(IP) Non-Linear Preconditioning Explained

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Linear preconditioning is an established research area, as it is testified by the 20th anniversary of this conference. Non-linear preconditioning is more recent and much less understood. A standard approach to obtain a good linear preconditioner is to start with a good stationary iterative method, like domain decomposition or multigrid, and then to use the associated operator as a preconditioner for a Krylov method, or in other words to use the Krylov method as an accelerator. Since the Krylov method finds in general a better polynomial than the stationary iterative method alone, it is worthwhile using Krylov acceleration in the linear case. In the non-linear case, one can proceed similarly: to obtain a good non-linear preconditioner, one starts with an effective fixed point iteration for solving the non-linear problem, like a non-linear domain decomposition method or the full approximation scheme in multigrid, and then instead of just running the fixed point iteration, one uses Newton's method to solve the preconditioned system at the fixed point. My first goal is to explain in detail how to obtain such non-linear preconditioners for Newton's method. My second goal is to show that while there are many cases where using Newton's method is beneficial to accelerate convergence, there are also cases where using Newton's method is detrimental, in great contrast to the linear case and Krylov methods. Newton's method can destroy convergence, and I will show a domain decomposition example where using Newton's method to accelerate the non-linear domain decomposition method leads to period doubling and chaos, while the domain decomposition method alone converges very well.

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