreases. On the other hand, while the profit of S^1 decreases due to its increased marginal cost (which is expected), the profit of S^2 in fact increases as the result of increased marginal cost of its competitor S^1. In economics, this phenomenon is in fact not surprising: in a competitive market (e.g., a Cournot or Bertrand game), when all other factors (especially $\gamma^1 = \gamma^2$) are the same, the marginal cost is what determines the competitiveness of each firm, affecting both its service production and profit.

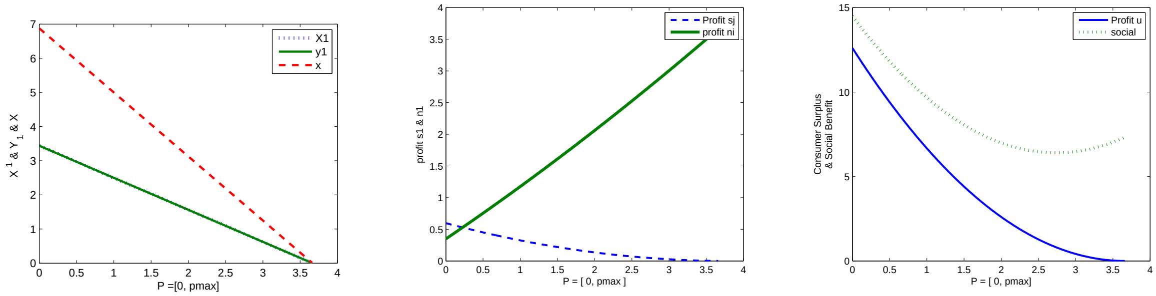
Effect of User Network Access Charge p. As seen from eq. (1), the user network access charge p affects the overall user demand for services, and consequently service production. The effect of varying p on service production is shown in Fig. 4(a), where all other parameters are set to the default values, and p varies from 0 to p_{max} (as determined by the other parameters). As $X^1 = X^2 = Y_1 = Y_2$, only the curves for X^1 and Y_1 are plotted, and both curvese coincide. As expected, increasing p suppresses service production by service providers, and thus the volume of services transported by network providers. As p varies, the corresponding profits for service and network providers are shown in Fig. 4(b). The profit of service providers obviously decreases with p. On the other hand, while increasing p reduces the revenue that network providers obtain from service providers this loss in revenue is offset by the increased revenue extracted from users. Hence, the overall profit of network providers increases[6]. Not surprisingly, increasing p also reduces consumer surplus, as shown in Fig. 4(c). In the same figure, we also plot the overall social welfare function, Π^{SW}. As argued in Section III-B, Π^{SW}, is a convex function of p. With the parameter setting of this example, its maximum is achieved when $p = 0$, which leads to the maximum market demand for services. But with $p = 0$, the profit of network providers is at its minimum.

[6]As an aside, we remark that this result reflects the reality of today's ISP market, where on a per-byte basis, users of an ISP pay a significantly higher fee than a service/content provider pays the ISP for transporting traffic. The same also applies to wireless cellular service providers.

Incentives for Service Innovation and Network Upgrade. Lastly, we investigate the incentives for service innovation and network upgrade. We use the default parameter values to compute the profits of service/network providers before service innovation/network upgrade. We consider the incentive for service innovation by service provider S. After service innovation, the market share of S^1 is increased to $\gamma^1 := \gamma^1 + 2\eta^1$ (and $\gamma_i^1 = \gamma_i^1 + \eta^1$, $i = 1, 2$), where $\eta^1 > 0$, and its marginal cost becomes $s^1 := s^1 + \sigma^1$. We compute the incentive function $I(\eta^1, \sigma^1)$ as we simultaneously vary η^1 in the range [0,5] and σ^1 in the range[-0.01,0.01]. The resulting incentive function $I(\eta^1, \sigma^1)$ is shown as a surface plot in Fig. 5. As argued in Section IV, we see that $I(\eta^1, \sigma^1)$ is indeed an increasing function in η^1, and a decreasing function in σ. In particular, when $\gamma^1 > 0$ and $\sigma^1 < 0$ (namely, as a result of service innovation, not only S^1's market share expands, but its marginal cost decreases), then $I(\eta^1, \sigma^1 > 0$. Hence, in such cases there are always incentives for a service provider to innovate. On the other hand, when $\sigma^1 > 0$ (i.e., S^1's marginal cost increases as a result of innovation), we find that $I(\eta^1, \sigma^1) > 0$ only when η^1 is sufficiently large. In other words, only when the increased revenue due to its expanded market share offsets the increased marginal cost is there an incentive for a service provider to innovate. Similar results are obtained for network upgrade, as shown in Fig. 6, and the same conclusion applies to network upgrade.

VI. Conclusions

In this paper we have developed a simple economic model to study the interaction and competition among service providers, network providers and users. The novelty of the model lies in combining Cournot and Bertrand competition to capture the co-dependent, interacting and competitive relationships among service providers and network providers. Using this model, we explored how these relationships can affect incentives for innovation on the part of network providers and service providers. Our work represents a modest first attempt in characterizing and modeling the intricate and complex interactions that exist between various actors in a "service-oriented" Internet. We plan to further expand this model



(a) Effect of p on individual and overall service production X^j and X, and service transport Y_i by network providers

(b) Effect of p on profits of service vs. network providers

(c) Effect of p on consumer surplus and overall social welfare

Fig. 4. Effects of p on service production, profits of service/network providers, consumer surplus and overall social welfare.

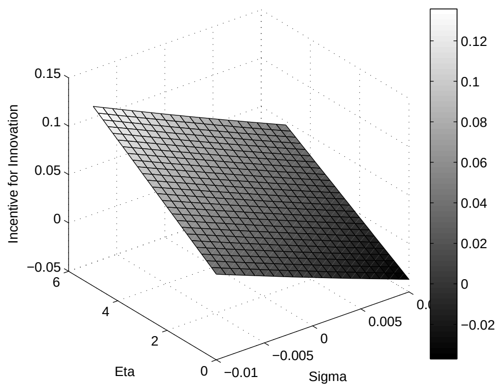


Fig. 5. Effect of σ and η on incentives for service innovation.

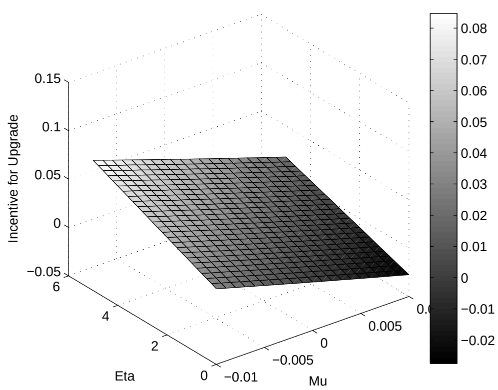


Fig. 6. Effect of μ and η on incentives for network upgrade.

to include more realistic inter-connection structures among network providers and service providers, and thereby more faithfully capture the complex interactive and competitive relationships among them. We would also like to incorporate competition among network providers for users, and study the dynamics of evolving relationships among these actors. Conducting empirical studies of these relations based on real data is also a research area of great interest to us.

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