

# HOSLIM: Higher-Order Sparse Linear Method for Top- $N$ Recommender Systems

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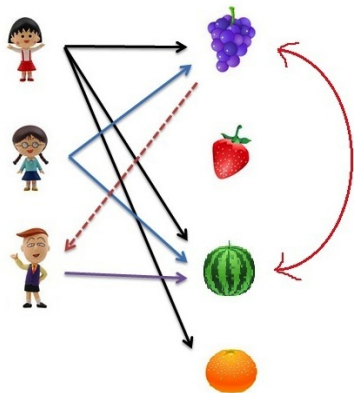
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# Notations

Def	Description
$u$	user
$i$	item
$n$	total number of users
$m$	total number of items
$R$	user-item purchase binary matrix, $(n \times m)$
$H_u$	set of items that the user has purchased

# Item-based Collaborative Filtering



- ▶ The user is recommended the items that have been copurchased the most in the dataset with the items he has purchased.
- ▶ The recommendation score of user  $u$  for item  $i$  is:

$$r_{ui} = \sum_{k \in H_u} \text{association score}(i, k)$$

- ▶ The association score between items  $i$  and  $k$  is either computed as the cosine( $r_{*i}, r_{*k}$ ) or as  $P(i|k)$ .
- ▶ Each item that the user has purchased independently contributes to  $r_{ui}$ .

# SLIM: Sparse Linear Method for Top- $N$ Recommender Systems

- ▶ SLIM learns a sparse aggregation coefficient matrix  $S(m \times m)$ , by solving an  $l_1$  and  $l_2$  regularized optimization problem.

$$\tilde{R} = RS$$

- ▶ SLIM casts the estimation of  $r_{ui}$  as a regression problem. This allows to capture some dependencies between the items.
- ▶ The recommendation score  $r_{ui}$  is computed as a sparse aggregation of items purchased by the user  $u$ :

$$r_{ui} = \sum_{k \in H_u} r_{uk} s_{ki}$$

- ▶ The optimization problem:

$$\begin{aligned} & \underset{s_j}{\text{minimize}} \frac{1}{2} \|\mathbf{r}_i - R\mathbf{s}_j\|_2^2 + \frac{\beta}{2} \|\mathbf{s}_j\|_2^2 + \lambda \|\mathbf{s}_j\|_1 \\ & \text{subject to } \mathbf{s}_j \geq 0 \text{ and } s_{jj} = 0 \end{aligned}$$

- ▶ SLIM outperforms other top- $N$  recommendation methods, in a wide variety of datasets.

## Limitation of the existing top- $N$ methods

- ▶ Both the old and the new top- $N$  recommendation methods capture only pairwise relations between the items.
- ▶ They do not capture higher-order relations.
- ▶ In some cases, purchasing a subset of the items significantly increases the likelihood of purchasing the rest.
- ▶ Ignoring this type of relations, when present, can lead to suboptimal recommendations!
- ▶ HOKNN was the 1st method that incorporated combinations of items (i.e. itemsets). The recommendation score is computed as:

$$r_{ui} = \sum_{k, m \in H_u} P(i|k, m)$$

However, in most datasets this method did not lead to significant improvements.



# Motivation

- ▶ SLIM improves upon  $k$ -NN.
- ▶ Could Higher-Order SLIM improve upon Higher-Order  $k$ -NN and SLIM?

Table : Extra Notations

Def	Description
$j$	itemset
$\sigma$	minimum support threshold
$p$	total number of itemsets
$l$	set of itemsets
$R'$	user-itemset purchase matrix, $(n \times p)$

# HOSLIM: Higher-Order Sparse Linear Method for Top- $N$ Recommendation

- ▶ The model:  $\tilde{R} = RS + R'S'$ 
  - ▶ Recommendation score: a sparse aggregation of both the **items** purchased by the user and the **itemsets** that it supports.

$$\tilde{r}_{ui} = \mathbf{r}_u^T \mathbf{s}_i + \mathbf{r}'_u^T \mathbf{s}'_i$$

- ▶  $S$ : the aggregation coefficient matrix corresponding to items. ( $m \times m$ )
- ▶  $S'$ : the aggregation coefficient matrix corresponding to itemsets. ( $p \times m$ )



## HOSLIM: The optimization problem

$$\underset{s_i, s'_i}{\text{minimize}} \frac{1}{2} \|\mathbf{r}_i - R\mathbf{s}_i - R'\mathbf{s}'_i\|_2^2 + \frac{\beta}{2} (\|\mathbf{s}_i\|_2^2 + \|\mathbf{s}'_i\|_2^2) + \lambda (\|\mathbf{s}_i\|_1 + \|\mathbf{s}'_i\|_1)$$

subject to  $\mathbf{s}_i \geq 0$   
 $\mathbf{s}'_i \geq 0$   
 $s_{ji} = 0$ , and  
 $s'_{ji} = 0$  where  $\{i \in \mathcal{I}_j\}$ .

- ▶ The optimization problem is solved using the BCLS library.

# Datasets

Name	#Users	#Items	#Non-zeros	Density
groceries	63,035	15,846	1,997,686	0.2%
synthetic	5000	1000	68,597	1.37%
delicious	2,989	2,000	243,441	4.07%
ml	943	1,681	99,057	6.24%
retail	85146	16470	820,414	0.06%
bms-pos	435,319	1,657	2,851,423	0.39%
bms1	26,667	496	90,037	0.68%
ctlg3	56,593	39,079	394,654	0.017%

# Experimental Evaluation

- ▶ 10×Leave-one-out Cross Validation
- ▶ Extensive search over the parameter space of the various methods was performed, in order to find the set of parameters that lead to the best results.
  - ▶  $k$ -nn: number of neighbors
  - ▶ HOKNN: number of neighbors, support threshold ( $\sigma$ )
  - ▶ SLIM:  $l_2$  parameter ( $\beta$ ),  $l_1$  parameter ( $\lambda$ )
  - ▶ HOSLIM:  $l_2$  parameter ( $\beta$ ),  $l_1$  parameter ( $\lambda$ ), support threshold ( $\sigma$ )
- ▶ Evaluation Metric: Hit Rate (HR)

$$HR = \frac{\#hits}{\#users}$$

## Results Outline

The experimental analysis focuses on answering the following questions:

- ▶ Do higher-order relations exist in real-world datasets?
- ▶ Incorporating them in modern top- $N$  methods could improve the recommendation quality?

Table : Itemset Statistics

Name	#Itemsets	$\frac{\text{itemset nnz}}{\text{original nnz}}$
groceries	551,333	7.97
synthetic	8,614	2.55
delicious	47,362	16.38
ml	895	1.94
retail	42,925	1.85
bms-pos	73,382	5.99
bms1	19,055	6.39
ctlg3	5,245	0.51

## Verifying the existence of Higher-Order Relations

We measured how prevalent are the itemsets with strong association between the items that comprise it (beyond pairwise associations).

- ▶ We found all frequent itemsets of size 3 with  $\sigma$  equal to 10. For each of the itemsets we computed the quality metric:

$$dependency_{max} = \frac{P(ABC)}{\max(P(AB)P(C), P(AC)P(B), P(BC)P(A))}$$

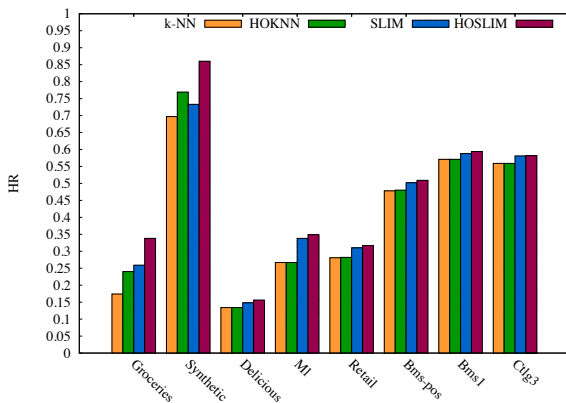
We considered only the itemsets with dependency higher than a specified threshold (2,5).

- ▶ We found out that there are datasets like groceries with high coverage of such itemsets (a large percentage of users having at least one itemset and a large percentage of non-zeros covered by at least one itemset.)

However, there are datasets like retail with low coverage.

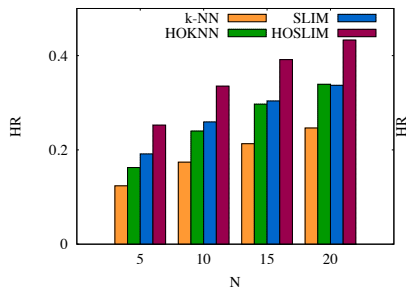
- ▶ The correlation coefficient of the improvement in performance in HOSLIM beyond SLIM with the product of affected users coverage and number of non-zeros coverage = 0.88

# Performance Comparison

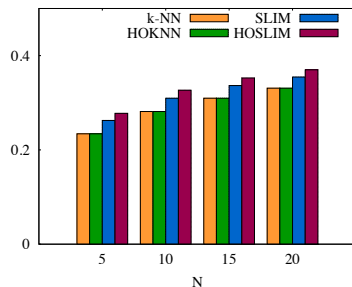


- ▶ The incorporation of higher-order information can improve the recommendation quality.
- ▶ The improvement percentage depends on the existence of higher-order relations in the dataset.

# Performance for different values of $N$



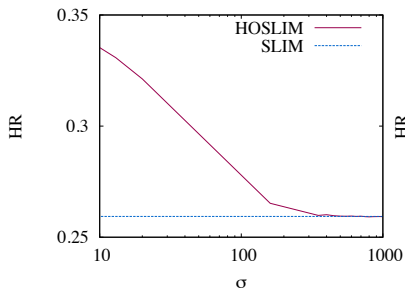
Groceries dataset



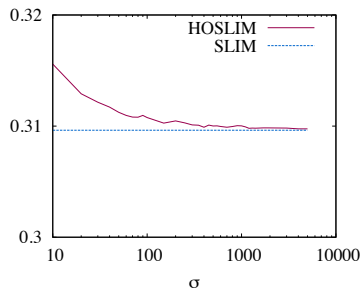
Retail dataset

- ▶  $N$  is small, as a user will not see an item at the 100th position.

## Sensitivity of the support of the itemsets



Groceries dataset



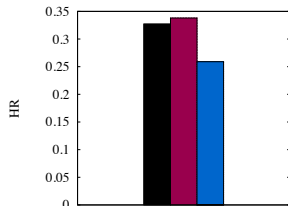
Retail dataset

- ▶ A low support means that HOSLIM benefits more from the itemsets, thus the HR is higher.

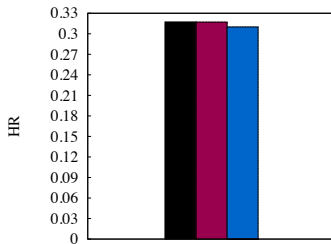


# Efficient Recommendation by controlling the number of non-zeros

constrained HOSLIM ■  
unconstrained HOSLIM ■  
SLIM ■



Groceries dataset



Retail dataset



$$nnz(S') + nnz(S_{HOSLIM}) \leq 2nnz(S_{SLIM})$$

- ▶ The cost of computing the top- $N$  recommendation list depends on the number of non-zeros in the model.

## Concluding Remarks

- ▶ Higher-order information exists in some real-world datasets.
- ▶ Its incorporation in modern top- $N$  methods could help the recommendation quality, especially when the dataset in question contains abundant higher-order information.

Thank you!